Transient development of laminar mixed convection flow in a vertical tube

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Abstract

The problem of the transient laminar mixed convection flow in a vertical tube subject to time-dependent inlet boundary conditions has been numerically investigated. Results have clearly shown that successive step changes of the fluid inlet temperature have tremendous effects on the flow structure and its transient behaviors. It has been observed that a multiple-cell structure may co-exist in the tube. Thus, as hot fluid is cooled by heat loss to ambient environment, a first recirculation cell is noticed on the tube wall; it continues to increase both in volume and intensity. Shortly after the beginning of the positive step change of fluid inlet temperature, a second recirculation cell appears on the tube centerline, which first increases in size but rapidly decreases and eventually vanishes farther downstream. It has been observed that both the duration and the amplitude of the temperature step changes have important effects on the dynamic structure of the flow and thermal fields.

1 Introduction

The laminar mixed convection flow, encountered in many thermal applications in engineering, has received a tremendous attention from researchers (a partial review of previous works may be found in [1,2]). For a vertical tube flow in particular, the pioneer works by Hanratty and colleagues [3,4] have clearly shown that the flow appears highly unstable and may undergo its transition to an unstable ‘non-laminar’ state (but not necessarily turbulent) at a relatively low Reynolds number. For a case of opposing buoyancy, the flow regimes with or without reversal have also been experimentally and numerically investigated.
In real industrial applications, however, a transient laminar flow may occur during normal operation or, most often, due to accidents. Unfortunately, such flow did not receive much attention in the past. With regard to the problem of an unsteady laminar mixed convection flow subject to time-dependent boundary conditions at the tube inlet, references are relevant. In a recent work, a similar problem of the flow in vertical pipe submitted to a step change of the inlet flow rate has been investigated; results have shown that both buoyancy and external heat transfer rate have an important effect not only on the internal flow, but also on the local Nusselt number as well.

In the present work, we are interested to study the structure of the hydrodynamic and thermal fields of a laminar mixed convection flow and its transient behaviours, with an emphasis on the occurrence of the flow reversal phenomenon due to the time-dependent boundary conditions at the tube entrance.

2 Mathematical formulation and numerical method

2.1 Governing equations

We consider the simultaneously developing upward flow of water in a vertical tube of diameter \(D=20\text{mm}\) and length \(L=800\text{mm}\), which is submitted to a constant and uniform convective conditions at its outer surface. The flow is assumed transient, laminar and axisymmetrical; the fluid is viscous and incompressible with constant physical properties except for its density that appears in buoyancy force terms (the Boussinesq’s assumptions); both the compression work and viscous dissipation are considered negligible. Under such assumptions, the governing equations are as follows:

\[
\nabla \cdot (\rho \mathbf{V}) = 0 \tag{1}
\]

\[
\frac{\partial (\rho \mathbf{V})}{\partial \tau} + \nabla \cdot (\rho \mathbf{V} \nabla \mathbf{V}) = -\frac{\partial p}{\partial R} + \nabla \cdot \left( \mu \nabla \mathbf{V} \right) - \frac{\mu V R^2}{R^2} \tag{2}
\]

\[
\frac{\partial (\rho V_Z)}{\partial \tau} + \nabla \cdot (\rho \mathbf{V} V_Z) = -\frac{\partial p}{\partial Z} + \nabla \cdot (\mu \nabla V_Z) + \rho g \beta (T - T_0) \tag{3}
\]

\[
\frac{\partial (\rho C_p T)}{\partial \tau} + \nabla \cdot (\rho \mathbf{V} C_p T) = \nabla \cdot (k \nabla T) \tag{4}
\]

where \(\mathbf{V} = (V_R, V_Z)\) is the fluid velocity vector; \(\tau\) and \(p\) are the time and pressure; \(\rho, \mu, k\) and \(C_p\) are the fluid density, dynamic viscosity, thermal conductivity and heat capacity, all evaluated at the reference temperature \(T_0\).

2.2 Boundary and initial conditions

The conservation equations (1-4) constitute a set of non-linear and highly coupled PDE and are subject to the following boundary and initial conditions. At the tube inlet, the fluid enters with a uniform axial velocity \(V_0\) and uniform temperature \(T_{Z=0}\) that follows a prescribed time-dependent variation, Fig. 1, where \(T_0\) is the fluid inlet temperature at time \(\tau = 0\), \(\Delta T\) and \(\Delta \tau_1\) are the imposed temperature step change and its corresponding duration. At the tube centreline, the conditions of axisymmetry prevail. On the tube wall, the usual non-slip conditions and convective boundary conditions with constant and uniform heat
transfer coefficient $h_0$ and ambient temperature $T_0$ are imposed. At the exit section, the so-called ‘pressure outlet’ condition with a prescribed surrounding pressure is used.

