Structural performance simulation using finite element and regression analyses

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Abstract

For large-scale structural optimization problems the time and number of finite element analyses required to achieve the optimum solution can be prohibitive. For these structures, with a limited number of design variables the ability of determining a performance parameter over the full range of the design variable variation with a small number of finite element analyses can be very valuable.

One of the main problems in performing finite element analysis based on the need of the optimization routine is that the full potential of each analysis to achieve a complete picture of the design space is never realized. By combining the information of several well-spaced analyses outside the optimization process, the full shape of the relevant performance parameters over the specified design range can be achieved within the required accuracy.

In this paper, in order to recover the stress and deflection responses necessary for the numerical optimization of large-scale structures an approach using finite element analyses in conjunction with regression analysis was employed to achieve the relationships over the design range. To study the validity and efficiency of this approach the structural performance of a pressure vessel cover plate is used as a test case.

1 Introduction

The desire to develop and manufacture a product that is of superior performance than its predecessor is a major driving force in engineering design. As a result, design tools that allow the attainment of these goals in a timely and economical fashion have become essential in the design process. Over the past forty years,
the development of numerical optimization techniques has been instrumental in this context.

Since the pioneering work by Schmit in 1960, the use of numerical optimization techniques in engineering design has gained widespread acceptance and popularity [1-9]. With increase in the scale and size of the structures the time and number of finite element evaluations becomes a determining factor in the ability of solving the optimization problem. In this work we study the accuracy and validity for the combined use of finite element and regression analyses as an approach for addressing this issue. The design of a pressure vessel cover plate was chosen as a test case to perform the study.

2 Structural performance criteria

Structural performance whether is linear or nonlinear static or dynamic is usually focused on the two major design criteria strength and stiffness. The strength design criteria is based on the simple premise that the load induced stress at the critical location of the structure should be less than or equal to a maximum allowable/permisssible stress. Similarly, the stiffness design criteria is based on the premise that the load induced deformations at the critical location should be less than or equal a maximum allowable value.

To study the ability of simulating these structural performance criteria using finite element and regression analysis the linear static performance of a pressure vessel cover plate [10,11], was considered. Table 1, represents the summary of the structural performance data obtained from finite element modeling and analysis of the cover plate. The stress plot of one of these analyses is presented in Figures 1. The finite element analysis of the cover plate was performed using ALGORE software package. For loading conditions the plate was subject to a uniformly distributed design pressure along the Z direction having a magnitude of 4.2 Mpa. Boundary conditions required that the plate’s edges be securely clamped. This meant that nodes along the plate’s periphery were restricted so as to experience no translation or rotation. The plate material is Isotropic SA-515-70 grade carbon steel.

<table>
<thead>
<tr>
<th>Analysis #</th>
<th>Thickness h, mm</th>
<th>Maxi. Principal Stress, MPa</th>
<th>Max Deflection, z,mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>1150</td>
<td>2.622</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>288</td>
<td>0.330</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>128</td>
<td>0.098</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td>72.2</td>
<td>0.042</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
<td>46.2</td>
<td>0.020</td>
</tr>
<tr>
<td>6</td>
<td>30</td>
<td>32.1</td>
<td>0.012</td>
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<td>7</td>
<td>35</td>
<td>23.5</td>
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</tr>
<tr>
<td>8</td>
<td>40</td>
<td>18.0</td>
<td>0.005</td>
</tr>
</tbody>
</table>
Eight finite element analysis cases were performed in terms of both maximum principal stress and deflection. In each case, a 3-D model was generated using 224 three-dimensional plate/shell elements with 210 nodes. Regression analysis was performed using the cover plate maximum stress and deflection finite element results.

Figure 1: Stress contour for cover plate with 10mm thickness.

Figure 2: Maximum principal stress versus thickness.
3 Regression model

Successful fitting of a regression model requires a careful analysis of the database for the problem under study. As part of this analysis, plotting the variables of interest is an invaluable first step in assessing the relationship between the independent and dependent variable(s).

In reference to the data presented in Table 1, and as a consequence of importance for numerical optimization purposes, the key variables of interest included: the thickness of the cover plate and the load induced maximum principal stress and deflection (response variables). In Figures 2 and 3, the scatter-grams of thickness versus maximum principal stress and maximum deflection from the finite element analyses are shown respectively.

![Figure 3: Maximum deflection versus thickness](image)

3.1 Maximum principal stress:

Based on the scatter plots of Figures 2 and 3, in conjunction with the theory of Flexure of Plates [12], a power regression model was proposed for both the maximum principal stress and deflection. In general, the power regression model [13] assumes the form:

\[ Y|x = \beta_0 x^{\beta_1} \quad \text{for } x > 0 \]  

Or,

\[ y_i = \beta_0 x_i^{\beta_1} e_i \quad \text{for } x > 0 \]

A logarithmic transformation is used as follows:

\[ \ln y_i = \ln \beta_0 x_i^{\beta_1} e_i \]

Or,

\[ \ln y_i = \ln \beta_0 + \beta_1 \ln x_i + \ln e_i \]
\[ y_i^* = \beta_o^* + \beta_1^* x_i^* + e_i^* \]  

(5)

Where,

\[ y_i^* = \ln y_i, \quad \beta_o^* = \ln \beta_o, \quad \beta_1^* = \beta_1, \quad x_i^* = \ln x_i \quad \text{and} \quad e_i^* = \ln e_i \]

Parameter estimates are \( \hat{\beta}_1 = \beta_1^* \) and \( \hat{\beta}_o = e^{\beta_o^*} \).

