The effect of aspect ratio on the development of convective motion in a bottom heated enclosure containing ice and water

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Abstract

A numerical study of the steady state flow in a rectangular enclosure with two vertical walls which are adiabatic and with two horizontal isothermal walls has been undertaken. The enclosure contains water and the upper wall is maintained at a uniform temperature that is below the freezing point of water while the lower wall is maintained at a uniform temperature that is above the freezing point of water. The upper portion of the enclosure is thus filled with ice and the lower portion is filled with water. The conditions considered in the present study are such there can be significant natural convection in the water and the effect of the density maximum that exists in the water at approximately 4°C can have a significant effect on this flow. The main aim of the study was to determine how far above 4°C the hot wall temperature can be before significant convective motion develops in the water. The governing equations have been expressed in dimensionless form and solved using a finite element procedure. The effect of the various governing parameters, and in particular the effect of the enclosure width-to-height ratio, on the value of the hot wall temperature at which convective motion commences has been considered.

Nomenclature

\[ A = \text{aspect ratio of enclosure, i.e. } \frac{W'}{H'} \]
1. Introduction

The flow in an enclosure with two vertical walls which are adiabatic and
with two horizontal isothermal walls has been considered. The enclosure contains water and the upper wall is maintained at a temperature that is below the freezing point of water while the lower wall is maintained at a temperature that is above the freezing point of water. The upper portion of the enclosure is thus filled with ice and the lower portion is filled with water. The flow situation considered is shown schematically in Fig. 1.

![Figure 1: Flow situation considered.](image)

The present study was undertaken in support of experimental studies of the nature of the ice that forms under various conditions. These studies require that there be no convective motion in the water during the freezing. Because water has a density maximum at approximately 4°C, the experiments are usually carried out with the hot wall at the bottom of the enclosure and with this wall kept at a temperature below 4°C. However, this limits the range of conditions that can be covered in the experimental work. The question therefore arises as to how far above 4°C can the hot wall temperature be before significant convective motion develops in the water. The present numerical study was undertaken, basically, to provide a partial answer to this question and in particular to determine whether the width-to-height ratio of the enclosure will have a significant influence on this result.

There have been many previous studies of solidification and melting of liquids in enclosures. Most of these studies have, however, been concerned with the evolution of the flow with time and have not been concerned with a detailed study of the effects of the various governing parameters on the final steady state for the case where there is under-cooling. A review of much of this work is given in [1] and [2]. A numerical study of the particular case of the freezing of pure water in a rectangular enclosure is described in [3], this paper also providing a review of past work on the subject. Nu-
numerical studies of steady state freezing of water in a rectangular enclosure are described in [4] and [5], for example. Experimental studies of freezing in an enclosure for situations in which the density maximum is important are described in [6], [7] and [8]. These existing studies do not give much information about the condition under which natural convective motion will be important under the circumstances here being considered and it is for this reason that the present study was undertaken. The present study is an extension of that described in [9] in which attention was restricted to a square enclosure.

2. Governing Equations and Solution Procedure

It has been assumed that if convective motion develops, the flow remains laminar and can be treated as two-dimensional and that the fluid properties can be assumed constant except for the density change with temperature which gives rise to the buoyancy forces. The relation between the density and the temperature was assumed to be:

\[ \frac{(\rho'_{\text{max}} - \rho')}{\rho'} = \alpha'(T' - T'_{\text{max}})^2 \]  

(1)

the subscript max referring to conditions at the maximum density temperature.

The solution for the water, in which the natural convection has been assumed to be important, has been obtained in terms of the stream function and vorticity, \( \psi' \) and \( \omega' \). The prime (') denotes a dimensional quantity.

The following dimensionless variables have then been defined:

\[ \phi = \phi'/\alpha, \quad \omega = \omega' H'^2/\alpha, \quad x = x'/H', \quad y = y'/H' \]  

(2)

\[ t = t'/\alpha H'^2, \quad T = (T' - T'_{\text{F}})/(T'_{H} - T'_{C}) \]  

(3)

where \( \alpha \) is the thermal diffusivity of the water, and \( T'_{F} \) is the freezing temperature. The prime (') denotes a dimensional quantity.

In terms of these dimensionless variables, the governing equations for the liquid flow are:

\[ \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\omega \]  

(4)

\[ \left( \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right) - \frac{1}{Pr} \left( \frac{\partial \psi \partial \omega}{\partial x \partial y} - \frac{\partial \psi \partial \omega}{\partial x} \frac{\partial \omega}{\partial y} + \frac{\partial \omega}{\partial t} \right) = -2Ra^* \frac{\partial T}{\partial x} \]  

(5)

\[ \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) - \left( \frac{\partial \psi \partial T}{\partial y} - \frac{\partial \psi \partial T}{\partial x} \frac{\partial T}{\partial y} + \frac{\partial T}{\partial t} \right) = 0 \]  

(6)

Here \( Ra^* \) is the modified Rayleigh number defined by:

\[ Ra^* = \frac{a'g'W'\beta(T'_{H} - T'_{C})^2}{\nu' \alpha'} \]  

(7)
The equation governing the temperature distribution in the solid phase is:

\[
\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{\partial T}{\partial t}
\]

(8)

The boundary conditions on the solution are as follows:

On all enclosure surfaces:

\[
\psi = 0, \quad \frac{\partial \psi}{\partial n} = 0
\]

(9)

where \( n \) is the coordinate measured normal to the surface considered.

On vertical side walls:

\[
\frac{\partial T}{\partial x} = 0
\]

(10)

On the bottom and top surfaces:

\[
T = T_H, \quad T = T_C (= T_H - 1)
\]

(11)

respectively.

