Non-linear analysis of heat transfer in burnishing rolling operation

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Abstract

This paper presents the physical and mathematical models of heat conduction in the burnishing rolling operation with an electric current. The variational equation of heat conduction in three dimensions for this case is proposed. Then, the finite elements method is used to obtain the solution. Algorithm to non-linear analysis and examples of calculation in the temperature field on the surface and outer layer zones are presented.

1 Introduction

One of the post-machining methods employed to form the outer layer, characterised by advantageous exploitation properties, is burnishing rolling with an electric current. The heating of parts is realised whilst burnishing, by means of an electric current passing the areas of contact between the burnishing element and the object (fig. 1) [1, 2, 4, 5].

The objective in this paper is to present the application of the variational methods to modelling the temperature field during the burnishing rolling operation with an electric current. A new thermo-elastic and thermo-visco-plastic material model is used. The model takes into account the history of the material and a possibility of the occurrence of a phase change in it, with temperature-dependent material properties. Then, application of the finite elements methods for obtaining the solution is shown. An exemplary scheme to a step-by-step numerical solution using one of the various integration methods is proposed. The stability and accuracy properties of the time integration schemes used the same way as in the structural
analysis [3]. The procedure has been implemented in the finite element computer program SYMUL-NAGN [7].

Figure 1: Diagram of the system of heat fluxes arising in characteristic volumes V and areas Σ during roller burnishing with current: Sp₁ and Sp₂, WW₁ and WW₂ - surface of the object and outer layer after previous treatment and after burnishing, respectively.

2 Mathematical incremental model of heat transfer

Burnishing rolling operation with electric current is a multiple non-linear thermodynamical process. While the process occurring the three of the most important nonlinearities: geometrical (a change of initial geometric of the body providing to non-linear dependence strain-displacement εijkl-uijkl), physical (non-linear dependence stress-strains σijkl-εijkl), thermal (non-linear law to heat transfer and boundary conditions) [6].

Therefore, we use the step-by-step incremental procedure, with updated Lagrangian formulations [3]. The basic approach in an incremental step-by-step solution is to assume that the solution for the discrete time t is known (here the temperature field ¹T) and that the solution for the discrete time t=t+Δt (incremental of the temperature field ¹ΔT) is required, where Δt is a suitably chosen time incremental. Therefore the temperature field at time τ is ¹T = ¹T + ¹ΔT, and could then proceed to the next time increment calculations t+2Δt.

We assume that the non-linear material obeys Fourier's incremental law of heat conduction in three dimensions in the global co-ordinate {z}, at typical step time t→τ=t+Δt:

\[
\text{div} \{\lambda(¹T) \text{grad} [¹ΔT(z, Δ t)] + ¹Δq_{v1}[z] + ¹Δq_{vd}[z] = ¹c(¹T) ¹ρ(¹T) ¹Δ T(z, Δ t)\},
\]

where ¹Δ T(z, Δ t) = ∂[¹Δ T(z, Δ t)]/∂t, ¹Δq_{v1}[z] and ¹Δq_{vd}[z] are the rate of incremental spatial heat sources generated by electrical current (heat of Joule's) and by visco-plastic deformation per unit volume V [1], ¹λ(¹T) is temperature-dependent heat conductivity coefficients corresponding to the principal axes zᵢ, ¹c(¹T) is heat capacity and ¹ρ(¹T) mass density at time t,
\[ \dot{\varepsilon}_{\text{VP}}(i, \dot{\varepsilon}_{\text{VP}}(i, \dot{T}) \text{ is accumulated yield stress, depend on the history of visco-plastic deformation and temperature, } \dot{\varepsilon}_{\text{VP}}(i, \dot{\varepsilon}_{\text{VP}}(i, \dot{T})/3), \]

\[ \dot{\varepsilon}_{\text{VP}}(i, \dot{\varepsilon}_{\text{VP}}(i, \dot{T})/3) \text{ are accumulated effective visco-plastic strain and strain rate. Expressions } \text{grad}[^{t}\Delta T(z,\Delta t)] \text{ and } \text{div}[^{t}\lambda(T)\text{grad}[^{t}\Delta T(z,\Delta t)]] \text{ in the equation (1) dependent of the co-ordinate system.} \]

Equation (1) must be considering with initial condition:

\[ T(z; t = t_0) = T_e(z), \quad z \in V = V_b + V_o, \quad (2) \]

where \( T_e \) is the environmental temperature at time \( t = t_0 \).

