Hydrodynamic and thermal behaviours of a nanofluid in a uniformly heated tube

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Abstract

In this study, we have numerically investigated the problem of forced convection flow and heat transfer of nanofluids, namely water-Al₂O₃ and Ethylene Glycol-Al₂O₃, flowing inside a uniformly heated tube. By assuming that the mixture behaves as a homogenous single-phase fluid, its apparent thermal properties may then be calculated as function of the corresponding properties of constituents as well as of their respective concentrations. Results have clearly shown the beneficial effects due to the inclusion of nanoparticles. In general, it has been observed that the heat transfer rate considerably increases with an increase of the particle concentration, for both laminar and turbulent flow regimes. Among the nanofluids tested, it has been found that Ethylene Glycol-Al₂O₃ offers a higher heat transfer enhancement. On the other hand, the wall shear stress has significantly increased with the presence of particles. Such adverse effect appears more pronounced for Ethylene Glycol-Al₂O₃ than for water-Al₂O₃ mixture.

Keywords: laminar flow, turbulent flow, heat transfer, nanofluid, nanoparticles, heat transfer enhancement, heat transfer augmentation, numerical simulation.

1 Introduction

The term ‘nanofluid’ refers to a new and special class of fluids constituted from a mixing of a continuous phase, a liquid in most cases, and a dispersed phase composed of ‘nanoparticles’ i.e. extremely fine metallic particles of size usually less than 100nm. The thermal properties of such nanofluid appear considerably augmented with respect to those of the base fluid. In fact, few experimental data have clearly shown that the apparent thermal conductivity of a nanofluid may increase as much as 20% comparing to that of the base fluid, for a relatively low
particle concentration, says 1-5% in volume [1-4]. Some recent experimental studies have eloquently demonstrated the superior heat transfer performance of nanofluids in confined tube flow [5,6]. Results from recent authors’ works, considering water-Al₂O₃ nanofluid, have also confirmed this interesting trend [7]. Such heat transfer enhancement appears to depend on several factors such as dimension, form and of course, particle loading and thermal properties of particles [8]. In short, the nanofluids can then be a very interesting alternative for high heat transfer rate applications, in particular in electronic cooling and micro-scale heat transfer. Unfortunately, in spite of their great potentials, they are still in their early development and more research works will, indeed, be needed in order to assessing their beneficial effects in various thermal applications. In this study, we have numerically investigated the influence due to the presence of nanoparticles on the flow and heat transfer behaviours in a uniformly heated tube.

![Figure 1: Geometric configuration of the problem under study.](image)

## 2 Mathematical formulation and numerical method

### 2.1 Governing equations

The problem studied consists of a forced flow of a nanofluid, namely Ethylene Glycol-Al₂O₃, inside a straight cylindrical tube of diameter 0.01m and length 1m. The latter is submitted to a constant and uniform wall heat flux. The flow is assumed steady, laminar and symmetrical with respect to a vertical plane passing through the tube main axis, Fig. 1. Due to the extreme size of particles, one may expect that the liquid-particle mixture may easily be fluidized and hence, it appears reasonable to consider that it would behave as a homogenous single-phase fluid [8]. Furthermore, by assuming thermal equilibrium and negligible slip between the phases, the apparent thermal properties of nanofluids can then be estimated using classical relationships derived for two-component mixture.

Under the above assumptions, the general governing equations are:

\[
\nabla \cdot (\rho \mathbf{V}) = 0 \quad (1)
\]

\[
\nabla \cdot (\rho \mathbf{V} \cdot \mathbf{V}_i) = -\frac{\partial p}{\partial X_i} + \nabla \cdot (\mu \nabla \mathbf{V}_i) + S_i, \quad i=1,2,3 \quad (2)
\]

\[
\nabla \cdot (\rho \mathbf{V} C_p T) = \nabla \cdot (k \nabla T) \quad (3)
\]
where \( V = (V_R, V_\theta, V_Z) \) is the fluid velocity vector; \( X_i \) refers to the spatial directions \( (X_i = R, \theta \) and \( Z) \); \( S_i \) are the remaining velocity-related stress terms due to the choice of cylindrical coordinate system.

