On 3D dynamic control of secondary cooling in continuous slab casting process

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Abstract

In this paper a 3D-model for simulation and dynamic control of the continuous slab casting process is presented. The diffusion convection equation with multiphase transition is used as a simulation model. The developed model is discretized by the finite element method and the algebraic equations are solved using the successive over relaxation (SOR) method. The optimal control method is used to control the secondary cooling. The numerical results are presented and analyzed.

Keywords: 3D-model, continuous casting, secondary cooling, optimal control.

1 Introduction

In the continuous slab casting process the molten steel is poured into a bottomless mold which is cooled with internal water flow. The cooling in the mold extracts heat from the molten steel and initiates the formation of the solid shell. The shell formation is essential for the support of the slab after mold exit. After the mold the slab enters into the secondary cooling area in which it is cooled by water mist sprays. The secondary cooling region is usually divided into cooling zones in which the amount of the cooling water can be controlled separately.

The control of cast cooling is of central importance in continuous slab casting process because it has a considerable influence on formation of cracks and other defects which can be formed in cast material. The cast should be cooled down according to a certain temperature field which depend on e.g. steel quality, cast shape, casting speed. Many numerical models for simulation of the casting process have been developed in recent years [5, 7]. Some of them have been applied to control and to optimize the casting process [3, 4, 8]. To our knowledge, all real-time industrial control applications are based on two dimensional models. In many
cases two dimensional models are sufficient for control purpose. However, nowadays the highly automated and instrumented casting machines allows the use of more sophisticated simulation and control models.

Our aim is to use the developed 3D-model in dynamic process control. We have considered the optimal control method to control the secondary cooling. The optimal control method can be very complicated and computationally time consuming. The optimal control method minimizes a cost function which is constructed by means of metallurgical cooling criteria. Several different cooling criteria can be used, e.g. the maximum length for the liquid pool, the maximum reheating and cooling rates on the slab surface, the minimum and maximum temperature at the unbending point and the maximum and minimum temperature on the surface along the casting machine [5]. The development work of our optimal control model is still ongoing. Therefore we use quite simple cost function in our numerical example.

2 Mathematical formulation of the state problem

Let $\Omega \subset \mathbb{R}^2$ be a rectangular domain $[0, L_X] \times [0, L_Y]$ and $\mathcal{V} = \Omega \times [0, L_Z]$ be a 3D domain. The boundary $\Gamma = \partial \mathcal{V}$ consist of parts:

$$
\Gamma_0 = \Omega \times \{0\}, \\
\Gamma_N = \{(x, y) \in \partial \Omega : x = 0 \lor y = 0\} \times [L_M, L_Z], \\
\Gamma_S = \{(x, y) \in \partial \Omega : x \neq 0 \land y \neq 0\} \times [0, L_Z] \cup \Omega \times \{L_Z\}, \\
\Gamma_M = \{(x, y) \in \partial \Omega : x = 0 \lor y = 0\} \times [0, L_M],
$$

where $L_M$ is the length of the mould, $L_Z$ is the length of the strand and $L_X, L_Y$ are the width and thickness of the calculation domain.

We define the temperature $T = T(x, y, z, t)$ dependent enthalpy function $H(T)$ and the Kirchoff’s temperature $K(T)$ by

$$
H(T) = \rho \left( \int_0^T c(\xi)d\xi + L(1 - f_s(T)) \right), \quad K(T) = \int_0^T k(\xi)d\xi,
$$

where $\rho, c(T), k(T)$, and $L$ are density, specific heat, thermal conductivity and latent heat, $f_s(T)$ is the solid fraction.
The mathematical model of the continuous casting problem can be written as:

\[
\begin{align*}
\frac{\partial H(T)}{\partial t} + v \frac{\partial H(T)}{\partial z} - \Delta K(T) &= 0 & \text{in } \mathcal{V} \times (0, t_f], \\
T &= T_0 & \text{on } \Gamma_0 \times (0, t_f], \\
\frac{\partial K(T)}{\partial n} + h(T - T_w) + \sigma \epsilon (T^4 - T_{ext}^4) &= 0 & \text{on } \Gamma_N \times (0, t_f], \\
\frac{\partial K(T)}{\partial n} &= 0 & \text{on } \Gamma_S \times (0, t_f], \\
\frac{\partial K(T)}{\partial n} &= Q & \text{on } \Gamma_M \times (0, t_f], \\
T(x, y, z, 0) &= T^0 & \text{in } \mathcal{V}.
\end{align*}
\]

Here $n$ is the unit vector of outward normal on $\partial \mathcal{V}$, $h$ is the heat transfer coefficient, $v$ is the casting speed, and $T_w, T_{ext}$ are known temperatures. The $\sigma$ is the Stefan-Boltzmann constant and $\epsilon$ is the emissivity. The cooling capacity $Q$ in the mould is known constant and $t_f$ is the simulation time.

### 3 Mesh approximation

We decompose $\Omega$ on a set of quadrilateral finite elements as shown on figure 2. Step sizes are smaller near left and down boundaries of $\Omega$, and all quadrilaterals are rectangular except those elements near the round corner. The 1D-domain $[0, L_Z]$ is divided into $n_z$ mesh points in $z$-direction. We decompose our 3D domain $\mathcal{V}$ in to a set of prismatic 3D finite elements with quadrilaterals on its cross section. Since the constructed mesh is Cartesian product of 2D mesh in $\Omega$ and 1D mesh on...
In time we discretize our problem by using semi-implicit mesh scheme. We approximate the term \( \left( \frac{\partial}{\partial t} + v(t) \frac{\partial}{\partial z} \right) H \) by using the characteristics of the first order differential operator [1, 6]. Namely, if \((x, y, z, t)\) is the mesh point on the time level \(t\) we choose \(\tilde{z} = z - \int_{t-\tau}^{t} v(\xi) d\xi\) and approximate this term by:

\[
\left( \frac{\partial}{\partial t} + v(t) \frac{\partial}{\partial z} \right) H \approx \frac{1}{\tau} \left( H(x, y, z, t) - H(x, y, \tilde{z}, t-\tau) \right).
\]

(2)

We denote \(\tilde{H}(x, t-\tau) = H(x, y, \tilde{z}, t-\tau)\). If \(\tilde{z} < 0\) then we set \(\tilde{H}(x, t-\tau) = H(x, y, 0, t-\tau)\).

The weak solution of the problem (1) is defined by integral identity

\[
\int_{V} (H(T) - \tilde{H})/\tau \, \eta dx + \int_{V} \nabla K(T) \cdot \nabla \eta dx
\]

\[
+ \int_{{\Gamma_N}} \{ h(T - T_w) + \sigma \epsilon (T^4 - T_{ext}^4) \} \, \eta d\Gamma_N + \int_{{\Gamma_M}} Q \, \eta d\Gamma_M = 0,
\]

(3)

for all test functions \(\eta \in V^0 = \{ \eta \in H^1(V) : \eta = 0 \text{ on } \Gamma_0 \}\). Here \(H^1(V)\) is the Sobolev space of first order and \(T = T(x) = T(x, y, z, j\tau), j \geq 1\).

Let \(V_h\) be a finite element approximation of the space \(H^1(V)\), \(V_h^{T_0} = \{ v_h \in V_h(V) : v_h = T_{0h} \text{ on } \Gamma_0 \}\) with \(T_{0h}\) being the \(V_h\)-interpolant of \(T_0\) and obviously defined subspace \(V_h^0\). Let further \(\varphi_1(x), \ldots, \varphi_n(x)\), be the basis functions in \(V_h\). Thus, the function

\[ T_h(x) = \sum_{i=1}^{n} T_i \varphi_i(x) \]