On the surface of the body a variety of boundary conditions are encountered in heat transfer analysis:

a) temperature conditions. The temperature may be prescribed at specific points in the surfaces, denoted by \( \Sigma_T \), and/or at the specific points in the volume of the body, denoted by \( V_T \) (conditions of I gender):

\[ T(z; t = t_k) = T_b(z; t_k), \text{ or } ^{t}\Delta T(z; \Delta t = \Delta t_k) = \Delta T_b(z; \Delta t_k), \quad z \in \Sigma_T, \]

\[ T(z; t = t_k) = T_b(z; t_k), \text{ or } ^{t}\Delta T(z; \Delta t = \Delta t_k) = \Delta T_b(z; \Delta t_k), \quad z \in V_T, \quad (3) \]

b) heat flow conditions. The heat flow input may be prescribed at specific points and surfaces of the body. These heat flow boundary conditions are:

- conditions of II gender:

\[ ^{t}\lambda_o(^{t}T)n \odot \text{grad}[^{t}\Delta T_o(z, \Delta t)] = b_o(^{t}\Delta q_{SI}[] + ^{t}\Delta q_{SU}[]), \quad z \in \Sigma_k, \quad (4) \]

\[ ^{t}\lambda_b(^{t}T)n \odot \text{grad}[^{t}\Delta T_b(z, \Delta t)] = b_b(^{t}\Delta q_{SI}[] + ^{t}\Delta q_{SU}[]), \quad z \in \Sigma_k, \]

- conditions of IV gender:

\[ ^{t}\lambda_o(^{t}T)n \odot \text{grad}[^{t}\Delta T_o(\cdot)] = [^{t}\Delta T_o(\cdot) - \\Delta T_b(\cdot)]/R_s(\cdot) =
\]

\[ = -^{t}\lambda_b(^{t}T)n \odot \text{grad}[^{t}\Delta T_b(\cdot)], \quad z \in \Sigma_k, \quad (5) \]

where \( R_s \) is the heat resistance in the surface contact (for ideal contact \( R_s = 0 \)), \( b_o \) and \( b_b \) is the heat division coefficients for burnishing element (b) and object (o), \( n \odot \text{grad}[^{t}\Delta T(\cdot)] \) is the scalar product, \( ^{t}\Delta q_{SI}[] \) and \( ^{t}\Delta q_{SU}[] \) are the rate of incremental surface heat sources generated by electrical current (heat of Joule’s) and fretting per unit surface \( S_\Sigma [1], \)

c) convection boundary conditions (conditions of III gender):

\[ ^{t}\Delta q_c[\cdot] \equiv \alpha_c(^{t}T)\cdot \Delta T = -^{t}\lambda_c(^{t}T)n \odot \text{grad}[^{t}\Delta T(z, \Delta t)], \quad z \in \Sigma_c, \quad (6) \]

where \( T_c \) is the temperature of the object or burnishing element and \( \alpha_c(^{t}T) \) is a temperature-dependent convection coefficient,

d) radiation boundary conditions (conditions of III gender):
\[ \Delta T = \frac{1}{\lambda} \nabla \cdot \mathbf{q} \]

where \( \lambda \) is the thermal conductivity of the material.

\[ T = T(x, y, z) \]

The variable \( T \) is the temperature field.

\[ \frac{\partial T}{\partial t} = \nabla \cdot \mathbf{q} \]

The heat flux \( \mathbf{q} \) is given by

\[ \mathbf{q} = -k \nabla T \]

where \( k \) is the thermal diffusivity.

\[ \mathbf{q} = \mathbf{q}(x, y, z, t) \]

The heat source term \( \mathbf{q} \) is determined by the heat generation and removal processes.

