Combined natural convection and radiation heat transfer in an open tilted cavity

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Abstract

Combined natural convection and radiation in open cavities have various applications ranging from passive solar heating of rooms to solar concentration. Earlier investigators have solved the problem considering only natural convection heat transfer in open cavities [1,2,3], but they do not include the interaction of radiation combined with natural convection heat transfer. This study presents a two dimensional computational model of the interaction of natural convection and radiation heat transfer in an open tilted square cavity containing air as a non participant fluid. The cavity has two opaque adiabatic horizontal walls and one opaque isothermal vertical wall. The momentum and energy equations were solved by finite differences using ADI technique. Coupled to this equations are the radiative energy flux boundary conditions. The average heat transfer coefficient was calculated for several values of the Grashof number for air (Pr=0.7) for different tilted angles, with the isothermal wall at 500 K and the open side at 300 K. The results indicate that the radiation and the tilted angle have great influence on the total amount of heat lost by the cavity as well as on the pattern of flow and temperature fields.

Keywords: natural convection, radiation heat transfer, open cavity, total Nusselt number.
1 Introduction

The heat transfer in cavities has been widely studied due to its importance in several engineering problems such as passive heating and cooling in buildings, cooling of electronics devices, and solar collectors among others.

Several studies have been done in cavities, in particular the natural convection in close cavities has been studied with great attention as can be seen in the recompilation work of Catton in 1978 [4] and Ostrach in 1988 [5]. However, with respect to open cavities, there are a small number of articles that deal with the subject and there are a fewer number that consider the combined effect of convection and radiation, Balaji [6]. Cabanillas in 2001 [3] made a widely revision of the reported articles on the subject for different conditions of the cavities. The present work is an effort to understand the basic phenomena of heat transfer that occurs naturally between a heated 2D open cavity and its surroundings when the convection and radiation mechanisms are taking into account for different title angles.

2 Description of the problem

Consider an open rectangular cavity with three walls; two adiabatic and one isothermal as it is shown in figure 1. The cavity is filled with air (Pr = 0.7), which is consider as an incompressible newtonian fluid with constant properties, except for the density in the buoyancy term of the energy equation which is allowed to vary (Boussinesq approximation). The fluid is initially static and with a dimensionless temperature of $T = 0$. At time $\tau = 0$ the isothermal wall is turned on with $T = 1$ and the heat transfer process starts by convection from the heated wall to the fluid, as well as by radiation from the heated wall to the adiabaic walls and the opening. The angle of the cavity ($\phi$) is fixed but it can take several values from 0° when the aperture of the cavity sees upwards to 180° when it sees downwards.

![Figure 1. Rectangular cavity with an open side.](image-url)
3 Governing Equations

The governing equations are those of the conservation of mass, momentum and energy for a square cavity with radiative exchange at the boundaries. Since the streamfunction-vorticity formulation is used, the continuity equation is automatically satisfied and the pressure is eliminated as a solution variable. The equations are expressed in a dimensionless form as:

\[ \nabla^2 \psi = -W \]  
\[ \frac{\partial W}{\partial t} + U \frac{\partial W}{\partial X} + V \frac{\partial W}{\partial Y} = \frac{1}{Gr^{1/2}} \left[ \nabla^2 W \right] + \left( \frac{\partial T}{\partial X} \sin \phi - \frac{\partial T}{\partial Y} \cos \phi \right) \]  
\[ \frac{\partial T}{\partial t} + U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} = \frac{1}{Pr \ Gr^{1/2}} \nabla^2 T \]

The velocities are related to the streamfunction by

\[ U = \frac{\partial \psi}{\partial Y}, \quad V = -\frac{\partial \psi}{\partial X} \]