### 2.2 Thermal properties of nanofluids

The thermal properties of the nanofluids under study have been evaluated using classical formulas for two-component fluid. Some experimental data were also used for computing their dynamic viscosities (subscripts \( nf, bf, p \) and \( r \) refer respectively to nanofluid, base fluid, particle and ‘nanofluid-to-base fluid’ ratio):

\[
\rho_{nf} = (1 - \phi)\rho_{bf} + \phi\rho_p \tag{4}
\]

\[
C_{P_{nf}} = (1 - \phi)C_{P_{bf}} + \phi C_{P_p} \tag{5}
\]

\[
\mu_r = \frac{\mu_{nf}}{\mu_{bf}} = 123\phi^2 + 7.3\phi + 1 \quad \text{for water-}\gamma\text{Al}_2\text{O}_3 \tag{6}
\]

\[
\mu_r = \frac{\mu_{nf}}{\mu_{bf}} = 306\phi^2 - 0.19\phi + 1 \quad \text{for Ethylene Glycol-}\gamma\text{Al}_2\text{O}_3 \tag{7}
\]

\[
k_r = \frac{k_{nf}}{k_{bf}} = \frac{k_p + 2k_{bf} - 2\phi(k_{bf} - k_p)}{k_p + 2k_{bf} + \phi(k_{bf} - k_p)} \quad \text{for water-}\gamma\text{Al}_2\text{O}_3 \tag{8}
\]

where \( \phi \) is the particle volume concentration. Eqns (6,7) for \( \mu_r \) have been obtained from some scarce experimental data [2]. It is very interesting to note, in particular, that for the apparent dynamic viscosity and the nanofluids studied, existing formulas such as those proposed by Einstein-Brinkman [9] and Batchelor [10] seem to strongly underestimate the reality; much more data will indeed be needed in this area. For nanofluid thermal conductivity, Eqn (8) has been obtained from a well-known Hamilton and Crosser’s model [11] assuming spherical particles. Note that such model, originally derived for micrometer-size particles, appears also to be appropriate for use with nanometre-size ones [8].

One can see that the problem under consideration may be characterized by a set of five dimensionless parameters, namely the Reynolds number \( Re = V_0D\rho/\mu \), the Prandtl number \( Pr = C_p\mu/k \), the particle volume concentration \( \phi \) and the property ratios \( k_p/k_{bf} \) and \( C_{pp}/C_{pbf} \).

### 2.3 Boundary conditions and numerical procedure

The fluid enters with a uniform axial velocity \( V_0 \) and temperature \( T_0 \). The constant and uniform heat flux \( q'' \) and the non-slip condition are prescribed on the tube wall. At the tube exit, the usual fully-developed flow conditions prevail.

The system of governing equations (1-3), which are non-linear and strongly coupled each other, has been successfully solved using a finite-control-volume-based numerical method where the power-law scheme has been used throughout
for computing heat and momentum fluxes, and the well-known SIMPLE-procedure for treating the velocity-pressure coupling [12, 13]. In order to ensure the accuracy as well as the consistency of numerical results with respect to a number of grid points, several non-uniform grids have been thoroughly tested, and the 32X24X155 grid - with respectively 32, 24 and 155 nodes along R,θ (covering 0° to 180°) and Z direction, with grid points highly packed near the domain boundaries - has been adopted.

The numerical model has been satisfactorily validated by comparing results as obtained by the computer code to analytical/numerical data for (i) a case of forced convection tube flow and heat transfer, and (ii) a case of mixed convection laminar flow of water in horizontal tube [14] (complete details of the grid sensibility study and code validation may be found in [15]).

The computer model has then been used to extensively carry out numerical simulations using water-Al₂O₃ and Ethylene Glycol-Al₂O₃ mixtures. As starting conditions, velocity and temperature fields as obtained for a case without particles were used. For subsequent calculations, a converged case of a particular value of φ has been employed as initial conditions. For most of the cases performed in this study, a converged solution has usually been achieved with a residue as low as 10⁻⁸ for all of the governing equations (1-3).