\[ \frac{\partial \mathbf{q}}{\partial t} = \nabla \cdot \mathbf{E} \]

The electric field \( \mathbf{E} \) is related to the heat source term \( \mathbf{q} \) through the electric current density \( \mathbf{J} \).

\[ \mathbf{J} = \sigma \mathbf{E} \]

where \( \sigma \) is the electrical conductivity.

\[ \mathbf{J} = \mathbf{J}(x, y, z, t) \]

The electric current density \( \mathbf{J} \) is determined by the electric field \( \mathbf{E} \) and the material properties.

\[ \nabla \cdot \mathbf{J} = \rho \]

The charge density \( \rho \) is determined by the electric field \( \mathbf{E} \) and the material properties.

\[ \nabla \cdot \mathbf{q} = \nabla \cdot \mathbf{J} + \nabla \cdot \mathbf{q}_s \]

The total heat flux \( \mathbf{q} \) is the sum of the conduction, convection, and radiation heat fluxes.

\[ \mathbf{q}_s = \mathbf{q}_s(t, x, y, z) \]

The radiation heat flux \( \mathbf{q}_s \) is determined by the radiation source term and the material properties.

\[ \mathbf{q}_s = \mathbf{q}_s(t, x, y, z) \]

The radiation source term \( \mathbf{q}_s \) is determined by the radiation temperature and the material properties.

\[ \Delta T = \frac{1}{\lambda} \nabla \cdot \mathbf{q} \]

The temperature change \( \Delta T \) is determined by the heat conduction and radiation processes.

\[ \Delta T = \Delta T(x, y, z) \]

The temperature change \( \Delta T \) is determined by the temperature field \( T \) and the material properties.

\[ \Delta T = \Delta T(t, x, y, z) \]

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The temperature change \( \Delta T \) is determined by the temperature field \( T \) and the material properties.
where a bar denotes "variation in", we obtain (because $\Delta T$ is the only variable):

$$
\delta[i_1 \Delta T, i_2 \Delta T', i_3 \Delta T] = \left\{ i_1 \Delta T' \right\}^T \left[ \lambda i_1 (i_1 T) \right] i_1 \Delta T \Delta T' + i_1 \Delta q_{VI} \left[ i_1 \Delta T \right] \Delta T \Delta T' + \left. \right|_{i_2 \Delta T} + \left. \right|_{i_3 \Delta T} + \left. \right|_{i_1 \Delta T}
$$

(11)

where:

$$
\left\{ i_1 \Delta T' \right\} = \left[ \frac{\partial (i_1 \Delta T)}{\partial z_1}, \frac{\partial (i_1 \Delta T)}{\partial z_2}, \frac{\partial (i_1 \Delta T)}{\partial z_3} \right], \left[ \lambda (i_1 T) \right] = \left[ \begin{array}{ccc}
\lambda_1 (i_1 T) & 0 & 0 \\
0 & \lambda_2 (i_1 T) & 0 \\
0 & 0 & \lambda_3 (i_1 T) \\
\end{array} \right].
$$

(12)

This equation (11) provides the basis for the finite element discretisations for obtaining the solution.

4 Implementation of the finite element method

Assume that the complete body under consideration has been idealised as an assemblage of finite elements, we have, at step time $t \rightarrow \tau$, for element $e$ and $m$:

$$
i_1 \Delta T^{(e)} (\cdot) = [i_1 \Delta T] \left[ i_1 \Delta T^{(e)} \right], \quad i_1 \Delta T^{(m)} (\cdot) = [i_1 \Delta T] \left[ i_1 \Delta T^{(m)} \right],
$$

$$
i_1 \Delta T^{(e)} (\cdot) = [i_1 \Delta T] \left[ i_1 \Delta \Theta^{(e)} \right], \quad i_1 \Delta T^{(m)} (\cdot) = [i_1 \Delta T] \left[ i_1 \Delta \Theta^{(m)} \right],
$$

$$
i_1 \Delta T^{(e)} (\cdot) = [i_1 \Delta T] \left[ i_1 \Delta \Theta^{(e)} \right], \quad i_1 \Delta T^{(m)} (\cdot) = [i_1 \Delta T] \left[ i_1 \Delta \Theta^{(m)} \right],
$$