is the finite element approximation of \(T(x)\), \(T = (T_1, \ldots, T_n)^t\) is the column vector of the unknown nodal values of function \(T_h(x)\). Below the notations are: \(H(T) = (H(T_1), \ldots, H(T_n))^t\), \(K(T) = (K(T_1), \ldots, K(T_n))^t\) and

\[ f_h(T_h) = \sum_{i=1}^{n} f(T_i) \varphi_i(x), \]

for every function \(f(T)\), which depends on \(T(x)\).
Approximation of the state problem (3) by finite element method is defined as
\[
\int_V \frac{H_h(T_h) - \tilde{H}_h}{\tau} \eta dx + \int_V \nabla K_h(T_h) \cdot \nabla \eta dx \\
+ \int_{\Gamma_N} \{ h(T_h - T_w) + \sigma \epsilon((T^4)_h - (T^4_{ext})) \} \eta d\Gamma_N + \int_{\Gamma_M} Q_h \eta d\Gamma_M = 0,
\]  
\tag{4}
\]  
for all test functions \( \eta \in V^0_h \). Discrete state problem (4) is equivalent to the system of nonlinear algebraic equations:
\[
M \frac{H(T) - \tilde{H}}{\tau} + AK(T) + B(h)(T - T_w) + B(T^4 - T^4_{ext}) + DQ = 0,
\]  
\tag{5}
\]  
where \( M, A, B(h), B \) and \( D \) are the \( n \times n \) matrices with entries
\[
m_{ij} = \int_V \varphi_j \varphi_i dx, \quad a_{ij} = \int_V \nabla \varphi_j \cdot \nabla \varphi_i dx,
\]
\[
b_{ij}(h) = \int_{\Gamma_N} h \varphi_j \varphi_i d\Gamma_N, \quad b_{ij} = \sigma \epsilon \int_{\Gamma_N} \varphi_j \varphi_i d\Gamma_N,
\]
\[
d_{ij} = \int_{\Gamma_M} \varphi_j \varphi_i d\Gamma_M.
\]

More precisely, let \( L(l, k) \) and \( K(s, t) \) be the global indices of any two mesh points, where \( k \) and \( t \) indicate the node number in \( z \)-direction and \( l, s \) in \( xy \)-plane, respectively. By definition of mass matrix \( M \) we have
\[
m_{LK} = \int_V \varphi_L \varphi_K dx = \left( \int_0^{LZ} \varphi_k(z) \varphi_l(z) dz \right) \int_{\Omega_h} \varphi_l(x, y) \varphi_s(x, y) dxdy
\]
\[= m_{kt} m_{ls}.\]

Therefore matrix \( M \) equal to Kronecker product of standard 1D mass matrix \( M^z \) and 2D mass matrix \( M^\Omega \). Similarly we can define for the stiffness matrix \( A \) the entries
\[
a_{LK} = m_{kt}^z a_{ls}^\Omega + a_{kt}^z m_{ls}^\Omega
\]
and for matrices \( B, B(h) \) and \( D \)
\[
b_{LK} = \sigma \epsilon m_{kt}^z m_{ls}^b,
\]
\[
b_{LK}(h) = h_k m_{kt}^z m_{ls}^b,
\]
\[
d_{LK} = m_{kt}^z m_{ls}^b.
\]

Temperature dependent enthalpy \( H \) and Kirchhoff transform \( K \) are defined as piecewise linear and continuous functions. For the solution of the system (5) the modified SOR-method [2] for points in \( V \) is used and for nonlinear part on \( \Gamma_N \) Newton-Raphson method is used with one inner iteration.
We note that in our method it is sufficient to construct only 2D- and 1D-matrices. Therefore the computational efficiency of our model is very good. Also the memory allocation requirements are not so strong than in the case of ordinary 3D-brick elements.

\[
\begin{align*}
\Gamma_0 & \quad \text{Mould} \\
Mould & \\
1 & n + 1 \\
2 & n + 2 \\
& \vdots \\
n & 2n \\
\end{align*}
\]

Figure 3: Schematic presentation of 3D-cooling zones for quarter of the slab.