The dimensionless variables were chosen as follow:

\[ X = \frac{x}{L}, \quad Y = \frac{y}{H}, \quad U = \frac{u}{U_c}, \quad V = \frac{v}{U_c}, \quad P = \frac{p}{\rho U_c^2}, \quad T = \frac{T - T_a}{T_H - T_a}, \quad t = t \frac{U_c}{H} \]

where

\[ U_c = \sqrt{g \beta \Delta T \ H} \]

The initial conditions in the cavity are the following,

For \( \tau = 0 \) and \( 0 < X < 1, \ 0 < Y < 1, \)
\[ W(X,Y,0) = 0, \quad \psi(X,Y,0) = 0, \quad U(X,Y,0) = V(X,Y,0), \]
\[ T(X,Y,0) = 0, \quad T(0,Y,0) = 1.0. \]

The hydrodynamic boundary conditions for the walls 1, 2 and 3 are given in terms of the streamfunction as:

for \( \tau = \tau \) and \( 0 < X < 1, \ 0 < Y < 1, \)
\[ \psi (X,0,\tau) = 0; \quad U(X,0,\tau) = V(X,0,\tau) = 0 \]
\[ \psi (X,1,\tau) = 0; \quad U(X,1,\tau) = V(X,1,\tau) = 0 \]
\[ \psi (0,Y,\tau) = 0; \quad U(0,Y,\tau) = V(0,Y,\tau) = 0 \]
The boundary conditions on vorticity are not known explicitly, but are determined by the Taylor series expansion of the streamfunction in the vicinity of the wall.

The following energy balances give the temperature boundary conditions for walls 1 and 3:

\[ q_{c1} = q_{r1} \]  \hspace{1cm} \text{(7)}

where \( q_{c1} \) is the heat flux gave by the wall to the fluid through conduction-convection and \( q_{r1} \) is the radiative heat exchange between that wall and the others including the opening side.

Thus, for wall 1 (adiabatic wall),

\[ k_3 \left[ \frac{\partial T}{\partial y} \right]_{y=0} = q_{r1} \]

defining \( N_r \) and \( Q_{r1} \) as

\[ N_r = \frac{\sigma T_1^4 L}{k_3 (T_H - T_1)} \]

\[ Q_{r1} = \frac{q_{r1}}{\sigma T_H^4} \]

and substituting, it is obtained the dimensionless form

\[ \left[ \frac{\partial T}{\partial y} \right]_{y=0} = N_r Q_{r1} \]  \hspace{1cm} \text{(8)}

Similarly, for wall 3 (adiabatic wall),

\[ q_{c3} = q_{c3} \], and

\[ \left[ \frac{\partial T}{\partial y} \right]_{y=0} = N_r Q_{r3} \]  \hspace{1cm} \text{(9)}

For wall 2 the thermal boundary condition is

\[ \text{For } 0 \leq Y \leq 1 \text{ and } X = 0, \ T = 1. \]  \hspace{1cm} \text{(10)}

The condition for the open side are as follow. The fluid entering the cavity has a zero temperature because it is the exterior temperature. For the fluid that leaves the cavity, it is assumed that convection dominates over conduction. It was found that the vorticity boundary conditions that are similar to the temperature ones
have a better convergence [7]. Also, for the stream function, it is assumed that \( \partial \psi / \partial X = 0 \), which causes that \( V = 0 \), this means that the streamlines at the cavity’s entrance are parallel. Thus, the open side boundary conditions are:

for \( 0 \leq Y \leq 1 \) and \( X = 1 \),

\[
\frac{\partial \psi}{\partial X} = 0, \quad V = 0, 
\]

(11)

if the fluid enters the cavity

\[ T = 0, \quad W = 0 \]  \hspace{1cm} \text{(12)}

if the fluid leaves the cavity,

\[
\frac{\partial T}{\partial X} = 0, \quad \frac{\partial W}{\partial X} = 0.
\]

(13)

To solve the radiative balance the radiosity formulation is used with the same grid that it is used in the convective problem. Thus, for each element of a wall the radiosity is given by

\[
J_k = \varepsilon_k + (1 - \varepsilon_k) \sum_{j=1}^{\text{NTP}} F_{kj} J_j
\]

(14)

where \( F_{kj} \) is the view factor from the k-element to the j-element.