(13)

where the superscript $(e)$ or $(m)$ denotes element $e$ or $m$, $\{ i_1 \Delta \Theta \}$ and $\{ i_1 \Delta \Theta \}$ are a vector of increments in the nodal point temperature and of increments in the nodal point temperature rate, at all $n$ nodal points, respectively:

$$
\{ i_1 \Delta \Theta \} = \left[ i_1 \Delta \Theta_1 , i_1 \Delta \Theta_2 , ..., i_1 \Delta \Theta_n \right], \quad \{ i_1 \Delta \Theta (\cdot) \} = \left[ i_1 \Delta \Theta_1 (\cdot), i_1 \Delta \Theta_2 (\cdot), ..., i_1 \Delta \Theta_n (\cdot) \right].
$$

(14)

The matrices $[i_1 \Delta \Theta (\cdot)]$ and $[i_1 \Delta \Theta (\cdot)]$, $[i_1 \Delta \Theta (\cdot)]$ now define the temperature increment and temperature increment gradients within element $e$ as a function of the nodal point temperature increment, respectively, and the matrix $[i_1 \Delta \Theta^{(m)} (\cdot)]$ is the surface temperature increment interpolation matrix.

Using the relation (13) and substituting into the variational equations (11) we obtain the discretised equations of heat transfer equilibrium:

$$
[i_1 C] \{ i_1 \Delta \Theta \} + ([i_1 K^e] + [i_1 K^r] + [i_1 K^w]) \{ i_1 \Delta \Theta \} = \{ i_1 \Delta Q \} + \{ i_1 \Delta Q^\prime \},
$$

(15)
where $[\mathbf{C}]$, $[\mathbf{K}]$, $[\mathbf{K}^\prime]$, $[\mathbf{K}^\prime\prime]$ are the convection and radiation, heat capacities, conductivity matrices and total nodal point conditions of IV gender, respectively:

\[
[\mathbf{C}] = \sum_{\varepsilon=1}^{E} \int_{t_{v}(\varepsilon)}^{t_{v+1}(\varepsilon)} \rho(\varepsilon) \mathbf{V}(\varepsilon) \mathrm{d}t \mathbf{V}(\varepsilon),
\]

\[
[\mathbf{K}] = \sum_{\varepsilon=1}^{E} \int_{t_{v}(\varepsilon)}^{t_{v+1}(\varepsilon)} \mathbf{B}(\varepsilon) \mathrm{d}t \mathbf{V}(\varepsilon),
\]

\[
[\mathbf{K}^\prime] = \sum_{m=1}^{S_C} \int_{t_{v}(m)}^{t_{v+1}(m)} \mathbf{H}(m) \mathrm{d}t \Sigma(m),
\]

\[
[\mathbf{K}^\prime\prime] = \sum_{m=1}^{S_k} \int_{t_{v}(m)}^{t_{v+1}(m)} \mathbf{H}(m) \mathrm{d}t \Sigma(m),
\]

\[
[\mathbf{K}^\prime\prime]\prime = \sum_{m=1}^{S_k} \int_{t_{v}(m)}^{t_{v+1}(m)} \mathbf{H}(m) \mathrm{d}t \Sigma(m),
\]

\[
[\mathbf{K}^\prime\prime]\prime = \sum_{m=1}^{S_k} \int_{t_{v}(m)}^{t_{v+1}(m)} \mathbf{H}(m) \mathrm{d}t \Sigma(m).
\]

$E$ is the total number of the finite elements in the system, $S_R$, $S_C$, $S_k$ are the number of the finite elements in the zones $\Sigma_R$, $\Sigma_C$ and $\Sigma_k$, respectively.

