Experimental validation of a model-based controller for a rectangular natural circulation loop

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Abstract

This paper presents some theoretical and experimental new results concerning the modeling and control of rectangular natural circulation loops. Despite the structural simplicity of these systems, which are used to cool a heat source by means of natural circulation of a fluid without mechanical pumping, they can exhibit unstable dynamics. The first part of this study is devoted to describing and experimentally validating a non-linear mathematical model, which has been derived by truncating the Fourier series expansion of the fluid temperature and of the function describing the system geometry and the heating conditions at the boundaries. In the second part the model has been used to design a set of controllers to stabilize the fluid motion in order to guarantee efficient heat removal. In particular, the proportional control strategy has been considered for a set of nominal heating powers, using both the fluid velocity and a temperature difference as the feedback variable. The results of simulations corresponding to each designed controller have been compared with the experimental trends, showing in all cases good agreement. In particular, the controllers have been proved to be capable of stabilising the system to the desired equilibrium point.

1 Introduction

Natural circulation loops, also called closed loop thermosyphon, are thermal devices which are usually devoted to the cooling of a heat source without mechanical pumping. In fact, density gradients established between the heat source, placed at the bottom, and a heat sink placed at the top governed the fluid
motion. This allows a drastic reduction of the probability of the cooling failure, which is the main reason to prefer natural to force convection in safe plants.

The simplest configurations of closed loop thermosyphons are those lying on a vertical plane, symmetrical with respect to a vertical axis and made of a heat source at the bottom, a heat sink placed on top. They may have or not two thermally isolated vertical legs connecting the heat exchanging sections. The heating boundary conditions typically reported in literature are: imposed temperature at the heating and cooling section [1], imposed heat power at the heating and cooling section [2] or mixed condition [3].

The stability of natural circulation loops mainly depends on the entity of the buoyancy, an hence on the vertical temperature difference. In fact, the growth of the vertical temperature difference $\Delta T$ corresponds to the acceleration of the flow until the system experiences chaotic oscillations [1-3]. Temperature oscillations and the associated inversions of the flow direction compromise the heat removal from the thermal source and should therefore be avoided. To Stabilisation of this kind of dynamic represents a main task in the field of natural circulation loops [4-6].

In order to design suitable control laws a reliable model of the system is needed. However the analytical model is non-linear and has distributed parameters. It is anyway possible to approximate this model by using a truncated Fourier series expansion of its variables. The geometry of the system and the heating conditions play an important role in the possibility of applying this method. Several results have been, in fact, reported for toroidal circuits without adiabatic legs and either with imposed temperature or imposed heat flux. In these cases the simplicity of the geometry makes it possible to obtain a system of three first-order non-linear ordinary differential equations. For rectangular circulation loops with adiabatic legs such a model has not been yet derived, though they represent the most common configuration in real applications.

Therefore, the concern of this paper was on the extension of the approach based on Fourier series expansion to the rectangular circulation loop modeling. This was achieved extending the results of the approach proposed for the generic geometry in [7]. The satisfactory behaviour of the model led to use it as the base for the design of a classical control strategy. Both the model and the controller were validated by means of tests performed on an experimental apparatus.

2 Experimental apparatus

The experimental natural circulation loop is depicted in Fig. 1 and its main dimensions are reported in Tab. 1. It consists of two horizontal heat transfer sections in copper, two vertical phirex tubes, four horizontal phirex tubes and four 90° phirex bends. The lower heating section consists of twelve independent electrical heating wires, providing 0.5 kW each, positioned around the copper tube, so that the system is able to provide up to 6 kW. The cooling section is a
coaxial heat exchanger. An expansion tank open to the atmosphere is installed, allowing the fluid expansion.