As initial conditions, we assume that at time $\tau=0$, a steady, isothermal and developing laminar forced flow of fluid at uniform temperature $T_0$ already exists in the domain.

One can see that the problem under consideration may be characterized by a set of three dimensionless parameters, namely the Reynolds number $Re = \frac{VD\rho}{\mu}$, the Prandtl number $Pr = \frac{C_p \mu}{k}$ and the Grashof number $Gr = \frac{\rho^2 g \beta (\Delta T) D^3}{\mu^2}$.

Figure 1: Successive step changes imposed on fluid inlet temperature.

### 2.3 Numerical method and code validation

The governing equations (1-4) have successfully been solved by using a ‘finite-control-volume-based’ numerical method [14], employing the power-law scheme to compute the heat and momentum fluxes and a fully implicit second-order temporal scheme for treating the transient terms [15]. The resulting ‘discretization equations’ have been sequentially solved by using a combination of an efficient multiple-and-alternate-sweeping and a ‘line-by-line’ technique. In order to ensure the accuracy of results and, more importantly, their independence with respect to the number of nodes, several non-uniform grids have been tested and a 80x800 grid – with respectively 80 and 800 nodes along R and Z directions and highly packed grid points near the tube wall and the entrance region – has been adopted. A time step varying from 0.01s to 0.05s has been found adequate for the physical problem under investigation. For monitoring of the convergence, we have essentially based on the residuals that result from the ‘discretized form’ of the governing equations. For all of the cases performed in this study, a converged solution was achieved with very low residuals, says at least $10^{-8}$ for all equations.
The computer model was successfully validated by comparing its results as obtained for the axial velocity profile and the local Nusselt number \( \text{Nu}_Z \) (\( \text{Nu}_Z = \frac{h_z D}{k} \) where \( h_z \) is the local averaged heat transfer coefficient) to available analytical and numerical data for (i) a classic case of developing laminar forced convection flow in a tube, Fig. 2, and (ii) a case of steady laminar mixed convection flow of air in a uniformly heated vertical tube, with \( \text{Gr} = 10^6 \) [16], Fig. 3. For both cases, the agreement may be qualified as very good.

![Figure 2: Comparison for \( V_z \) at \( R = 0.45D \) (case of developing laminar forced convection flow).](image)

![Figure 3: Comparison for \( \text{Nu}_Z \) (case of laminar mixed convection flow of air in vertical tube, \( \text{Gr} = 10^6 \)).](image)
3 Results and discussion

In order to study the transient behaviours of the flow and thermal fields under the perturbation due to the time-dependent fluid inlet temperature, results as obtained for four typical cases are considered: **Case 1** $\Delta T=10^\circ C$, $\Delta \tau_1=1s$; **Case 2** $\Delta T=15^\circ C$, $\Delta \tau_1=1s$; **Case 3** $\Delta T=10^\circ C$, $\Delta \tau_1=4s$ and **Case 4** $\Delta T=15^\circ C$, $\Delta \tau_1=4s$; other common parameters are: $Re=300$, $h_0=5W/m^2K$ and $T_0=20^\circ C$. The maximum Grashof number corresponding to $\Delta T=15^\circ C$ is estimated to be $2.65\times10^5$.

Results have clearly shown that step changes of the fluid inlet temperature have produced very interesting effects on the flow structure and its transient evolution. During the period $\Delta \tau_1$ where a negative step change of fluid temperature is imposed, the cold fluid, after being forced into the tube, is heated up along the tube wall and accelerated while flowing upwards; in the core region, fluid still remains at lower temperature. This is a classic case of buoyancy-assisted flow. However, as cold fluid continues to flow farther downstream while the positive step change of fluid inlet temperature is initiated, the reverse trend is produced and a very interesting phenomenon has been observed. Figure 4 shows, for example, the flow structure as obtained for **Case 1** and three specific times, says 2s, 7.5s and 11s immediately after the beginning of this positive step change of fluid temperature (note that the radial coordinate was exploded in order to get fine details of the flow structure). Thus, as hot fluid (the one just admitted into the tube) is cooled by heat transfer towards the tube wall, we clearly observe the onset of the first recirculation cell that is attached to the tube wall near its inlet. As hot fluid is continuing to enter the tube, this recirculation cell rapidly grows both in size and in intensity; at times 7.5s and 11s for example, it is nearly extended to $Z=0.16m$ and $Z=0.25m$, respectively. It is very interesting to observe that such recirculation zone clearly exhibits its 'multi-cellular' structure. At time 7.5s in particular, we can notice the striking presence of the second recirculation cell that is located on the tube centreline. The formation of this cell is due, in fact, to the existence of an isolated mass of colder fluid therein i.e. the one that results from the first period $\Delta \tau_1$ and since then, it continues flowing downstream. As fluid temperature inside this recirculation zone is lower than the rest nearby, buoyancy force acts again the main flow. The second cell increases at first in size, but rapidly decays and completely vanishes farther downstream, as it is gradually heated up by surrounding fluid. Thus, at time 11s, it is almost reduced to quarter of its corresponding size at 7.5s. It should be noted that during all that time, the first recirculation cell i.e. the one attached to the wall, continues steadily to grow as long as the opposed-buoyancy conditions persist.