The nodal point increment heat flow input vector $\{\mathbf{\Delta Q}\}$ is given by:

\[
\{\mathbf{\Delta Q}\} = \{\mathbf{\Delta Q}_{\mathbf{V}D}\} + \{\mathbf{\Delta Q}_{\mathbf{V}I}\} + \{\mathbf{\Delta Q}_{\mathbf{S}I}\} + \{\mathbf{\Delta Q}_{\mathbf{S}p}\},
\]

where:

\[
\{\mathbf{\Delta Q}_{\mathbf{V}D}\} = \sum_{\varepsilon=1}^{E} \int_{t_{v}(\varepsilon)}^{t_{v+1}(\varepsilon)} \mathbf{V}(\varepsilon) \mathrm{d}t \mathbf{V}(\varepsilon),
\]

\[
\{\mathbf{\Delta Q}_{\mathbf{V}I}\} = \sum_{\varepsilon=1}^{E} \int_{t_{v}(\varepsilon)}^{t_{v+1}(\varepsilon)} \mathbf{V}(\varepsilon) \mathrm{d}t \mathbf{V}(\varepsilon),
\]

\[
\{\mathbf{\Delta Q}_{\mathbf{S}I}\} = \sum_{m=1}^{S_C} \int_{t_{v}(m)}^{t_{v+1}(m)} \mathbf{H}(m) \mathrm{d}t \Sigma(m),
\]

\[
\{\mathbf{\Delta Q}_{\mathbf{S}p}\} = \sum_{m=1}^{S_k} \int_{t_{v}(m)}^{t_{v+1}(m)} \mathbf{H}(m) \mathrm{d}t \Sigma(m).
\]

The vector $\{\mathbf{\Delta Q}\}^I = \{\mathbf{\Delta Q}\}^I_{\mathbf{S}p} + \{\mathbf{\Delta Q}\}^I_{\mathbf{V}I}$ of the boundary conditions of I gender is given by:

\[
\{\mathbf{\Delta Q}\}^I_{\mathbf{S}p} = \sum_{m=1}^{S_T} \int_{t_{v}(m)}^{t_{v+1}(m)} \mathbf{H}(m) \mathrm{d}t \Sigma_T(m),
\]

\[
\{\mathbf{\Delta Q}\}^I_{\mathbf{V}I} = \sum_{\varepsilon=1}^{E_T} \int_{t_{v}(\varepsilon)}^{t_{v+1}(\varepsilon)} \mathbf{V}(\varepsilon) \mathrm{d}t \mathbf{V}(\varepsilon),
\]

where $S_T$ and $E_T$ are the number of the finite elements in the zones $\Sigma_T$ and volume $V_T$, respectively.

5 Algorithm of numerical analysis

Use the principles of step-by-step integration scheme [3] in this section is presented for the solution of transient heat transfer problems. To approximate the
velocity component \( \{^t \Delta \hat{\Theta} \} \) in term of \( \{^t \Delta \Theta \} \) in the equation (15) we can use the Euler method (with the temperature varies linearly over the time interval \( \Delta t \)) and the direct integration methods: the Houbolt method, the Wilson \( \Theta \) method, and the Newmark method. In this paper, an example to using the Houbolt method to solution the equation (15) is showed. The Houbolt method reduces directly to a static analysis. The following finite approximate of the velocity component \( \{^t \Delta \hat{\Theta} \} \) is employed:

\[
\{^t \Delta \hat{\Theta} \} = a_1 \{^t \Delta \Theta \} - a_2 \{^t \Theta \} + a_3 \{^{t-\Delta t} \Theta \} - a_4 \{^{t-2\Delta t} \Theta \},
\]

where:

\[
a_1 = \frac{11}{(6\Delta t)}, \quad a_2 = \frac{7}{(6\Delta t)}, \quad a_3 = \frac{3}{(2\Delta t)}, \quad a_4 = \frac{1}{(3\Delta t)},
\]

are the integral constants.