Based on the proposed power regression model of Equation (2), parameter estimation required a logarithmic transformation of the database, parameter transformation, formulation of the power regression equation, and finally a check on model utility. In the following sections the results obtained for both the maximum principal stress and deflection models are presented. For maximum principal stress, consistent with Equation (3), the required logarithmic transformation of the database is shown in Table 2.

Using the MINITAB statistical software package, the linear regression analysis of the data in Table 2, was performed. In particular, the regression analysis was based on:

\[ x = \ln(h) \quad \text{and} \quad y = \ln (\sigma) \]

(6)

The MINITAB output regression equation is:

Figure 4: Maximum principal stress fitted curve.
\[ y = 10.3 - 2.00 \times \]  

(7)

Based on Eq. (5) the appropriate parameter transformation yields

\[ \hat{\beta}_0 = e^{\hat{\beta}_0} = e^{10.2652} = 28716 \quad \text{and} \quad \hat{\beta}_1 = \hat{\beta}_1 = -2.0 \]  

(8)

Substituting the parameter estimates of Equation (8) into Equation (2) yields:

\[ \sigma = 28716h^{-2.0} \]  

(9)

Consistent with the theory of flexure of plates [12], the obtained regression equation confirms that the maximum principal stress is inversely proportional to the square of the cover plate’s thickness. Figure 4, shows the fitted principal stress model and suggests a good fit.

<table>
<thead>
<tr>
<th>Test Case #</th>
<th>Thickness h, mm</th>
<th>Regressor Transform ln (h)</th>
<th>Maximum P. stress σ, Mpa</th>
<th>Response Transform ln (σ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>1.60944</td>
<td>1150</td>
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<tr>
<td>2</td>
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<td>2.30258</td>
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<td>4.85203</td>
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</tr>
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<td>3.15700</td>
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<tr>
<td>8</td>
<td>40</td>
<td>3.68888</td>
<td>18.0</td>
<td>2.89037</td>
</tr>
</tbody>
</table>

### 3.2 Maximum deflection:

For maximum deflection, consistent with Equation (3) the required logarithmic transformation of the database is shown in Table 3.

<table>
<thead>
<tr>
<th>Test Case #</th>
<th>Thickness h, mm</th>
<th>Regressor Transform ln (h)</th>
<th>Maximum Deflection z, mm</th>
<th>Response Transform ln (z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>2.622</td>
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<td>3</td>
<td>15</td>
<td>2.70805</td>
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<td>8</td>
<td>40</td>
<td>3.68888</td>
<td>0.005</td>
<td>-5.29832</td>
</tr>
</tbody>
</table>

Using MINITAB, the linear regression analysis of the data in Table 3 was performed. In particular, the regression analysis was based on:

\[ x = \ln(h) \quad \text{and} \quad y = \ln(z) \]  

(10)
The MINITAB regression equation is:
\[ y = 5.80 - 3.00 \times \]  

Based on Equation (5) the appropriate parameter transformation yields
\[ \hat{\beta}_0 = e^{\hat{\beta}_0} = e^{5.80418} = 331.68 \quad \text{and} \quad \hat{\beta}_1 = \hat{\beta}_1 = -3.0 \]  

Substituting the parameter estimates of Equation into Equation (2) yields:
\[ z = 331.68h^{-3.0} \]  

Figure 5: Maximum deflection fitted curve.

Consistent with the Theory of Flexure of Plates [12], the obtained regression equation confirms that the maximum deflection is inversely proportional to the cubic power of the cover plate’s thickness. Figure 5, shows the fitted model and suggests a good fit.

3.3 Residual analysis

As a final diagnostic test, verifying the validity of the regression models a MINITAB normal probability plot was generated to test the normality assumption. The results obtained for both regression models are shown in Figures 6 and 7, respectively. The normal probability plots of both Figures 6
and 7, do not suggest a serious departure from a straight-line, thereby implying that the normality assumption, in both cases, does not appear to be violated [14].

![Stress normal probability plot](image1)

**Figure 6:** Stress normal probability plot.

![Deflection normal probability plot](image2)

**Figure 7:** Deflection normal probability plot.

4 Conclusions

The main objective of this work was to study the validity of using regression analysis to reduce the number of finite element analyses needed during structural
optimization processes. For large-scale with limited number of design variables the ability of determining a performance parameter over the full range of the design variable variation with small number of finite element analysis can be very valuable. To achieve this goal a few finite element analyses need to be performed outside the optimization process to define the design space over the entire range of the design variables variation within the required accuracy.

By performing the finite element analyses outside the optimization process independent of the need of the optimization routine the full potential of each analysis, to achieve a complete picture of the design space, can be realized. By combining the information of several well-spaced analyses the full shape of the relevant performance parameters over the specified design range can be defined.

The regression analysis study for the pressure vessel cover plate test case performed in this paper demonstrated that for one design variable, which is the case for most sheet metal structures, good results could be achieved. With reasonable number of finite element analyses the stress and deflection responses necessary for numerical optimization can be achieved with reasonable accuracy over the entire specified design range.

References