Similar flow structure and transient behaviors have also been observed for other cases simulated in this study, in particular **Case 2** (Fig. 5) and **Case 3** (Fig. 6) by which interesting effects due to parameters $\Delta T$ and $\Delta \tau_1$ may be studied. Thus, by carefully scrutinizing Figures 4 and 5, one may see that with an increase of the amplitude $\Delta T$ from $10^\circ C$ to $15^\circ C$, the previously discussed recirculation cells have considerably increased in size and in intensity. For the one located on the centreline and time 7.5s in particular, its size is almost triple
of that observed for Case 1; it considerably extends both in R and Z directions, beginning at an axial position much closer to the tube entrance, says at Z=0.05m. With further increase of time, says at 11s, it becomes much more extended in Z-direction under the entrainment effects of the main flow. It is very interesting to note that this recirculation cell persists much longer (in time) for Case 2 than for Case 1, which is obvious since much colder fluid exists therein. For the other recirculation cell on the tube wall, its increase of size and intensity also appears very important. Thus, at time 11s for example, it occupies a much larger and thicker zone near the tube wall (its thickness is almost one third of tube radius).

![Figure 4: Development of flow structure for Case 1 (ΔT=10°C, Δτ₁=1s).](image-url)
The above effects observed on these recirculation cells, the one attached to the wall as well as that located on the tube centreline, are obviously due to the stronger opposed-buoyancy forces that are directly linked to the increase of ΔT.

On the other hand, by scrutinizing Figures 4 (Case 1) and 6 (Case 3), one can see that the duration Δτ₁ also has important effects on the flow field and its transient behaviours. Although a ‘multi-cellular’ flow structure still exists for both cases, there are notable differences between them. Thus, with an increase of the parameter Δτ₁ from 1s to 4s, it is observed that the location of the second cell,
i.e. the one on the tube centreline, is slightly pushed downstream in Case 3 since more cold fluid is now admitted into the tube for a longer duration $\Delta \tau_1$. There is no substantial change regarding size and intensity of this recirculation zone, and as for Case 1, it tends to rapidly vanish in time. However, for the first recirculation cell attached to the tube wall, remarkable changes may be noticed. In general, one can observe that its volume and intensity have considerably augmented. Thus, for time 11s for example, it occupies a much larger region, extending from about $R=0.006m$ to the tube wall and from $Z=0.02m$ to nearly 0.2m along the axial direction. Such interesting and striking behaviour may be
explained by the fact that with a longer duration $\Delta \tau_1$, the corresponding column of cold fluid (near $10^\circ$C) that has been introduced into the tube during the period $\Delta \tau_1$ is consequently more important. Following the imposition of the positive step change of fluid inlet temperature, the first recirculation cell is initiated on the tube wall near the inlet (Fig. 6, time 2s), and it goes into a strong interaction with the above column of cold fluid. In fact, the latter actually ‘feeds’ the recirculation cell. Note that in the near-wall region, fluid velocity has already been considerably reduced by opposed-buoyancy effects; this could greatly facilitate such ‘back-flow’ interaction. By contrast, for Case 1, such interaction seems not to be important, probably because of the fact that the mass of cold fluid is not large enough and consequently, it would be easier to be heated up and entrained downstream.

4 Conclusion

In this paper, the problem of transient mixed laminar convection flow in a vertical tube subjected to successive step changes of fluid inlet temperature has been numerically investigated. Results have clearly shown that both the amplitude and duration of the step changes have drastic effects on the flow structure and its transient development. It has been observed that multiple recirculation cells may co-exist within the tube: the first cell, located near the wall, steadily increases in size and intensity, and the second one, observed on the centerline, tends to decay and vanishes farther downstream. Both the cells considerably increase with an augmentation of the step amplitude. On the other hand, an increase of the step duration tends to rend more important the recirculation cell near the tube wall.

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References


