Study on model predictive control of plate-fin heat exchangers

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Abstract

Heat exchangers are important pieces of equipment to transfer heat between process fluid media. In some processes the working fluid temperatures must be exactly controlled. The common control method such as PID feedback control might not provide high control quality because such a control is based on the deviation between inputs and outputs, and therefore the disturbances in the outputs cannot be eliminated if disturbances in the inputs occur. To overcome the disadvantages of traditional control methods, a model predictive control method is developed. A mathematical model for dynamic behaviour of one-dimensional flow multistream plate-fin heat exchangers is proposed to calculate the adjustment parameters. A model predictive control method is put forward by introducing a feed-forward controller. Experiments are carried out to compare the model predictive control with the PID feedback control method, which shows that the adjustment time and fluctuations of the controlled outputs of model predictive control are much smaller than those of PID feedback control.

Keywords: heat exchanger, dynamic simulation, model predictive control.

1 Introduction

The problem of modelling and control of heat exchangers has received much attention since 1960’s. Heat exchangers are usually designed to perform a given heat duty under their nominal operation conditions. Unfortunately, fluctuations in supply stream temperatures and stream flow rates frequently occur, and these disturbances affect the outlet temperatures of the streams. So it is necessary to
control the outlet temperatures of streams to satisfy the requirements of the process.

PID feedback control method is commonly used to adjust the flow rates of some streams to maintain the outlet temperatures of streams at their target values. The performance of the PI-control system for heat exchangers was investigated by Takahashi [1]. Koppel studied the optimal control of a tubular heat exchanger [2]. He converted the basic partial differential equation into an equivalent ordinary differential equation with delay time and obtained time optimal control law using the maximum principle. Unbehauen and Schmid [3] applied DDC algorithm to the temperature control of a tubular heat exchanger. The experimental results showed that a PID controller with anti-windup is the best mean to control the heat exchanger under different operating conditions.

Du et al [4] applied the lumped parameter model in parallel flow plate-fin heat exchangers to offers the dynamic behaviour of the apparatus. They found that the transfer functions of plate-fin heat exchangers to the disturbances in inlet fluid temperature and mass flow rates could be described in a general form as

$$G = Ke^{-\tau_s}/[(1+\tau_1s)(1+\tau_2s)]$$

The coefficients appearing in the transfer functions were determined by experiments or numerical calculation. They used this transfer function to control a four-stream plate-fin heat exchanger, and obtained good control result.

The above mentioned control method are based on the traditional control techniques which use the deviation of the outlet stream temperatures from their target values as the input of the controller. The controller will function only when the outlet stream temperatures of the exchanger deviate from their target values. Since the time constant of an industrial heat exchanger is usually very large, such a control strategy would either yield unacceptable fluctuation in the outlet stream temperatures to be controlled or need a long time to reach the steady state.

In order to reduce the adjustment time and fluctuation, the adjustment can be given immediately as the disturbances occur. A mathematical model is established to predict the dynamic response of heat exchanger and provide the values of the adjustment parameters to the controller.

## 2 Mathematical model

Consider a multistream plate-fin heat exchanger operating at a steady state and then undergoes a transient process which is caused by sudden changes in mass flow rates and/or inlet flow temperatures. As a transient process begins, the inlet fluid temperatures can further vary with time but the mass flow rates do not change any more.

It is assumed that the heat transfer coefficient and specific heat capacities of the fluid and the wall material are constant. There is no heat conduction resistance perpendicular to the plate and fin surfaces and no heat loss to the environment. The heat conduction in the solid material in the flow direction can be neglected. The mathematical description can be obtained from energy
balances of the fluids, separating plates and fins. The resulting system of dimensionless partial differential equations are summarized below:

\[
B_{i,j} \frac{\partial^2 \theta_{i,j}}{\partial \bar{x}^2} + W_{i,j} \frac{\partial \theta_{i,j}}{\partial \bar{x}} = \frac{U_{p,i,j}}{2} \left( \theta_{p,i,j} + \theta_{p,i+1,j} - 2\theta_{i,j} \right) + U_{f,i,j} \left( \int_0^1 \theta_{f,i,j} \, d\bar{y} - \theta_{i,j} \right) 
\]

(1)

\[
B_{p,i,j} \frac{\partial \theta_{p,i,j}}{\partial \bar{\tau}} = \frac{U_{p,i,j}}{2} \left( \theta_{i,j} - \theta_{p,i,j} \right) + \frac{U_{p,i-1,j}}{2} \left( \theta_{i-1,j} - \theta_{p,i,j} \right) + \gamma_{f,i,j} \frac{\partial \theta_{f,i,j}}{\partial \bar{y}} \bigg|_{\bar{y}=0} - \gamma_{f,i-1,j} \frac{\partial \theta_{f,i-1,j}}{\partial \bar{y}} \bigg|_{\bar{y}=1}
\]