For a given wall formed by n elements, the net heat transferred by radiation (\( q_r \)) will be given by

\[
q_{r1} = J_1 - q_1
\]

(15)

where \( q_{r1} \) is the net heat for wall 1, \( J_1 \) is the radiative energy that leaves that wall and \( q_1 \) is the amount of radiation received on wall 1.

The average Nusselt number is given by

\[
\text{Nu}_t = \text{Nu}_c + \text{Nu}_r
\]

(16)

Where \( \text{Nu}_c \) is the Nusselt number for convection and \( \text{Nu}_r \) is for radiation, which are defined by
Given by convection heat transfer from the same wall. Then, the total Nusselt number is

\[ \frac{q_{\text{rad}}}{q_{\text{conv}}} \] 

Where \( q_{\text{rad}} \) is the heat transferred by radiation by the heated wall and \( q_{\text{conv}} \) is the convection heat transfer from the same wall. Then, the total Nusselt number is given by

\[ \frac{\text{Nu}_c}{\text{Nu}_r} = 1 + \frac{q_{\text{rad}}}{q_{\text{conv}}} \]

4 Method of Solution

Governing equations (1)-(3), along with the radiative flux equations and their initial and boundary conditions define the problem completely. The equations are all coupled through the boundary conditions and/or the variables. Forward time and central space differences approximated the derivatives of the partial differential equations (2) and (3). The ADI (Alternating-Direction-Implicit) finite difference technique was used. The streamline equation (1) was solved by the method of Paceman and Rachford [8]. The radiosity equations were solved by the method of successive approximation. The integrals were evaluated by the Simpson’s rule. The net radiative heat fluxes were calculated from the radiative net heat flux equation (15).

4.1 Validation

The numerical solution was validated with convergence and stability tests and by a comparison with reported results in the literature. The convergence and stability tests that are detailed described by Cabanillas [3] showed that the numerical scheme was stable in a wide range of Grashof numbers (Gr). To validate the numerical code of the present work, for the convective case only, the results published by other authors were considered. Table 1 shows a comparison between those Nusselt numbers (Nu) obtained by different authors including the present study for different Rayleigh numbers (Ra) when the cavity is 90° oriented.

The great similarity between these values show that the numerical procedure and the code are correct for the case analysed. It is important to mention that Chan-Tien [9] and Mohamad [10] used the SIMPLER numerical method in primitive variables (with very little variation between them).

For a \( \text{Gr} = 10^6 \) and \( \text{Pr} = 1.0 \), Angirasa [7], Comini [11] and Balaji [6] reported Nu values of 14.39, 14.9 and 14.4 respectively. These Nu values are in agreement with the 14.5 found in the present study.
Table 1. Nusselt number comparison for the convective case. The cavity is oriented 90°.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1 x 10³</td>
<td>1.07</td>
<td>1.31</td>
<td>1.10</td>
</tr>
<tr>
<td>1 x 10⁴</td>
<td>3.41</td>
<td>3.44</td>
<td>3.07</td>
</tr>
<tr>
<td>1 x 10⁵</td>
<td>7.69</td>
<td>7.41</td>
<td>7.16</td>
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<tr>
<td>1 x 10⁶</td>
<td>15.00</td>
<td>14.36</td>
<td>14.50</td>
</tr>
<tr>
<td>1 x 10⁷</td>
<td>28.60</td>
<td>28.60</td>
<td>26.80</td>
</tr>
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</table>

Also, considering only natural convection heat transfer, the Nu obtained for different cavity orientations with Gr = 1.5 X 10⁴ and Pr = 0.7 (Ra=1x10⁴) were compared. Table 2 shows the reported values by Mohamad [10] and the ones obtained in this study. The agreement between the values is evident.