Substituting (20) into (15) and arranging all known vectors on the right-hand side, we obtain for the solution of \( \{^t \Delta \Theta \} \):

\[
{[\tilde{K}]} \{^t \Delta \Theta \} = {^t \Delta \tilde{Q}^t} + {^t \tilde{Q}^t},
\]

where:

\[
{[\tilde{K}]} = a_1{[C]} + a_2{[K^c]} + a_3{[K^t]} + a_4{[K^iv]},
\]

\[
{^t \Delta \tilde{Q}^t} = {^t \Delta Q^t} + {^t \Delta Q^t},
\]

As shown in (24) the solution of \( \{^t \Delta \Theta \} \) required knowledge of \( \{^t \Theta \} \), \( \{^{t-\Delta t} \Theta \} \) and \( \{^{t-2\Delta t} \Theta \} \). Although the knowledge of \( \{^0 \Delta \Theta \} \) and \( \{0 \Delta \hat{\Theta} \} \) is useful to start the Houbolt integration scheme, it is more accurate to calculate \( \{^\Delta \Theta \} \) and \( \{^\Delta \hat{\Theta} \} \) by some other means; i.e., we employ special starting procedures. One way of the proceeding is to integrate (15) for the solution of \( \{^\Delta \Theta \} \) and \( \{^\Delta \hat{\Theta} \} \) using a different integration scheme, possibly a conditionally stable method such as the central difference scheme such as the central difference scheme with a fraction of \( \Delta t \) as the time step. The complete algorithm used in the integration is given in Table 1.

### Table 1: Step-By-Step Solution Using Houbolt Integration Method

<table>
<thead>
<tr>
<th>Step-By-Step Solution Using Houbolt Integration Method</th>
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</thead>
<tbody>
<tr>
<td>A. Initial Calculations:</td>
</tr>
<tr>
<td>1. Form matrix ([C], [K^c], [K^t], [K^iv])</td>
</tr>
<tr>
<td>2. Initialise ({^0 \Theta }) and ({^0 \hat{\Theta} })</td>
</tr>
<tr>
<td>3. Select time step (\Delta t) and calculate integration constants:</td>
</tr>
<tr>
<td>(a_1 = \frac{11}{(6\Delta t)}, \quad a_2 = \frac{7}{(6\Delta t)}, \quad a_3 = \frac{3}{(2\Delta t)}, \quad a_4 = \frac{1}{(3\Delta t)}).</td>
</tr>
</tbody>
</table>
5. Calculate effective matrix \( [\mathbf{T}_e] \):

\[
[\mathbf{T}_e] = a_e [\mathbf{C}] + [\mathbf{T}_e] + [\mathbf{K}_e^T] + [\mathbf{T}_e^T] + [\mathbf{K}_e^T].
\]

6. Triangularize \( [\mathbf{T}_e] \):

\[
[\mathbf{T}_e] = [\mathbf{L}][\mathbf{D}][\mathbf{L}^T].
\]

---

\section*{B. For Each Time Step}

1. Calculate effective heat load at time \( t \):

\[
\{\mathbf{i} \tilde{Q}_e\} = [\mathbf{C}](a_{i,2} \{\mathbf{i} \Theta\} - a_3 \{\mathbf{i} \Delta \Theta\} + a_4 \{\mathbf{i} \Delta \Theta_{-\Delta t}\}).
\]

2. Calculate effective incremental heat load at time step \( t \rightarrow t+\Delta t \):

\[
\{\mathbf{i} \Delta \tilde{Q}_e\} = \{\mathbf{i} \Delta Q\} + \{\mathbf{i} \Delta \tilde{Q}_{-\Delta t}\}.
\]

3. Partition of the heat equilibrium equation for two blocks, we can write the problem in the form:

\[
\begin{bmatrix}
[\mathbf{T}_1] & [\mathbf{T}_{12}] \\
[\mathbf{T}_{12}^T] & [\mathbf{T}_{22}]
\end{bmatrix}
\begin{bmatrix}
\{i \Delta \Theta_{1}\} \\
\{i \Delta \Theta_{2}\}
\end{bmatrix}
= \begin{bmatrix}
\{i \Delta \tilde{Q}_{1}\} \\
\{i \Delta \tilde{Q}_{2}\}
\end{bmatrix}
+ \begin{bmatrix}
\{i \tilde{Q}_{1}\} \\
\{i \tilde{Q}_{2}\}
\end{bmatrix},
\]

where vectors \( \{i \Delta \Theta_{1}\} \), \( \{i \Delta \tilde{Q}_{1}\} \) are knowns and \( \{i \Delta \Theta_{2}\} \), \( \{i \Delta \tilde{Q}_{2}\} \) are not known.