(2)

\[
B_{f,i,j} \frac{\partial \theta_{f,i,j}}{\partial \bar{\tau}} = \gamma_{f,j,i} \frac{\partial^2 \theta_{f,i,j}}{\partial \bar{y}^2} + U_{f,i,j} \left( \theta_{i,j} - \theta_{f,i,j} \right)
\]

(3)

together with the boundary conditions:

\[
\theta_{i,j} = \begin{cases} 
\theta_{m,k}(\bar{\tau}), & \bar{x} = \bar{x}_{m,k} \text{ or } \bar{x} = \bar{x}_{out,k} \\
\theta_{i-1,j}, & \bar{x} \neq \bar{x}_{m,k} \text{ and } \bar{x} \neq \bar{x}_{out,k}
\end{cases}
\]

(4)

\[
\bar{y} = 0: \quad \theta_{f,i,j} = \theta_{p,i,j}
\]

(5)

\[
\bar{y} = 1: \quad \theta_{f,i,j} = \theta_{p,i+1,j}
\]

(6)

initial conditions:

\[
\bar{\tau} = 0: \quad \theta_{i,j} = \theta_{i,j}^*, \quad \theta_{p,i,j} = \theta_{p,i,j}^*, \quad \theta_{f,i,j} = \theta_{f,i,j}^*
\]

(7)

where \( \theta_{i,j}^*, \theta_{p,i,j}^* \) and \( \theta_{f,i,j}^* \) are the original steady-state temperature distributions in the fluids, separating plates and fins, respectively.

### 3 Analytical solution

After Laplace transform and eliminate the fin and separating plates temperatures, the governing equation system in the Laplace domain is obtained and is expressed in a matrix form as:

\[
\frac{d\tilde{T}}{d\bar{x}} = \tilde{A} \tilde{T} + \tilde{A} \tilde{T}^*
\]

(8)
The elements of the coefficient matrices $A$ and $\overline{A}$ are:

\[
a_{(i-1)m+j,(i-1)m+j} = -\frac{1}{W_{i,j}} \left\{ sB_{i,j} + (1-\eta'_{i,j}) \frac{sB_{f,i,j}U_{f,i,j}}{U_{f,i,j} + sB_{f,i,j}} + \left( U_{p,i,j} + \eta'_{i,j}U_{f,i,j} \right) \left[ 1 - \frac{1}{2} \left( h_{(i-1)m+j,(i-1)m+j} + h_{[(i+1)-1]m+j,(i-1)m+j} \right) \right] \right\}
\]

(9a)

\[
a_{(i-1)m+j} = \frac{U_{p,i,j} + \eta'_{i,j}U_{f,i,j}}{2W_{i,j}} \left( h_{(i-1)m+j,(i-1)m+j} + h_{[(i+1)-1]m+j,(i-1)m+j} \right) \quad (l = 1, \cdots, mn; l \neq (i-1)m + j)
\]

(9b)

\[
\overline{a}_{(i-1)m+j,(i-1)m+j} = \frac{1}{W_{i,j}} \left\{ B_{i,j} + (1-\eta'_{i,j}) \frac{B_{f,i,j}U_{f,i,j}}{U_{f,i,j} + sB_{f,i,j}} + \frac{1}{2} \left( U_{p,i,j} + \eta'_{i,j}U_{f,i,j} \right) \left( h_{(i-1)m+j,(i-1)m+j} + h_{[(i+1)-1]m+j,(i-1)m+j} \right) \right\} + \frac{1}{s} \left( \eta'_{i,j} - \eta_{i,j} \right) U_{f,i,j} \left( 1 - \frac{1}{2} \left( h_{(i-1)m+j,(i-1)m+j} + h_{[(i+1)-1]m+j,(i-1)m+j} \right) \right) \}
\]

(10a)

\[
\overline{a}_{(i-1)m+j} = \frac{1}{2W_{i,j}} \left( U_{p,i,j} + \eta'_{i,j}U_{f,i,j} \right) \left( h_{(i-1)m+j,(i-1)m+j} + h_{[(i+1)-1]m+j,(i-1)m+j} \right) - \frac{1}{s} \left( \eta'_{i,j} - \eta_{i,j} \right) U_{f,i,j} \left( h_{(i-1)m+j,(i-1)m+j} + h_{[(i+1)-1]m+j,(i-1)m+j} \right) \quad (l = 1, \cdots, mn; l \neq (i-1)m + j)
\]

