Thermal flow in glaciers: Application to the Lys Glacier (Italian Western Alps)

A. Deponti, L. De Biase & V. Pennati
Environmental Science Department, University of Milano–Bicocca, Italy

Abstract

In the present work the thermal flow in a section of Lys Glacier, in Italian Western Alps, is investigated. This work represents the application of the model presented in De Biase and Deponti [1]. The velocity field is calculated assuming ice to be a perfectly plastic material. With this input data, the energy conservation equation is solved by the Finite Difference Method. In order to accurately model the complex geometry of the glacier section a regular but non–uniform grid has been used. General Finite Difference Formulae of second order accuracy have been used. The model has been applied to a section of Lys Glacier taking into account all the particular atmospheric conditions of the site and making two different hypotheses for the geothermal flux value at the base.

1 Introduction

Middle latitude glaciers are often located in regions with relevant anthropogenic development; ice cores extracted from these sites can thus give information about the direct anthropogenic impact on the atmosphere. In particular glaciers on the Alps, situated in Western Europe, a densely populated and highly industrialized region, represent good sites for studying the direct anthropogenic impact on the European environment.

Drilling operations call for cold temperature, well below 0°C so that the percolation due to melting is reduced. For this reason drilling sites have to be over 4000 m a.s.l. and a good knowledge of the temperature field of the site is required. In this context mathematical modelling represents a powerful tool for glaciological studies.

In 1996 two bore–holes were drilled in the ice of Monte Rosa in the Italian Western Alps. The first bore–hole was a test drilling on Colle Gnifetti (4480 m a.s.l.);
the second one was on Colle del Lys (4240 m a.s.l.). The second bore–hole was drilled in correspondence of the central line and gave a 80 m deep core. The Lys Glacier is a polithermal glacier: cold conditions can be found in the upper part while warm conditions can be found in the lower part. The drilling sites are situated in the upper part (cold conditions) where the temperature is $-11^\circ C$ at the depth of 15 meters and is close to the average atmospheric annual temperature; in this part of the glacier the accumulation rate is $1600 (\pm 100) \text{ Kg m}^{-2} \text{ a}^{-1}$. On the other hand, in the lower part of the glacier (warm conditions) the temperature is close to the pressure melting point and melt water is present during the Spring–Summer–Fall period [2].

In this work, the model proposed by De Biase and Deponti [1] is applied to a section of Lys Glacier in order to evaluate the temperature field.

## 2 The physical problem

The steady–state velocity field is calculated under the assumption of perfect plasticity of ice. In this case the effective stress acting inside the glacier is constant and the velocity field takes into account the effect of accumulation and ablation on the surface. If there is accumulation or ablation, in fact, ice must submerge or emerge to maintain a steady–state condition. The velocity vector component parallel to the surface is [3]:

$$u = \frac{b_n}{h} x + 2|b_n| \sqrt{1 - \left(1 - \frac{z}{h}\right)^2} + c$$  \hspace{1cm} (1)

while the component perpendicular to the surface is:

$$w = -\frac{b_n}{h} z$$  \hspace{1cm} (2)

where:

- $b_n$ is the net balance of accumulation/ablation;
- $h$ is the surface elevation;
- $c$ is an integration constant depending on boundary conditions.

The general form of the thermal energy conservation law is:

$$\frac{\partial \theta}{\partial t} = \kappa \nabla^2 \theta + \nabla \kappa \cdot \nabla \theta - \mathbf{v} \cdot \nabla \theta + \frac{Q}{\rho C}$$  \hspace{1cm} (3)

where:

- $\theta$ is the temperature;
- $t$ is the time;
- $\kappa$ is the thermal diffusivity;
- $\mathbf{v}$ is the velocity vector;
- $Q$ is the heat production term;
- $\rho$ is the density;
- $C$ is the thermal capacity.
The thermal diffusivity $\kappa$ is temperature–dependent; since the temperature varies in space, also $\kappa$ varies in space. Nevertheless, the thermal diffusivity of ice is 37 m$^2$a$^{-1}$ at 0°C and 58 m$^2$a$^{-1}$ at −60°C. Such a variation of $\kappa$, in combination with small temperature gradients, makes the term $\nabla\kappa \cdot \nabla \theta$ negligible in comparison to the others in equation 3 [3]. Assuming steady–state conditions, the equation of thermal energy conservation can be rewritten as:

$$\kappa \nabla^2 \theta - \mathbf{v} \cdot \nabla \theta + \frac{Q}{\rho C_p} = 0$$

(4)