**Table 1. Main loop dimensions.**

<table>
<thead>
<tr>
<th>Measure (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loop height</td>
</tr>
<tr>
<td>Loop width</td>
</tr>
<tr>
<td>Loop inner diameter</td>
</tr>
<tr>
<td>Heating section length</td>
</tr>
<tr>
<td>Cooling section length</td>
</tr>
</tbody>
</table>

![Figure 1: The Experimental system.](image)

The whole system is equipped with eight calibrated ($\pm 0.1$ K) $T$-thermocouples located as indicated in Fig. 1 by the measuring points $T\#$. An inductive flow meter is inserted in the main loop while another inductive flow meter and an electromechanical servo-valve are inserted to measure and regulate cooling flow rate, by means of an external computer. During the tests the heat power supplied is controlled by a power board connected to the external computer via a serial port. This system makes it possible to impose the desired values for both the heat flux in the lower heating section and the flow rate of the coolant in the upper section. All the sensors are connected to a data acquisition board in the external computer. A software tool was implemented in a LabView environment that makes it possible to monitor and record all the data of each experiment and to perform the control action. The sampling frequency adopted was 1 Hz.

**3 The mathematical model**

The aim of this section is to give a mathematical description of the behaviour of the natural circulation loop represented in Fig. 1, having generic height and width, indicated respectively as $L$ and $L_I$, and a constant inner tube diameter.
equal to $r$. In the following, $x$ represents an abscissa parallel to the loop pipes, with a positive direction corresponding to the clockwise path through the loop and an arbitrarily chosen origin in the bottom left corner of the loop.

The analytical model, obtained by writing mass, momentum and energy balances for the whole loop, is given as:

$$\frac{dv(t)}{dt} + \frac{1}{r} \left( \frac{v}{2r} \right)^{\frac{q}{2}} = -\frac{\beta}{2(L + L_i)} \int f(T - T_0) f(x) dx \quad \forall x = 0 \quad (1)$$

$$\frac{\partial T}{\partial t} + v(t) \frac{\partial T}{\partial x} = h(x) + a \frac{\partial^2 T}{\partial x^2} \quad T(x,0) = T_0(x) \quad (2)$$

where $v$ is the fluid velocity, $d$ and $b$ are the parameters describing the friction law, $v$ is the cinematic viscosity, $\beta$ is the volumetric expansion coefficient, $T$ is the fluid temperature, $T_0$ is the reference temperature, $a$ is the thermal diffusivity of the fluid. The function $f(x) = dz/dx$ describe the system geometry and is approximated by the piecewise function:

$$f(x) = \begin{cases} 1 & 0 < x < L \\ 0 & L < x < L + L_i \\ -1 & L + L_i < x < 2(L + L_i) \\ 0 & 2L + L_i < x < 2(L + L_i) \end{cases} \quad (3)$$

whereas the function $h(x)$ describes the heating conditions at the boundaries:

$$h(x) = \begin{cases} 0 & 0 < x < L \\ -\frac{2}{\rho_0 c_p} \frac{\dot{m} c_p \Delta T_c}{2\pi r L_i} & L < x < L + L_i \\ 0 & L + L_i < x < 2L + L_i \\ \frac{2}{\rho_0 c_p} q & 2L + L_i < x < 2(L + L_i) \end{cases} \quad (4)$$

where $q$ is the heat flux and $\Delta T_c$ is the inlet-outlet temperature difference of the cooling section. The heat extracted in an elementary area is $\dot{m} c_p \Delta T_c / 2\pi r L_i$, where $\dot{m}$ and $c_p$ are the mass flow rate and the specific heat of the cooling fluid.

As demonstrated in [7], a model formally identical to that expressed by eqns (1)-(2) can be reduced by means of the following Fourier series expansions of the known function $f(x)$ and $h(x)$ and of the variable $T(x,t)$:

$$T(x,t) - T_0 = \sum_{k \in K} a_k(t) e^{\frac{i2\pi}{2(L + L_i)} kx} \quad (5)$$

$$h(x) = \sum_{k \in K} b_k e^{\frac{i2\pi}{2(L + L_i)} kx} \quad (6)$$

$$f(x) = \sum_{k \in K} c_k e^{\frac{i2\pi}{3(L + L_i)} kx} \quad (7)$$
where \( K, J \subset \mathbb{Z}^* = \mathbb{Z} \setminus \{0\} \). The substitution of eqns (5)-(7) in eqns (1)-(2) leads to the following system of infinite ordinary differential equations:

\[
\frac{d^2 v(t)}{dt^2} + \frac{1}{r} \left( \frac{v}{2r} \right)^{\alpha/d} v^{2-d} = g \beta \sum_{k \in K \cap J} \rho_k(t) \cdot \bar{e}_k, \tag{8}
\]

\[
\frac{da_k(t)}{dt} + \left[ \frac{d}{(L + L_i + L_o) k^2} \right] a_k(t) = b_k; \quad k \in K \cap J \tag{9}
\]

with \( \bar{e}_k(t) = a_{-k}(t); \quad \bar{b}_k = b_{-k}; \quad \bar{c}_k = c_{-k} \).