Table 2. Nusselt number comparison for the convective case at different cavity’s title angles.

<table>
<thead>
<tr>
<th>Tilt</th>
<th>Nu Mohamad [10]</th>
<th>Nu Present study</th>
</tr>
</thead>
<tbody>
<tr>
<td>10°</td>
<td>2.57</td>
<td>2.82</td>
</tr>
<tr>
<td>30°</td>
<td>3.34</td>
<td>3.43</td>
</tr>
<tr>
<td>60°</td>
<td>3.70</td>
<td>3.69</td>
</tr>
<tr>
<td>90°</td>
<td>3.44</td>
<td>3.10</td>
</tr>
</tbody>
</table>

The radiative calculation procedure was validated for the isothermal cavity without convection. The typical case of a cavity with three isothermal walls at the same temperature and with an emissivity of 1 (ε = 1), that is, the cavity behaves like a black body, is considered. If the temperatures of the walls are 500 °C and the surrounding are at 300 °C, then it is expected that the radiative heat leaving the cavity is 3,084.48 W/m², while the code calculates a value of 3,084.4818 W/m². This result shows a 0.2 % difference between the calculated value with a standard procedure and the procedure which divides each wall of the cavity with
30 segments. When the radiative part is coupled with the convective part, both work with the same number of surface subdivisions. This subdivisions are determined by the mesh size used.

**5 Results**

The parameters used in the simulations were $T_w = 500$ K (wall temperatures), $T_a = 300^\circ$K (outside air temperature), $Pr = 0.7$, emissivity of 1 ($\varepsilon = 1.0$) for all the three walls of the cavity and $\varepsilon = 1.0$ for the cavity’s open side. The tilted angle of the cavity was varied from 0° (upwards cavity) to 180° (downwards cavity) with increments of 45°. The results presented correspond to the steady state.

Figure 2 shows a plot of the Nu numbers vs cavity’s tilt angles for a $Gr = 10^5$. The values of $Nu_c$, $Nu_r$, and $Nu_t$ are plotted. The effect of the orientation angle of the cavity can be analyzed in three different ways. The first one is about the magnitude between the two components: $Nu_c$ can be greater than $Nu_r$, at least for the extreme case considered (where the walls’ emissivity was assumed 1, $\varepsilon = 1.0$); for smaller values of $\varepsilon$ it is expected that $Nu_r$ must be smaller. Second, it exists a maximum for the convective component of the Nu which occurs before 90°.

![Figure 2. Nu_c, Nu_r, and Nu_t numbers vs cavity’s tilt angles for a Gr = 10^5.](image)

For tilt angles between 0° and 90° the cavity is looking upward and this facilitated the convective movement of the air caused by the floating forces. For
greater angles the convection is strongly diminished getting a minimum at 180° with \( \text{Nu}_c \) close to unit. Third, the radiative component of the \( \text{Nu} \) is not constant and it present a small variation, this indicates the coupling of the to heat transfer mechanism in the cavity.

Figure 3 shows a comparison of isotherms (right) and streamlines (left) for \( \text{Gr} \) of \( 10^5 \) and \( \phi = 0°, 45°, 90°, 135°, 180° \). From this figure, the following can be observed:

1. When \( \phi = 0° \) (aperture upwards), the stream function presents the formation of two cells turning in opposite ways, showing that the cold fluid is entering at the center of the cavity and leaving at the walls.

2. For tilt angles between 45° and 135°, streamlines presents the formation of fluid cells. These orientations allow the fluid to circulate inside the cavity having the buoyancy as the driving force.

3. When \( \phi = 180° \) (aperture downwards), the fluid that has been heated and has low density can not move upwards in the vertical direction (stagnant fluid), this causes the stratification of the thermal boundary layer that fulfill the cavity. The \( \text{Nu}_c \) presents a minimum value, as mentioned before. However, an interesting effect is observed in the streamlines where appears four cells which turn in opposite directions, allowing an incipient convective heat transfer to the exterior.

