Heat and mass forced convection in a composite fluid-porous annular enclosure

M. Benzeghiba & S. Chikh
Institut de Génie Mécanique,
USTHB, Algiers, Algeria

Abstract

The objective of this study is the analysis of heat and mass transfer characteristics and flow patterns, which develop in an annular cavity between a rotating inner cylinder and stationary outer one. A vertical saturated porous layer inserted in the enclosure divides the cavity in two fluid compartments. The interfaces between the fluid and porous medium are permeable, allowing the fluid to flow from one fluid region to another through the porous zone. The effects of fluid characteristics, centrifugal force, aspect ratio and thermophysical parameters of the porous filter on heat and mass transfer expressed by average Nusselt and Sherwood numbers are investigated and documented. The results show that the development of Taylor vortices is deeply damaged by decrease of permeability in the porous matrix and yields a reduction of Nusselt and Sherwood numbers. The definitions of primary and secondary flows, normal and anomalous modes should be therefore either revised or dropped out in interpreting the flow structures arising from an interaction between the centrifugal and Coriolis forces on one hand, and the boundary viscous force on the other.

1 Introduction

During the last several decades, the theory of flows between rotating cylinders has been extensively developed, with attention generally focused on fluid columns of infinite length such that effects of the end plates may be neglected [1]. Later, the problem of Taylor vortices in short annular enclosures has been the subject of a great number of experimental and numerical investigations in
order to better understand several heat transfer phenomena encountered in many practical applications, such as cooling of turbine rotors or electrical motor shafts; other applications include cooling of high speed gas bearing, rotating condensers for sea-water distillation and space-craft power plant. Most of these studies dealt with cavities fully filled by fluid as the one carried out by Lee and Minkowycz [2]. However, few studies have considered combined heat and mass transfer. Lee and Howell [3] were among the first to analyse the transport of heat and solute by forced convection in a porous medium bounded by a flat plate. They showed that Sherwood number is maximum around the region where the porous medium is attached. Recently, Khellaf and Lauriat [4] have carried out a deep numerical study to analyse the response time of a differentially heated fluid and taking into account the rotation of one cylinder. The work of Muralidhar [5] has reported a review of heat transfer studies in annular cavities completely filled by a fluid or a porous material. On the other hand, combined heat and mass transfer by forced convection in fluid or porous enclosures has been the topic of other investigations such as the ones of Wang and Du [6] and De Farias Neto et al. [7].

In the present study, steady state laminar forced convection is considered in a vertical annulus subject to horizontal thermal and concentration gradients. The annular space contains two fluid zones separated by a saturated porous matrix with permeable porous-fluid interfaces.

2 Problem statement and formulation

The physical problem under consideration consists of two vertical concentric cylinders of height H with a gap width d. Two adiabatic and impermeable horizontal disks close the annular cavity at the bottom and top ends (Figure 1). The enclosure is filled with a Newtonian fluid constituted by a solvent and a solute in a weak concentration. A porous matrix of thickness E and saturated with the same fluid divides the cavity in two separate fluid regions. Horizontal thermal and concentration gradients are applied in the radial direction. The inner cylinder is rotating, inducing centrifugal effects on the flow. Fluid properties are assumed constant and viscous dissipation, as well as Soret and Dufour effects negligible. A local thermal equilibrium is established within the porous matrix, which is homogeneous and isotropic. The fluid motion in the porous region is described by the Darcy-Brinkman-Forchheimer equation.

With these assumptions, the dimensionless governing equations are written for the whole domain as follows:

\[ \nabla \vec{V} = 0 \]  
\[ \left( \frac{1}{\varepsilon} \right) \nabla \vec{u} = -\frac{\partial p}{\partial x} + \left( \frac{R_v}{\varepsilon} \right) \frac{1}{Re} \nabla^2 \vec{u} - \left( \frac{1}{Re Da} \right) \vec{u} + \frac{C_f}{\sqrt{Da}} |\vec{V}| \vec{u} \]  
\[ \left( \frac{1}{\varepsilon} \right) \nabla \vec{V} = -\frac{\partial p}{\partial r} + \left( \frac{1}{\varepsilon} \right) \frac{w^2}{r} + \left( \frac{R_v}{\varepsilon} \right) \frac{1}{Re} \left( \nabla^2 \vec{v} - \frac{\vec{v}}{r^2} \right) - \left( \frac{1}{Re Da} \right) \vec{v} + \frac{C_f}{\sqrt{Da}} |\vec{V}| \vec{v} \]
The associated boundary conditions in a non-dimensional form are:

- At \( r_i = l / (K_r l) \)
  - \( \vec{V} = 0 \)
  - \( \theta = 0.5 \)
  - \( S = 0.5 \)

- At \( r_e = K_r / (K_r l) \)
  - \( \vec{V} = 0 \)
  - \( \theta = -0.5 \)
  - \( S = -0.5 \)

- At \( z = 0 \) and \( z = \Lambda = H/d \)
  - \( \vec{V} = 0 \)
  - \( \frac{\partial \theta}{\partial z} = 0 \)
  - \( \frac{\partial S}{\partial z} = 0 \)

At the permeable interfaces, continuity of the variables and their derivatives is considered.

\[ u_f = u_p \quad v_f = v_p \quad w_f = w_p \]
Heat and mass transfers are evaluated with Nusselt and Sherwood numbers. The average Nusselt number is computed for this geometry at the inner cylinder as in Khellaf and Lauriat [4].

\[
Nu = \frac{r_i}{A} \ln \left( \frac{K_r}{\theta} \right) \left[ \frac{1}{\theta} \left( \frac{\partial \theta}{\partial r} + u \theta \right) \right] dz
\]  

Based on the similarity between heat and mass transfer [7], Sherwood number is written as:

\[
Sh = \frac{r_i}{A} \ln \left( \frac{K_r}{\theta} \right) \left[ \frac{1}{\theta} \left( \frac{\partial \theta}{\partial r} + u \theta \right) \right] dz
\]  

3 Numerical procedure

The set of equations (1) to (6) with the corresponding boundary conditions are discretized by use of the control volume method based on the Power Law Differencing Scheme (PLDS). The SIMPLER algorithm, Patankar [8], is adopted to treat the coupling between velocity and pressure. The solution is assumed to be reached during the procedure, when the maximum relative error on the dependant variables \( \theta \) (velocity components, pressure, temperature and concentration) is less than \( 10^{-3} \).

\[
Max \left| \frac{\phi^{n+1} - \phi^n}{\phi^n} \right| \leq 10^{-3}
\]  

A grid sensitivity has been conducted and showed that a 51x51 grid was sufficient to describe accurately this transfer phenomenon in forced convection. A mesh refinement test has shown that a 71x71 grid yields only a variation of 0.21% in the average Nusselt and Sherwood numbers.

Table 1. Comparison of average Nusselt numbers

<table>
<thead>
<tr>
<th>Pr</th>
<th>Present study</th>
<th>Nu Khellaf and Lauriat [4]</th>
<th>Relative difference in %</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.71</td>
<td>1.069</td>
<td>1.069</td>
<td>0.0</td>
</tr>
<tr>
<td>5</td>
<td>2.057</td>
<td>2.055</td>
<td>0.1</td>
</tr>
<tr>
<td>10</td>
<td>2.685</td>
<td>2.667</td>
<td>0.3</td>
</tr>
<tr>
<td>100</td>
<td>6.254</td>
<td>6.165</td>
<td>1.44</td>
</tr>
<tr>
<td>200</td>
<td>8.090</td>
<td>7.862</td>
<td>2.9</td>
</tr>
<tr>
<td>300</td>
<td>9.389</td>
<td>9.054</td>
<td>3.6</td>
</tr>
</tbody>
</table>
The computer code has been validated in the fully fluid case by setting $\varepsilon=1$ and $\text{Da}\to\infty$. The comparison showed a perfect agreement with numerical results [4]. A difference less than 3.6% in Nusselt number has been obtained.

4 Results and discussion

Effects of several geometrical and thermophysical parameters on the flow patterns and heat and mass transfer coefficients are analysed and documented. Results are presented for the following ranges $0 \leq \text{Re} \leq 450$, the aspect ratio $0.25 \leq A \leq 10$, the porous filter permeability $10^{-5} \leq \text{Da} \leq 10^2$ and a porosity of 0.4. The porous matrix occupies 30% of the annular gap which is filled by humid air ($\text{Pr}=0.71$ and $\text{Le}=0.86$). A moderate curvature of the cavity is considered with a radius ratio $K_r=5$.

![Streamlines, Isotherms, Isoconcentrations](image)

Figure 2: Streamlines, Isotherms and Isoconcentrations, $\Theta$ and $S$: $(-0.5, 0.05, 0.5)$, $\text{Re}=100$, $A=1$, $R_1=0.35$, $R_k=R_v=1$. 