4 Control of secondary cooling

The secondary cooling region is divided into cooling zones (see figure 3) in which the values of heat transfer coefficients, \(h_i\), can be controlled separately in each cooling zone. The optimal control method is used for optimizing the secondary cooling on the boundary of the steel slab.

In optimal control method our aim is to minimize a cost function which is constructed by means of metallurgical cooling criteria. We can formulate our optimal control problem in the following way:

Find \(h^* \in U_{ad}\) such that

\[
J(T(h^*)) = \min_{h \in U_{ad}} J(T(h)),
\]

where \(U_{ad} = \{h(t) | h_{min} \leq h(t) \leq h_{max}\}\).

Our cost function, \(J\), has the form

\[
J(T) = \frac{1}{2} \int_{L_M}^{L_Z} (T - T_{tar})^2 dz,
\]

where \(T_{tar}\) is the target temperature.
To solve the optimal control problem (6) we use the steepest descent method

\[ h^{i+1} = h^i + \rho_{opt} p^{i+1}, \]  

where \( p^{i+1} \) is the steepest descent direction and \( \rho_{opt} \) is the optimal step size.

In order to find the steepest descent direction we construct and solve the adjoint state problem using the method of Lagrange multipliers. We construct the Lagrange function

\[ L(T, \lambda, h) = (\Psi(T, h), \lambda) + \tilde{J}(T), \]  

where

\[
\Psi(T, h) := M \frac{H(T) - \tilde{H}}{\tau} + AK(T) + B(h)(T - T_w) + B(T^4 - T_a^4) + DQ
\]

and

\[
\tilde{J}(T) = \frac{1}{2} \sum_{i=1}^{nz} \tilde{l}_i (T_i - T_{i\text{tar}})^2
\]

is the approximation of the cost function \( J \). We denote the length of the \( i \)-th element by \( \tilde{l}_i \). Differentiation of equation (8) with respect to variable \( T \) gives

\[
\left( \frac{1}{\tau} M (H'(T) \cdot \partial T) + A(K'(T) \cdot \partial T) + B(h) \cdot \partial T + B(4T^3 \cdot \partial T) , \lambda \right) + J'(T) \cdot \partial T = 0
\]

and we derive a system of linear algebraic equations

\[
\frac{1}{\tau} H'M \lambda + K' A \lambda + B(h) \lambda + 4T^3 B \lambda = -J'.
\]  

After solving the adjoint state problem (9) the steepest descent direction

\[
p_j = \begin{cases} 0, & (h_j = 0 \land I'_j > 0) \lor (h_j = h_{max} \land I'_j < 0) \\ -I'_j, & \text{otherwise}, \end{cases}
\]

\[
I'(h) = \left( \frac{\partial \Psi}{\partial n}(T, h), \lambda \right) + \frac{\partial \tilde{J}}{\partial h}(T) = (B(1)(T - T_w), \lambda)
\]

is found.

The optimal step size in the equation (7) is still unknown. We use the following algorithm to determine \( \rho_{opt} \):

**Step 1** \( \rho_0 = \frac{\rho_{max}}{2^{n+1}} \)

If \( J(T(h(\rho_0)))) < J(T(h(0))) \) GOTO Step 2;

Else \( \rho_{opt} = 0 \) END;

**Step 2** For \( i = 1, 2, \ldots, n \)

\[ \rho_i = (2^i + 1) \rho_0 \]

\[ \rho_{opt} = \rho_i \]

If \( J(T(h(\rho_i)))) < J(T(h(\rho_{i-1}))) \) CONTINUE;

Else \( \rho_{opt} = \rho_{i-1} \) END;
5 Numerical example

Here we present some numerical results for solving a continuous casting problem. All numerical experiments were run in PC with Pentium 4 processor of 2.66 GHz and 512 MB of RAM.