3 Results and discussion

In this paper, some significant results as obtained for Ethylene Glycol-Al₂O₃ nanofluid are presented and discussed with emphasis on the effects due to the inclusion of nanoparticles. The Reynolds number was fixed to be 6.3; the wall heat flux is q''=5000 W/m² and the fluid inlet temperature is T₀=290K.

3.1 Development of temperature field

Figure 2 illustrates at first the axial development of wall temperature for various particle concentrations. It is clearly observed that with the presence of particles, the fluid temperature at the wall has considerably decreased. Thus, for example near the tube exit, T_W has decreased almost by 17K between case φ=0 and φ=10%. Such beneficial effect, which obviously indicates a better heat transfer at the tube wall, is consistently observed all along the tube length, and appears more pronounced towards the tube end. The same effect has also been noticed on the internal thermal field. Figure 3 shows, in particular, the influence of the parameter φ on the fluid radial temperature profile at the exit section. One can observe that the fluid temperature clearly decreases with an augmentation of particle concentration φ, in particular near the tube wall where highest effect is noticed, indicating here again a better heat transfer rate at the tube wall. Near the tube axis however, it is observed that fluid temperature has slightly increased by few degrees between case φ=0% and φ=10%. These results have clearly indicated the beneficial effects due to nanoparticles, effects that can be explained by the fact that with the presence of these particles, thermal properties of a resulting mixture have been considerably improved. Thus, for case φ=10% in
Figure 2: Effect of particle concentration on axial development of $T_w$.

Figure 3: Effect of particle concentration on fluid temperature profile.
particular, values of \( \rho C_p \) and \( k \) have augmented respectively by almost 16\% and 32\% with respect to the corresponding values of case \( \varphi=0\% \). A nanofluid has, indeed, far greater thermal capacity than a conventional base fluid.

### 3.2 Influence of particle loading on heat transfer and wall friction

Figure 4 clearly shows that an inclusion of nanoparticles has provided an appreciable improvement of heat transfer rate at the tube wall. Thus, for a particle volume concentration of 7.5\% for example, the ‘nanofluid-to-base fluid’ ratio of heat transfer coefficients \( h_r \) (defined as \( h_{nf}/h_{bf} \)) has in fact increased by a factor of \( \approx 1.8 \) at the tube end. The ratio \( h_r \) is always higher than 1 along the tube length; for \( \varphi=7.5\% \), \( h_r \) varies from \( \approx 1.4 \) at the inlet to 1.8 at the exit. Also, \( h_r \) steadily increases with an augmentation of \( \varphi \). One can also observe that for low particle loading, the ratio \( h_r \) varies as a linear function of the axial position \( Z \). Although a direct comparison with other data and results was not possible due to the lack of such information, it is very interesting to note that similar behaviour regarding the heat transfer enhancement due to particles has also been observed experimentally for other types of nanofluid (see in particular [5,6]).

Figure 5 shows the overall effect of particles on the average heat transfer as obtained for Ethylene Glycol-Al\(_2\)O\(_3\) as well as for water-Al\(_2\)O\(_3\) mixture (\( \bar{h}_r \) is the ‘nanofluid-to-base fluid’ ratio of averaged heat transfer coefficients, defined as \( \bar{h}_r = \frac{\bar{h}_{nf}}{\bar{h}_{bf}} \)). The beneficial effects due to the presence of nanoparticles appear obvious as \( \bar{h}_r \) considerably increases with an increase of particle loading. For \( \varphi=7.5\% \) for example, \( \bar{h}_r \) has values 1.5 and 1.4, respectively, for Ethylene Glycol-Al\(_2\)O\(_3\) and water-Al\(_2\)O\(_3\); while for \( \varphi=10\% \), this ratio has increased to 1.8 and 1.6 respectively for the same fluids. The heat transfer improvement appears clearly better in case of Ethylene Glycol-Al\(_2\)O\(_3\). We have also observed that for low particle concentration, says for \( \varphi \leq 2.5\% \), \( \bar{h}_r \) remains near unity and is practically identical for the tested nanofluids.