Table 3 shows \( \text{Nu} \) with its components \( \text{Nu}_c \) and \( \text{Nu}_r \) for different values of \( \text{Gr} \) and varying the tilt angle of the cavity. This table shows that the effect of the tilt angle on \( \text{Nu}_c \) mentioned before for \( \text{Gr} = 10^5 \) is repeated for the other \( \text{Gr} \) studied. It is notorious the increase of the \( \text{Nu}_c \) with the increase of the \( \text{Gr} \). It is important to mention that for some tilt angles it was not possible to converge the numerical solution getting an oscillated behavior of the \( \text{Nu}_c \). It is intended to study in the near future this issue. In this table the \( \text{Nu}_c \) presented are the average with their corresponding standard deviation.

Table 3. \( \text{Nu} \), \( \text{Nu}_c \) and \( \text{Nu}_r \) for different \( \text{Gr} \) at different tilt angles of the cavity.

<table>
<thead>
<tr>
<th>Angle in degrees</th>
<th>10^4</th>
<th>10^5</th>
<th>10^6</th>
<th>10^7</th>
<th>10^8</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Nu</td>
<td>Nu</td>
<td>Nu</td>
<td>Nu</td>
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<tr>
<td>0°</td>
<td>1.04</td>
<td>4.24</td>
<td>0.78</td>
<td>5.42</td>
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<tr>
<td>45°</td>
<td>2.83</td>
<td>4.8</td>
<td>0.63</td>
<td>4.79</td>
<td>0.2</td>
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<tr>
<td>90°</td>
<td>2.44</td>
<td>4.69</td>
<td>1.13</td>
<td>6.46</td>
<td>1.06</td>
</tr>
<tr>
<td>135°</td>
<td>1.28</td>
<td>4.42</td>
<td>0.7</td>
<td>2.63</td>
<td>0.85</td>
</tr>
<tr>
<td>180°</td>
<td>1.45</td>
<td>4.45</td>
<td>1.00</td>
<td>2.57</td>
<td>1.12</td>
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</table>
Figure 3. Comparison of isotherms (right) and streamlines (left) for \( \text{Gr} = 10^5 \) and \( \phi = 0^\circ, 45^\circ, 90^\circ, 135^\circ, 180^\circ \).
6 Conclusions

A transient two-dimensional mathematical model of a combined natural convection and radiation in an open square cavity containing air as a laminar and non-participating fluid has been presented. Two opposite walls were adiabatic and other opposite to the cavity opening was isothermal. The convective part of code was validated by comparison with the results of other authors.

From the results, and according with previous studies, there is a high dependence of the convective Nusselt number with respect to the Grashof number. Also, there is a high dependence of the convective Nusselt number with respect to the tilted angle of the cavity. In most cases, the angle that best help the natural convection process is 45°. In complementary form, the angle that best reduces the natural convection process is 180°. Therefore, the lowest values of the Nu occur when the cavity is downwards, while the higher values occur, in most cases, when the tilt angle is 45°.

In all cases analyzed, with ε = 1.0, the Nu was greater than Nu indicating that the radiative transport in the heat transfer from the cavity can not be neglected. It is reasonable to assume that for other values of ε this number will change, however, it is demonstrated how high the radiative heat transfer can have. Also, it is important to mention that the Nur is not constant, showing the coupling of the radiative and convective heat transfer. More research is needed to investigate this coupling.

Nomenclature

<table>
<thead>
<tr>
<th>Latin</th>
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<tr>
<td>G</td>
<td>α</td>
<td>Gravity, m/s²</td>
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<td>Gr</td>
<td>β</td>
<td>Grashof number, gβΔT L³/ν²</td>
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<tr>
<td>H</td>
<td></td>
<td>Height of cavity, m</td>
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<tr>
<td>Ji</td>
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<td>Ψ</td>
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References