In order to integrate eqns (8)-(9), it is necessary to evaluate the Fourier series expansion coefficient, depending on the geometry and the heating conditions. In the case of the rectangular loop herein considered, the coefficients \( c_k \) of the \( f(x) \) expansion were calculated and are result from the following equation:

\[
c_k = \frac{1}{2 \pi k \ell} \left[ (1 - \cos k \gamma + isink \gamma)(1 - \cos k \pi) \right] \tag{10}
\]

where the following substitutions were considered:

\[
\phi = \frac{\pi x}{L + L_i}; \quad \gamma = \frac{\pi L}{L + L_i} \tag{11}
\]

Note that \( c_k \) vanish when \( k \) is even.

Acting analogously to calculate the coefficient \( b_k \) of the expansion of \( h(x) \), it is possible to obtain:

\[
b_k = \frac{\Gamma}{2 \pi k \ell} (-1 - \cos k \gamma + isink \gamma) \quad \text{when } k \text{ is odd} \tag{12}
\]

\[
b_k = \frac{\Gamma}{2 \pi k \ell} (1 - \cos k \gamma + isink \gamma) \quad \text{when } k \text{ is even} \tag{13}
\]

where, together with eqn (11), the following substitutions were considered:

\[
\Gamma = \frac{2}{\rho_c \ell \alpha \tau} \left( \frac{m_c \Delta T}{2 \pi \tau L_i} + q \right) \quad \Gamma_i = \frac{2}{\rho_c \ell \alpha \tau} \left( \frac{m_c \Delta T}{2 \pi \tau L_i} - q \right) \tag{14}
\]

The coefficients \( a_k \) of the expansion \( T(x,t) \) are complex and can be expressed as \( a_k(t) = a_k(t) + i\beta_k(t) \).

Arresting the Fourier series expansion at the third mode and applying the method of residuals, it is possible to rewrite equations eqns (8)-(9), with the substitution of the various coefficients as:

\[
\gamma(t) = -\frac{1}{r} \left( \frac{v}{2r} \right)^{\alpha/d} v^{2-d} + 2g \beta \sum_{k \in K \cap J} \rho_k(t) \cdot \bar{e}_k \tag{15}
\]

\[
\dot{\alpha}_i(t) = -a - \frac{\pi^2}{(L + L_i)^2} \alpha_i(t) + \frac{\pi}{L + L_i} v(t) \beta_i(t) + \frac{\Gamma}{2 \pi} \sin \gamma \tag{16}
\]

\[
\dot{\beta}_i(t) = -a - \frac{\pi^2}{(L + L_i)^2} \beta_i(t) - \frac{\pi}{L + L_i} v(t) \alpha_i(t) + \frac{\Gamma}{2 \pi} (1 + \cos \gamma) \tag{17}
\]
\[ \dot{\alpha}_2(t) = -a \frac{4\pi^2}{(L + L_\gamma)^2} \alpha_2(t) + \frac{2\pi}{L + L_\gamma} v(t) \beta_1(t) + \frac{\Gamma}{4\pi} \sin 2\gamma \]

\[ \dot{\beta}_2(t) = -a \frac{4\pi^2}{(L + L_\gamma)^2} \beta_2(t) - \frac{2\pi}{L + L_\gamma} v(t) \alpha_1(t) - \frac{\Gamma}{4\pi} (1 - \cos 2\gamma) \]

\[ \dot{\alpha}_1(t) = -a \frac{9\pi^2}{(L + L_\gamma)^3} \alpha_1(t) + \frac{3\pi}{L + L_\gamma} v(t) \beta_1(t) + \frac{\Gamma}{6\pi} \sin 3\gamma \]

\[ \dot{\beta}_1(t) = -a \frac{9\pi^2}{(L + L_\gamma)^3} \beta_1(t) - \frac{3\pi}{L + L_\gamma} v(t) \alpha_1(t) + \frac{\Gamma}{6\pi} (1 + \cos 3\gamma) \]

This system of seven ordinary differential equations represents the mathematical model, approximated to the third mode, of the dynamics of rectangular natural circulation loops.