On the interface between the water and ice the following conditions apply:

\[
\psi = 0, \quad \frac{\partial \psi}{\partial n} = 0, \quad T = 0, \quad \left. \frac{\partial T}{\partial n} \right|_l = \left( \frac{k_s}{k_f} \right) \left. \frac{\partial T}{\partial n} \right|_s
\]

(12)

where the subscripts \( l \) and \( s \) refer to conditions on the liquid and solid sides of the interface respectively.

The above dimensionless equations, i.e. eqs. (4) to (6) and eq.(8), have been solved using a finite element procedure. The solution was started by assuming that there is no motion in the water layer and that the one-dimensional pure conduction solution applied. The solution was continued in dimensionless time for a sufficiently long dimensionless time to ensure that either no convective motion would develop or until a steady state flow had developed. As the solution progresses, the interface position is locally modified according to the difference between the calculated rates of heat transfer at the interface on the solid and liquid sides, the element shapes being adaptively modified to follow the changing interface shape.

The solution for the temperature distribution allows the local heat transfer rate over the upper and lower surfaces to be determined. The local heat transfer rate distribution can then be integrated to give the mean heat transfer rates for the upper and lower surfaces. The mean heat transfer rate has been expressed in terms of the mean Nusselt number, \( Nu \), based on the overall temperature difference and on the enclosure height, i.e.:

\[
Nu = \frac{q' H'}{k' (T'_H - T'_C)}
\]

(13)
Calculations have been carried out with various nodal point numbers and distributions and the results obtained indicate that the results presented in this paper are grid-independent to better than 1 per cent.

Now under some conditions there is no convective motion in the liquid, the basic purpose of this study being to determine when this is the case. In this situation the interface between the solid and the liquid is flat and parallel to the top and bottom walls (see Fig. 2) and the heat transfer in both the solid and the liquid is by conduction so that:

$$q' = \frac{k'(T'_H - T'_P)}{H'_t} = \frac{k_r k'(T'_P - T'_C)}{H'_s}$$  \hspace{1cm} (14)

where $H'_t$ and $H'_s$ are the thickness of the liquid and solid layers as indicated in Fig. 2. Using this equation and noting that:

$$H'_t + H'_s = \frac{H'}{H'_t}$$

it follows by using:

$$T_H - T_C = 1$$

that when there is no motion in the liquid:

$$Nu = 4 - 3T_H$$  \hspace{1cm} (15)

3. Results

The solution has the following parameters:

- the enclosure aspect ratio, $A$
- the modified Rayleigh number, $Ra^+$. 

Figure 2: Situation when there is no convective motion in the liquid.
the Prandtl number, \( Pr \)
- the ratio of the thermal conductivities of the solid and the liquid, \( k_r \)
- the dimensionless hot bottom wall temperature, \( T_H \)
- the dimensionless temperature at which the maximum density occurs, \( T_{max} \).

Results have been obtained here for a Prandtl number of 12 and a conductivity ratio, \( k_r \), of 4. The value of the dimensionless maximum density temperature, \( T_{max} \), basically will be determined by the difference between the hot and cold wall temperatures because \( T'_{max} - T'_F \) is approximately 4 °C for water, \( T'_F \) being the solidification temperature.

Figure 3 shows typical variations of Nusselt number with dimensionless hot wall temperature. It will be seen that even at dimensionless temperatures well above the dimensionless maximum density temperature there is no significant convective motion. However, at the points indicated as P in Figure 3, significant motion develops with the result that the thickness of the ice layer decreases and that of the water layer increases which leads to an intensification of the liquid motion. As a result, there is a sharp increase in Nusselt number at point P. The dimensionless hot wall temperature at which significant convective motion commences will be designated as \( T_{HP} \).

![Graph](image.png)

**Figure 3:** Typical variation of mean Nusselt numbers with dimensionless hot wall temperature.

The basic aim of the present study is of course to determine \( T_{HP} \) since
below this value of the dimensionless wall temperature there is no motion in the liquid.

The effect of dimensionless enclosure aspect ratio, maximum density temperature, and modified Rayleigh number on the value of $T_{HP}$ has been studied. Typical variations of $T_{HP}$ with enclosure aspect ratio for a fixed Rayleigh number for various values of $T_{max}$ are shown in Fig. 4. Similarly, typical variations of $T_{HP}$ with $T_{max}$ for various enclosure aspect ratios for a fixed Rayleigh number are shown in Fig. 5 while typical variations of $T_{HP}$ with $Ra^*$ for various enclosure aspect ratios for a fixed value of $T_{max}$ are shown in Fig. 6. The results given in these figures clearly show that

![Figure 4: Typical variations of dimensionless hot wall temperature at which convective motion commences with enclosure aspect ratio.](image)

convective motion does not develop in the liquid until the dimensionless hot wall temperature is considerably higher than the dimensionless maximum density temperature and that the value of the aspect ratio has a much weaker effect on $T_{HP}$ than does the value of $Ra^*$ or the value of $T_{max}'$. Figures 4, 5 and 6 together can be used to estimate the conditions when the liquid motion will occur.

4. Conclusions

The results of the present study indicate that convective motion does not develop until the lower hot wall temperature is well above the maximum density temperature. The results also show that the enclosure aspect ratio has a relatively weak effect on the dimensionless hot wall temperature at
Figure 5: Typical variations of dimensionless hot wall temperature at which convective motion commences with dimensionless maximum density temperature.

which convective motion develops. The results obtained in this study can be used to determine the actual temperature at which significant convective motion will develop.

5. Acknowledgments

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6. References


Figure 6: Typical variations of dimensionless hot wall temperature at which convective motion commences with modified Rayleigh number.