The heat production term in glaciers is represented by the strain heating: the acting force per unit area is represented by the effective stress ($\sigma_e$) (defined as the square root of the second invariant of the stress deviator tensor), while the total displacement is represented by the effective strain rate ($\dot{\varepsilon}_e$) (defined as the square root of the second invariant of the strain rate tensor). The relation between stress and strain rate for ice is non–linear:

$$\dot{\varepsilon}_e = A\sigma_e^n$$

(5)

where $A$ and $n$ are empirical parameters; $n$ is usually set equal to 3 [4], while the magnitude order of $A$ is $10^{-15}$ s$^{-1}$ kPa$^{-3}$. Using the constitutive relation for ice (equation 5) the heat production term gets:

$$Q = A\sigma_e^{n+1}$$

(6)

3 The numerical method

A reference system in which the $x$–axis is tangent to the earth surface and directed downhill and the $z$–axis is perpendicular to the earth surface and directed upward is considered. The problem has been solved by means of the Finite Difference method. The domain has been discretized by means of a structured but non–uniform grid. In the interior of the domain the grid is uniform, but, since the upper and lower boundaries do not follow the grid lines, non–uniform grid spacing is to be considered near such boundaries. Discretized algebraic equations are written for each internal grid point; these equations are obtained by classical Finite Difference Formulae in the interior of the domain and by Generalized Finite Difference Formulae [5] near the boundaries; the boundary conditions are discretized by Generalized Finite Difference Formulae. All the formulae are second order accurate.

4 Application to the Lys Glacier

The model has been applied to the Lys Glacier on Monte Rosa in the Italian Western Alps. A vertical section of the central line in the accumulation area has been considered. In order to obtain bedrock and surface topography, data from a geophysical survey [2] are interpolated by sixth degree polynomials.

In figure 1 the flow–lines calculated by equations 1 and 2 are shown. The mean annual accumulation rate of 2.7 m per year has been considered.
Since we are considering a steady–state model and we are not interested in the daily and seasonal temperature fluctuations in the surface layers, the model considers the 10 m depth layer as the upper boundary; this is indeed the layer where the temperature fluctuations vanish. On this boundary a condition on the temperature value is imposed as shown in figure 2; the lapse rate has been calculated from field measures (personal communication of Rossi G.C., 2002). It can be noticed that the temperature variation with respect to altitude is somewhat higher then the mean atmospheric vertical lapse rate.

At the lower boundary, we impose the vertical derivative to equal the estimated geothermal flux. Two simulations have been made: the first considering the world average geothermal flux of 55 mW m\(^{-2}\), the second considering a tenth of the world average geothermal flux. It is indeed reasonable to consider a low geother-
mal flux in Monte Rosa because of the geology and the past history of the permafrost (see also [6]).

The left boundary corresponds to the bore–hole drilled in 1996. The bore–hole is located on the ice divide near the Italian–Swiss border. Temperature measures have been collected and they show that in the bore–hole a decrease with depth occurs as shown in figure 3 (personal communication of Rossi G.C., 2002). This unusual feature is caused, in our opinion, by the convection of cold matter from the highest part of the ice divide. We simulated this phenomenon by imposing a suitable value for the normal derivative on the left boundary which simulates an outgoing heat flux.

The right boundary corresponds roughly to the limit where ice starts melting at some period; this is the end of the section since we are not interested in modelling the temperate part of the glacier. Since surface and bedrock slopes are not too different here, we impose the derivative along the average slope direction to be null.

With the calculated velocity field and the described boundary conditions we solved the thermal energy conservation equation for the Lys Glacier. Results are shown in figures 4 and 5: the former shows the results obtained by using the world average value of the geothermal flux while the latter shows the results obtained by using a lower geothermal flux value. It can be noticed that the temperature decreases with depth, while in ice sheets [7] and in an idealized parallel–sided section the temperature increases with depth (see the results of the application of the model to a parallel–sided section in [1]). This is caused by the fast flow regime and by the strong surface temperature variation. In this case, in fact, the dominant process is convection of cold matter from the upper part of the glacier, while in a slow flow regime the dominant process is thermal diffusion. When the geothermal flux is low, there is almost no heat emission from the bedrock so that the temper-
Figure 4: Temperature profile (°C) in the Lys Glacier considering the world average value of geothermal flux.

Figure 5: Temperature profile (°C) in the Lys Glacier considering a tenth of the world average value of geothermal flux.

Temperature is decreasing with depth throughout the domain. When the geothermal flux is relevant, on the other hand, a heat flux from beneath occurs; moreover near the bedrock the flow is slow because of friction, and diffusion becomes dominant so that a temperature increase with depth occurs.

5 Conclusions and future work

Thanks to the Generalized Finite Difference Formulae used for the space discretization of the energy equation, the model can be applied to domain of complex geometry such as a section of the Lys Glacier.

The application of the model to the real case of the Lys Glacier in the Italian
Western Alps shows interesting features. The process that dominates heat transfer in a valley glacier such as Lys Glacier is convection of cold matter from the higher part of the glacier. Moreover the temperature field is strongly influenced by the geothermal flux value at the base. Unfortunately field data are too few to completely confirm the value of the results but they are reasonable and feasible.

References