An important physical assumption which ensures the existence of equilibrium points, is that the global heat power supplied to the fluid in its passage through the heat source is extracted in its passage through the heat sink. This leads to \( \int h(x) = 0 \) and therefore \( \int h(x)dx = 0 \) and, from eqn (14), \( \Gamma = 0 \).

In order to validate the model it is necessary to compare its simulation with measurements taken on an experimental loop. To this end, it is necessary to reconstruct the temperature function \( T(x,t) \) from the variables of the model \( \alpha(t) \) and \( \beta(t) \), i.e. the real and imaginary parts of its coefficients. This is easily performed according to eqn (5), arresting the sum at the third mode:

\[ T(x,t) = T_o + 2\alpha_1(t) \cos \frac{\pi}{L + L_\gamma} x - 2\beta_1(t) \sin \frac{\pi}{L + L_\gamma} x + 2\alpha_2(t) \cos \frac{\pi}{L + L_\gamma} 2x - 2\beta_2(t) \sin \frac{\pi}{L + L_\gamma} 2x + \]

\[ + 2\alpha_3(t) \cos \frac{\pi}{L + L_\gamma} 3x - 2\beta_3(t) \sin \frac{\pi}{L + L_\gamma} 3x \]

### 4 Model validation and analysis

In order to validate the model several simulations were compared with experimental data. The following parameters were adopted: \( \beta = 5.040 \cdot 10^{-4} \text{C}^{-1} \), \( \rho = 1000 \text{kg/m}^3 \), \( c_p = 4186 \text{kJ/kg} \cdot \text{K}^{-1} \), \( \nu = 1.002 \cdot 10^{-6} \text{m}^2 \cdot \text{s}^{-1} \). They correspond to the fluid properties at a reference temperature of \( T = 55 \text{C} \). The parameters \( a \), \( b \) and \( d \) are strongly dependent on the fluid motion condition and were computed for four different working conditions, as shown in Tab 2.

<table>
<thead>
<tr>
<th>Power</th>
<th>a [m$^2$/s]</th>
<th>b</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;900 W</td>
<td>0.004</td>
<td>2</td>
<td>0.5</td>
</tr>
<tr>
<td>900W ≤ 1600 W</td>
<td>0.0002</td>
<td>36</td>
<td>0.9</td>
</tr>
<tr>
<td>&gt;1600 W</td>
<td>0.0002</td>
<td>2</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Figure 2 reports the comparisons for experimental and simulated velocity (left column) and temperature difference $\Delta T_{25} = T_2 - T_5$ (right column) for operating condition $P = 2200$ W; in particular, the first row reports the experimental time series whereas the second row reports the time series simulated by means of the model and temperature reconstruction rule. These plots give a clear indication of the capability of the model of reproducing the dynamical behaviour of the experimental rectangular natural circulation loop, for both the system outputs. In fact, there is satisfactory evidence that the waveforms characterising the experimental time series are analogous to those of the simulated time series. Also, amplitude and frequencies characterising the system and the model are, in the overall, comparable.

![Experimental velocity, P=2200W](image1)

![Simulated velocity, P=2200W](image2)

![Experimental temperature difference AT25, P=2200 W](image3)

![Simulated temperature difference AT25, P=2200 W](image4)

Figure 2: Experimental (first row) and simulated (second row) time series of $v$ (left column) and $\Delta T_{25} = T_2 - T_5$ (right column) for $P = 2200$ W.

![Experimental attractor of AT25, P=2100W](image5)

![Simulated attractor of AT25, P=2100W](image6)

Figure 3: Comparison of experimental (on the left) and simulated (on the right) attractor of $\Delta T_{25} = T_2 - T_5$ for heat power $P = 2100$ W.
The comparisons in phase space for the same operating condition described in Fig. 2 is reported (only for $\Delta T2.5$) in Fig. 3. The left and right columns report the experimental and simulated attractor (for $P=2100\ W$).

Reminding that it is possible to draw just qualitative consideration on the similarity existing between the experimental and simulated attractors, the similarities characterising in both cases the morphological structures of the experimental and simulated attractors confirm, once again, that the model performs satisfactorily the description of the system dynamics.