(10b)

The solution of eqn. (8) can then be written as

\[
\overline{T} = \mathbf{U} e^{-\mathbf{L}^\tau \mathbf{T}} dx' + \int \mathbf{U} e^{-\mathbf{L}(\overline{T}' - \dot{x}')} \mathbf{U}^{-1} \overline{\mathbf{A}} \mathbf{U}^\ast e^{\mathbf{L}^\tau \mathbf{D}^\ast} \mathbf{U}^\ast dx'
\]

(11)

where $\lambda_l$ ($l = 1, \cdots, mn$) are the eigenvalues of the $mn$th-order square coefficient matrix $\mathbf{A}$ and $\mathbf{U}$ is a $mn$th-order square matrix whose columns are the eigenvectors of the corresponding eigenvalues. $\mathbf{H}$ and $\overline{\mathbf{H}}$ are coefficient matrices which give the relationship between the temperatures of plates and fins, $\mathbf{T}_p = \mathbf{H}\overline{T} + \overline{\mathbf{H}}\mathbf{T}^\ast$. They depend on the stream arrangement in the exchanger. After integration one has

\[
\overline{T} = \mathbf{U} e^{-\mathbf{L}^\tau \mathbf{T}} \mathbf{D}^\ast + \mathbf{V} e^{\mathbf{L}^\tau \mathbf{D}^\ast}
\]

(12)

in which

\[
\mathbf{V} = \left\{ \overline{v}_{k,l} \right\} = \left\{ \sum_{i=1}^{mn} \frac{u_{i,l} \overline{v}_{i,j}}{\lambda_i - \lambda_l} \right\}, \quad \overline{\mathbf{V}} = \left\{ \overline{\overline{v}}_{k,l} \right\} = \mathbf{U}^{-1} \overline{\mathbf{A}} \mathbf{U}^\ast
\]

(13)
The coefficient matrix $D$ should be determined by the boundary conditions.

$$D = W^{-1}F$$  \hspace{1cm} (14)$$

where $W = \{w_{(i-1)m+j,l}\}$ $(i = 1, \cdots, n; j = 1, \cdots, m; l = 1, \cdots, mn)$ is a $mn$th-order square matrix,

$$w_{(i-1)m+j,l} = \begin{cases} u_{(i-1)m+j,l}e^{x_{i,j,k}} , & \overline{x}_j = \overline{x}^{in}_m, \text{or} \overline{x}_{j+1} = \overline{x}^{in}_m \\ u_{(i-1)m+j,l} - u_{(i-1)m+j-1,l} \end{cases} e^{x_{i,j,k}}, \quad \overline{x}_j \neq \overline{x}^{in}_m \text{ and } \overline{x}_j \neq \overline{x}^{out}_m$$  \hspace{1cm} (15)$$

$F$ is a $mn$th-order vector,

$$f_{(i-1)m+j} = \begin{cases} \hat{\theta}^{in}_m - \sum_{l=1}^{mn} v_{(i-1)m+j,l} e^{x_{i,j,k}d^*_l}, & \overline{x}_j = \overline{x}^{in}_m, \text{or} \overline{x}_{j+1} = \overline{x}^{in}_m \\ \sum_{l=1}^{mn} (v_{(i-1)m+j-1,l} - v_{(i-1)m+j,l}) e^{x_{i,j,k}d^*_l}, & \overline{x}_j \neq \overline{x}^{in}_m \text{ and } \overline{x}_j \neq \overline{x}^{out}_m \end{cases}$$  \hspace{1cm} (16)$$

For such a solution it is difficult or even impossible to find its inverse transform analytically. Therefore the numerical inverse techniques have to be used to obtain the response in the real time domain. Here the fast Fourier transform (FFT) algorithm is used to get the analysis solution.

**4 Model predictive control**

PID control is a common method in industry heat exchangers, which can meet the basic process demand. However, on the basis of feedback control, this method cannot produce high control quality because of time delay and over-adjustment. So a model predictive control is presented here to overcome above disadvantages and realize the “zero defect” operation of heat exchangers.
In the model predictive control strategy the feed-forward model controller is introduced as shown in fig.1. When a disturbance occurs, the dynamic response of heat exchanger is predicted and heat exchanger operation can be adjusted by exerting adverse compensation in assistant fluid to keep the outlet parameters of objective fluid to its target value.

To verify the present control method, a series of experimental works are done. The equipment is shown in fig. 2. The working fluids are water. The fluid temperatures in water tanks are maintained at their given values with heaters. A two-stream plate-fin heat exchanger made of aluminium is investigated here. The structure parameters are shown in table 1.