The dimensions of our geometry are: $L_X = 0.675\,\text{m}$, $L_Y = 0.105\,\text{m}$, $L_Z = 33\,\text{m}$ and the corresponding number of mesh points are $nx = 41$, $ny = 11$, $nz = 200$. The rest of essential input parameters are as follows: $L_M = 0.9\,\text{m}$, $L = 261\,\text{kJkg}^{-1}$, $\rho = 7312\,\text{kgm}^{-3}$, $T_0 = 1808.15\,\text{K}$, $T_w = 293\,\text{K}$, $T_{ext} = 293\,\text{K}$, $\epsilon = 0.8\times10^{-3}$, $\delta = 5.67\times10^{-8}\,\text{Wm}^{-2}\text{K}^{-4}$, $v = 1.4\,\text{m/min}$, radius $R = 0.005\,\text{m}$, average mould cooling $\dot{Q} = 1229\,\text{kwm}^{-2}$, solidus $T_s = 1763.15\,\text{K}$, liquidus $T_l = 1793.15\,\text{K}$.

The secondary cooling region is divided into 17 water cooling zones. There are eight center and eight corner cooling zones on the wide face of the slab and one cooling zone on the short face of the slab (see figure (3)).

The correctness of our optimal control model is verified in our first numerical test. Our aim is to optimize the secondary cooling in the wide face of the slab. Therefore, we define a piecewise linear target temperature profile along the center-line (Target1) and the cornerline (Target2) of the slab. The target temperatures at the end of each cooling zone are presented in the table 1. We note that the cooling zone 17 on the short face of the slab has constant water cooling, $h_{17} = 0.65$.

<table>
<thead>
<tr>
<th>zone</th>
<th>Target1 [$\text{C}$]</th>
<th>zone</th>
<th>Target2 [$\text{C}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1070</td>
<td>9</td>
<td>900</td>
</tr>
<tr>
<td>2</td>
<td>1020</td>
<td>10</td>
<td>850</td>
</tr>
<tr>
<td>3</td>
<td>970</td>
<td>11</td>
<td>790</td>
</tr>
<tr>
<td>4</td>
<td>960</td>
<td>12</td>
<td>780</td>
</tr>
<tr>
<td>5</td>
<td>950</td>
<td>13</td>
<td>770</td>
</tr>
<tr>
<td>6</td>
<td>940</td>
<td>14</td>
<td>760</td>
</tr>
<tr>
<td>7</td>
<td>930</td>
<td>15</td>
<td>750</td>
</tr>
<tr>
<td>8</td>
<td>900</td>
<td>16</td>
<td>740</td>
</tr>
</tbody>
</table>

The calculated corner temperatures at time 2500s are shown in the figure 4. We see that the difference between target and calculated surface temperatures is minimized on the whole length of the slab.

Secondly, we check if our control model fulfils real-time calculation requirements. We want to verify that our control model can fulfill real-time computation requirements not only on the steady state situation, but also when a heavy change in casting parameters happens. In our second numerical test the simulation model...
Figure 4: Calculated temperatures using 3D optimal control model. Left graph shows target and calculated surface temperatures on the centerline and on the cornerline of the wide face. Right graph shows calculated corner temperatures.

Figure 5: Computational times in every timestep.

is run to a steady state situation and then the casting speed is linearly dropped from 1.4m/min to 0.8m/min within 15s. From the figure 5 we see that a heavy increase in calculation times appears instantly after the casting speed is dropped. However, the calculation time stays below 5s in every timestep. We remark that timestep $\tau = 5s$.

6 Conclusions

Our numerical test results verify the correctness of our 3D optimal control model and also the dynamic control requirements were fulfilled. The development of
more sophisticated cost function based on metallurgical criteria is one of the most important and challenging task of our future work. We will also focus on improving the numerical methods involving both state and minimization problem.

References