In this study, we have also attempted to investigate the effect of particles on heat transfer behaviour in turbulent flow using the well-known \( k-\varepsilon \) turbulent model [16]. Some preliminary results as obtained for water-Al\(_2\)O\(_3\) nanofluid are presented and discussed. From Figure 6, one can see the same interesting trend regarding the heat transfer enhancement by using particles. Thus, it is observed that the ratio \( \bar{h}_r \) also increases appreciably with particle concentration. For \( \varphi=7.5\% \) for example, \( \bar{h}_r \) has values 1.4 and 1.52, respectively, for \( Re=10\,000 \) and 50\,000. For low value of \( \varphi \), says for \( \varphi \leq 2.5\% \), \( \bar{h}_r \) appears identical regardless \( Re \). It is very interesting to mention that a similar influence of the Reynolds number in turbulent flow of nanofluids has also been experimentally observed by others [5].
Figure 4: Effect of particles on heat transfer coefficient ratio $h_r$.

Figure 5: Effect of particles on averaged heat transfer coefficient ratio $\bar{h}_r$. 
Figure 6: Effect of particles on averaged heat transfer coefficient ratio $\bar{h}_r$ for water-$\text{Al}_2\text{O}_3$ nanofluid in turbulent regime.

Figure 7: Effect of particles on averaged wall shear stress ratio $\bar{\tau}_r$. 
As we may see, the inclusion of nanoparticles has provided tremendous improvement of heat transfer for both laminar and turbulent flow regimes. Unfortunately, such inclusion has produced some adverse effect on the wall shear stress. Figure 7 shows, in particular, the influence of particle loading $\varphi$ on the ‘nanofluid-to-base fluid’ ratio of averaged shear stress $\overline{\tau}_r (\overline{\tau}_r = \frac{\tau_{nf}}{\tau_{bf}})$. It is clearly observed that the ratio $\overline{\tau}_r$ has considerably increased with an augmentation of parameter $\varphi$. Thus, for $\varphi=7.5\%$ for example, $\overline{\tau}_r$ has as values, 2.7 and 2.15, respectively, for Ethylene Glycol-Al$_2$O$_3$ and water-Al$_2$O$_3$ nanofluid; while for an extreme value of $\varphi$, says $\varphi=10\%$, $\overline{\tau}_r$ has increased to nearly 4 and 2.8 for the same tested fluids. Such adverse effect on wall friction, which clearly appears more pronounced for case of Ethylene Glycol-based nanofluid, is somewhat expected as it is obvious that the inclusion of nanoparticles will consequently and drastically affect the apparent viscosity of a resulting nanofluid. For Ethylene Glycol-Al$_2$O$_3$ in particular, the ‘nanofluid-to-base fluid’ viscosity ratio $\mu_r$ (Eqn 7) has increased from 1 for $\varphi=0\%$ to 1.19, 1.8, 2.71 and to 4, respectively, for $\varphi=2.5\%, 5\%, 7.5\%$ and 10\%.

4 Conclusion

In this work, the heat transfer enhancement by using nanofluids has been numerically investigated, considering two different liquid-nanoparticle mixtures, namely Ethylene Glycol-Al$_2$O$_3$ and water-Al$_2$O$_3$, flowing inside a uniformly heated tube. Results have clearly shown that, in general, the inclusion of particles has provided a considerable heat transfer enhancement both for laminar and turbulent flow regimes. It has been observed, however, that the presence of particles has created an adverse effect on the wall shear stress that has appreciably increased with particle loading. Among the nanofluids studied, Ethylene Glycol-Al$_2$O$_3$ clearly appears to offering a better heat transfer enhancement than water-Al$_2$O$_3$. It is also the one for which a more drastic and adverse effect of wall shear stress has been observed.

Acknowledgements

The authors wish to sincerely thank the Natural Sciences and Engineering Research Council of Canada, the ‘Ministère de l’Éducation du Québec’ and the ‘Université de Moncton’ for financial support to this project. Thanks are also due to the Faculty of Engineering of the ‘Université de Moncton’ for allowing computing facilities.
References