4. Solve for temperature increment vector \( \{i \Delta \Theta_{2}\} \) at time step using the modified Newton-Raphson iteration method:

\[
[\mathbf{T}_e]_1 \{i \Delta \Theta_{1}\} = \{i \Delta \tilde{Q}_{1}\} \}
\]

5. Substituting vector \( \{i \Delta \Theta_{1}\} \) into equation:

\[
\{\mathbf{i} \tilde{Q}_{2}\} = [\mathbf{T}_{12}] \{i \Delta \Theta_{1}\} + [\mathbf{T}_{22}] \{i \Delta \Theta_{2}\} - \{i \tilde{Q}_{2}\},
\]

and solve we obtain the vector \( \{i \Delta \tilde{Q}_{2}\} \).

6. Calculate the temperature vector at the end of iteration \( i \):

\[
\{\mathbf{i} \Theta\} = \{\mathbf{i} \Theta\}^{[i-1]} + \{i \Delta \Theta\}^{[i]}.
\]

7. Calculate vector \( \{i \Delta \Theta\} \) at step time:

\[
\{i \Delta \Theta\} = a_1 \{i \Delta \Theta\} - a_2 \{\mathbf{i} \Theta\} + a_3 \{i \Delta \Theta\} - a_4 \{i \Delta \Theta\}.\]

---

\section*{6 Examples of solution}

The analysis has been carried out using the following data: type of part - a roller, diameter of roller \( d = 30 \) mm; roughness profile of the transverse surface in accordance with projection case I with the parameters \( R_s = 0.142 \) mm; \( \alpha_r = \alpha_r' = 55^\circ; \)
p_r=0.5 rpm; burnishing by means of a D=60 mm diameter roller-shaped element; depth of burnishing g=0.071 mm; burnishing feed p_n=p_r=0.5 rpm; velocity of burnishing rolling V_n=1.5 m/s; intensity of electrical current I=0,100,300,500 and 700 A; h_r=0.363; heat conductivity coefficients λ(a)=42-0.012ΔT W/(mK), λ(b)=13.2207-0.0032ΔT W/(mK); resistivity ρ(l(a))=15(1+ 0.00196ΔT)10^-8 Wm, ρ(l(b))=57.6(1+0.00282ΔT)10^-8 Wm; mass density ρ=7850/(1+3αΔT) kg/m³; heat capacity c=484+0.01ΔT J/(kgK); coefficient of fretting μ=μ_0(1-0.003V_n)(1-0.000015ΔT); heat resistance in the surface contact R_s=0 (ideal contact) and R_s=0.094 m²K/W yield stress σ_y = 924(0.066+ε(VP)ₗ₁)₀.₁₈ × (1+ε(VP)ₗ₁)₀.₁₅ (1,737−0.043' T½). We demonstrate the non-linear analysis in the figure 2 and 3.

Figure 2: Field of temperature increment in the contact zone for I=450 A, V_n=0.95 m/s and μ_o=0.02 [3].

Figure 3: Distribution of resultant temperature inside the burnishing element and the object in axis z₂ for ideal contact (a) and with heat resistance in the surface contact (b).
7 Conclusions

In this paper we have presented a possibility to apply the variational and finite element methods for the analysis of heat conduction in burnishing rolling operation with electric current. We have presented an efficient procedure for the finite element solution of problem with a new thermo-elastic/thermo-visco-plastic material model. The model takes into account the history of the material and a possibility of phase change in it to occur [1]. The solution procedure is based on Houbolt integration method. The solution procedure uses one time step size for the calculation of nodal point temperatures. The material model and solution procedure has been implemented in the finite element computer program SYMUL-NAGN [7]. Numerical results are reported for two test problems.

References