The next step towards the design of a controller consisted of analysing the linearised model at the equilibrium points corresponding to a set of heating power values. It should be observed that for each input power three different equilibrium points were obtained. Two are perfectly symmetrical and corresponds to opposite fluid velocity, while the third correspond to null fluid velocity and is always an unstable point. The two other equilibrium points are stable with low power values (around 1100 W) and unstable at greater values.

5 Control design

The main purpose of the control action is to avoid temperature and velocity oscillations in the circuit which compromise efficient heat removal from the system. The adopted design procedure considered the heating power as input.

The control action was designed in order to drive the system to an equilibrium point. The control strategy was very simple and consisted simply of a proportional feedback on the velocity. The equilibrium conditions were derived from the model for a set of given nominal heating powers and the controller provided deviations around this power. The calculation were performed considering the cooling equal to the heating power, so that $I_c=0$.

The choice of the fluid velocity as control variable was made because literature on toroidal circulation loops lacks of controller of this kind. Also, while the controllers proposed for toroidal loops use as feedback variable the temperature difference between given section of the loop [4-6], in the present case the other variables appearing in the model are the real and imaginary parts of the Fourier coefficient of the temperature.

A further consideration must be made; in fact, under the previous assumption $I_c=0$, eqns (18)-(19), expressing the dynamics of variables $\alpha(t)$ and $\beta(t)$, identify in the linearized system a non-controllable subsystem which is stable. The global system is thus stabilizable and the influence of this subsystem can be neglected in designing the controller. This was also confirmed for the nonlinear system by performing a wide set of simulations.

The design, simulations and experiments were performed using a wide set of nominal heating power values. In order to make the presentation more concise we will only give the results referring to a power of 1800W. In all the experiments and simulations the control action was activated after a well developed oscillation, in order to show the controller capability not only in keeping the system stable, but also in stabilising it from an unstable condition. The gain was computed on the linearized system, tested and refined on the ODE simulator and
finally tested on the experimental system. A general criterion adopted was to obtain good settling times without large overshoots, in order to achieve suitable robustness for the system. The control law was therefore:

\[ \Gamma = \Gamma_{\text{nom}} + k_p (\text{ref} - \text{v}) \]

with

\[ \Gamma_{\text{nom}} = \frac{2}{\rho_c c_r} \left( \frac{P_{\text{nom}}}{\pi r L} \right) \]

and \( P_{\text{nom}} \) is the nominal heating power describing the operating condition.

Figure 4: Simulated velocity, \( \text{v} \), and temperature difference at the heating section, \( \Delta T_{25} \), for proportional control on velocity (\( P_{\text{nom}}=1800 \) W).

Fig. 4 shows the trend of the simulated controlled fluid velocity, \( \text{v} \), and the corresponding inlet-outlet temperature difference at the heating section, \( \Delta T_{25} \), at the with a nominal power of 1800W. As can be observed, the control action is activated after 1000s and after a transient both variables converges to their respective steady state values.

Figure 5: Experimental velocity, \( \text{v} \), and temperature difference at the heating section, \( \Delta T_{25} \), for proportional control on velocity (\( P_{\text{nom}}=1800 \) W).

The same controller was implemented on the real system and the corresponding trends of the fluid velocity and temperature difference are given in Fig. 5. In this experiment the control action was activated at \( t=700s \). The benefit of the control action can be noticed in the transient as a great reduction in the oscillations of the temperature difference, which can even overcome 60 °C without the controller (causing the fluid to be, at least for a few seconds, over the boiling temperature). Moreover, all fluid inversions are avoided, thus ensuring a correct heat removal.
6 Conclusion

In this work a low-order model and a control strategy for a rectangular natural circulation loop were derived. The method adopted allows a low-order non-linear model to be obtained from approximation of Navier-Stokes equations, which is in accordance with the experimental system, as confirmed by the comparison between simulations and experimental measurements. The model was used to design a simple proportional controller on the velocity aiming to ensure the stabilization of the fluid motion and efficient heat removal. The controller was designed for a set of nominal heating powers. The results of simulations of the controller were compared with the experimental trend, showing a good agreement. In all cases the controller was able to stabilise the system to the given equilibrium point.

References