![Figure 2: Experimental equipment.](image)

<table>
<thead>
<tr>
<th>Stream</th>
<th>Fin type</th>
<th>Fin Size High-Pitch - Thickness (mm)</th>
<th>Length (mm)</th>
<th>Width (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>rectangular</td>
<td>6.5 – 4.2 – 0.6</td>
<td>970</td>
<td>170</td>
</tr>
<tr>
<td>B</td>
<td>rectangular</td>
<td>6.5 – 4.2 – 0.6</td>
<td>970</td>
<td>170</td>
</tr>
</tbody>
</table>

Table 1: The parameters of two-stream plate-fin heat exchanger.

<table>
<thead>
<tr>
<th>Stream</th>
<th>Flow rate(kg/s)</th>
<th>Inlet temperature(°C)</th>
<th>Outlet temperature(°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.16</td>
<td>31</td>
<td>26.302</td>
</tr>
<tr>
<td>B</td>
<td>0.13</td>
<td>22</td>
<td>27.714</td>
</tr>
</tbody>
</table>

Table 2: Work condition of two-stream heat exchanger.

The work condition of the two-stream heat exchanger is shown in table 2. In order to compare the model predictive control method with PID control method, three operation conditions are conducted: (1) The inlet flow rate of stream B has a negative step change from 0.13kg/s to 0.1kg/s, (2) The inlet temperature of stream B changes from 22°C to 19.3°C. (3) The inlet flow rate of stream B has a step change from 0.13kg/s to 0.153kg/s, and after 14 seconds, the
temperature of stream B changes from 22°C to 20.3°C. Two control methods are taken to keep the outlet temperature of objective fluid B at its target value. The experimental results of these two methods are shown in fig. 3.

- **PID control**
- **Model control**

Flow rate of stream B has a step change from 0.13kg/s to 0.1kg/s

Inlet temperature of stream B changes from 22°C to 19.3°C

Inlet flow rate of stream B has a step change from 0.13kg/s to 0.153kg/s and after 14 seconds the temperature of stream B changes from 22°C to 20.3°C

Figure 3: The comparison of PID control results with Model control results.
The results show that the model predictive control method with feed-forward controller can obtain good control results than PID control method with feedback controller. When the inlet flow rate or temperature of objective fluid changes, or even these two parameters change simultaneously, the dynamic mathematics model is used to predict the outlet temperature response and provide the adjustment to keep the objective fluid parameters at their target values. That means, before the disturbance of inlet parameters influences the outlet parameters, the adjustment is added to suppress this influence. So the delay time of model predictive control method is much shorter than that of PID control method. The over adjustment value of model predictive control method also decreases.

5 Conclusions

The PID feedback control is widely used in processes. However, such a controller functions only after the outputs to be controlled deviate from their target values. For heat exchangers which usually have a large value of time constant, such a control strategy might not be able to provide high control quality because of time delay and over-adjustment. A model predictive control strategy is presented to overcome above disadvantages. The key point of model predictive control method is modelling of heat exchangers. A mathematical model which describes the dynamic response of multistream plate-fin heat exchangers is proposed.

A two-stream plate-fin heat exchanger is investigated experimentally. The model predictive control strategy is realized by introducing feed-forward controllers. As disturbances in the inlet temperatures and mass flow rates of the streams occur, the outputs of the feed-forward controllers are calculated with the present mathematical model. The experiments show that the model predictive control method is better than PID control method. The delay time of the model predictive control method is much shorter than that of PID control method and the over adjustment of model predictive control method also decreases.

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Nomenclature

$B$ ratio of heat capacity to total wall heat capacity in the heat exchanger, dimensionless.

$F$ heat transfer area, $m^2$

$h$ fin height, $m$

$L$ length of heat exchanger, $m$

$U$ parameter, $U = \frac{\alpha F}{W}$ dimensionless

$W$ thermal flow rate, $W/K$

$\bar{x}$ dimensionless coordinate, $\bar{x} = x / L$, dimensionless

$\bar{y}$ dimensionless coordinate, $\bar{y} = y / h$, dimensionless

$\alpha$ heat transfer coefficient, $W/m^2K$

$\gamma$ dimensionless lateral heat conductivity of fins,

$\bar{\tau}$ dimensionless time

$\theta$ dimensionless temperature, $\theta = \frac{T - T_{\text{min}}}{(T_{\text{max}} - T_{\text{min}})}$, dimensionless

$x$ coordinate in flow direction, $m$

$y$ coordinate in the direction of fin height, $m$

Subscript

$f$ fin

$in$ inlet

$p$ separating plate

References