© 2002 WIT Press, Ashurst Lodge, Southampton, SO40 7AA, UK. All rights reserved. 
Web: www.witpress.com Email witpress@witpress.com

ISBN 1-85312-906-2
Streamlines, isotherms and isoconcentrations are first presented in figures 2 and 3. For a cavity with an aspect ratio of 1, a pair of contra-rotative Taylor vortices appears in centre of the cavity. Due to centrifugal forces the fluid is ejected from the inner to the outer cylinder in the centre of the enclosure, and the viscous forces make the fluid flows inward near the top and bottom disks. Consequently, a distortion in the temperature and concentration fields is observed at midheight of the cavity close to the inner cylinder as shown in figure 2. As permeability of the porous matrix is reduced, the flow is mostly confined in the inner fluid zone and a very weak motion is observed in the outer fluid region. Indeed, in the upper and lower regions of the cavity the flow is in both inward and outward directions yielding what is called an anomalous mode.

Figure 3: Streamlines, Isotherms and Isoconcentrations, $\theta$ and $S$: (-0.5,0.05,0.5), $Re=300$, $A=1$, $R_1=0.35$, $R_k=R_c=1$. 
Figure 3 illustrates the apparition of an instability at high rotation speed \( Re=300 \); the flow loses its symmetry. The introduction of the porous filter cancels the instability and the two fluid cells have their centres close to each other.

Angular velocity is plotted in figure 4 for different Reynolds numbers. One can notice that disymmetry appears at \( Re=250 \) in the upper zone of the cavity where the instability occurs.

![Angular velocity distribution in fluid case, \( w(0, (0.1), 1), A=1 \).](image)

Figure 4: Angular velocity distribution in fluid case, \( w(0, (0.1), 1), A=1 \).

Average heat and mass transfer coefficients are displayed in figure 5. At a low rotation of the inner cylinder (\( Re<50 \)), Nusselt and Sherwood numbers are constant and this characterises a mode of transfer by pure diffusion. Over a certain value of \( Re \) which depends on the permeability of the porous matrix both heat and mass transfer coefficients increase with \( Re \). This value is around 50 for the fluid case and 100 when the porous filter is inserted in the enclosure. In fact, the insertion of the porous matrix reduces Nusselt and Sherwood numbers except in the diffusive regime where \( Nu \) is increased by approximately 20%.

![Evolution of heat and mass transfer coefficients as functions of \( Re \), \( A=1, R_k=R_v=1 \).](image)

Figure 5: Evolution of heat and mass transfer coefficients as functions of \( Re \), \( A=1, R_k=R_v=1 \).
Figure 6: Isotherms, Streamlines and Isoconcentrations, $\theta$ and $S$: (-0.5, (0.1), 0.5), $Re=300$, $Da=10^3$, $R_k=R_v=1$. 

$10^5 \Psi : (-40, (5), 40) \quad 10^5 \Psi : (-8, (1), 8) \quad 10^5 \Psi : (-80, (8), 80) \quad 10^4 \Psi : (-60, (5), 70)$

$A=0.5 \quad A=1 \quad A=4 \quad A=8$
Taller cavities and the effect of the aspect ratio (A) are presented in figure 6. The anomalous multicellular flow pattern persists up to a moderate value of A (A=4). At this value vortices of square shape with a wavelength similar to the one given by the linear stability theory of Taylor [1] and Chandrasekhar [9] are obtained. This implies that the end effects are negligible for this aspect ratio when a porous filter is inserted. Over the value of A=4, the multicellular pattern is damaged due to the porous matrix resistance to the flow.

Figure 7 illustrates the average Nusselt and Sherwood numbers versus the aspect ratio A. The jump from n to n+2 cells during the bifurcation process is clearly depicted in the fluid case. As n increases, each couple of contra-rotative cells transports the same heat and mass flux from inner to outer cylinder. For instance, at the different bifurcation points, the additional cells do not perturb the initial system. The effect of the permeability is also illustrated. At Da=10^3, a considerable decrease in Nu and Sh is observed at the first bifurcation 2-4 cells, but then the variation is dumped even if the periodic structure remains. However, at Da=10^5, the fluid does not penetrate easily the porous zone.

Figure 7: Representation of Nu and Sh Versus A for Re=300, Rk=Rv=1.

5 Conclusion

A numerical analysis is carried out to investigate the effect, on combined heat and mass transfer and flow structure, of the introduction of a porous filter in a vertical annular cavity.

Results showed that the porous matrix cancels the effect of the centrifugal instability and allows a centrifugal bicellular flow more appropriate in a filtration process. By varying parameters such as Re (rotation of inner cylinder), A (aspect ratio of the cavity) or Da (permeability of porous filter), anomalous modes of the flow structure are depicted and the multicellular flow pattern is damaged at a relatively small aspect ratio in comparison to the fluid case.
References


