RTM preform permeability identification by an iterative inverse technique

D. Dinescu, H. Sol, K. Hoes
Dept. Mechanics of Materials and Constructions, Vrije Universiteit Brussel (VUB), Brussels, Belgium

Abstract

The simulation of the resin flow through a porous medium is a very important aspect for the design of a high-performance composite part and needs accurate input data – in this case the permeability of the preform. However, the permeability measurement requires a laborious procedure, which cannot be considered nowadays as being standardised. A new method for the identification of the fibre reinforcement permeability in the Resin Transfer Moulding process for composite materials has been implemented in a user-friendly computer software. The method belongs to the category of inverse methods, which allows the estimation of model parameters. The comparison between the experimental and the computed parameters is used as convergence criteria in the iterative inverse technique. The paper presents the permeability tensor calculation (the unknown input parameter) and the conditions that have to be imposed on the software in order to obtain an accurate result. The rapid identification is assured both by a fast mathematical model and by a new sensor-based set-up which can perform the measurement of the flow front arrival times ten times faster than a traditional procedure using a mold with a transparent upper half.

1 Introduction

Resin Transfer Moulding (RTM) is considered as one of the most performant processes belonging to the family of Liquid Composite Moulding techniques. It can provide high-performance and high-quality composite parts for various industrial and domestic applications (aircraft, marine, telecommunications, automotive, sport, household products etc.). A successful RTM production cycle depends on the design factors and on the process parameters. A “key –
parameter” is the fabric permeability whose identification is still a research topic for a lot of scientists around the world.

The traditional in – plane methods for permeability measurement use an experimental set – up where the resin is injected into a fibre reinforcement and the flow front is monitored via a transparent upper – plate (e.g. Adams and Rebenfeld [1], [2], Trochu, Gauvin [3]). The measurement is suitable for uni – directional or bi – directional flow, at a constant pressure or constant flow rate. The identification procedure is based on the methodology proposed by Adams and Rebenfeld [1], [2] that models the flow through porous media using Darcy’s Law. Permeabilities for the principal flow directions of the fabric $K_x$ and $K_y$ can be obtained in this way by taking into account the flow front position on these two perpendicular directions X and Y.

A new technique for the permeability tensor computation, based on an inverse method will be presented hereafter. The proposed method belongs to a broader class of mixed numerical / experimental methods for parameter estimation. As applications one can mention those presented by Sol [4] where the method was applied for anisotropic thin plate rigidities and a non – destructive procedure (Resonalyser) for stiffness identification for orthotropic materials was developed. De Visscher [5] implemented a new feature in the Resonalyser: the identification of damping properties of thin orthotropic plates.

2 Theoretical background

The principle of a mixed numerical / experimental technique (MNET) or inverse method is based on the comparison of two models: experimental and mathematical (numerical or analytic). The material property estimation is made using an iterative procedure, which updates the output parameters of the mathematical model according to the experimental output parameters. The correlation between parameters is obtained by minimisation of a Cost function:

$$C(p) = (m - y_r(p)) \cdot W_{rs}^{(m)} \cdot (m - y_s(p)) + (p_r^{(0)} - p_r) \cdot W_{rs}^{(p)} \cdot (p_s^{(0)} - p_s)$$

where:
- $C(p) : R^{NP} \rightarrow R$ is a Cost function yielding a scalar value
- $p$ is a (NP x 1) column containing the NP material parameters
- $p^{(0)}$ contains the initial estimates for the material parameters
- $y$ is a (NM x 1) column containing the NM computed output values using $p$ parameter column
- $m$ contains the (NM x 1) measured output values
- $W^{(m)}$ is a (NM x NM) weighting matrix applied on the difference between the measured column and the output column
- $W^{(p)}$ is a (NP x NP) weighting matrix for the difference between the initial parameter column $p^{(0)}$ and the $p$ parameter column
The minimisation of the Cost function is described by Sol [4] and the final parameter estimation formula is:

$$\{p\}^{j+1} = \{p\}^j + \left(2[c_c] + [S]^T[c_R][S]\right)^{-1}\left\{[S]^T[c_R]\{R\}^{\exp} - \{T\}^j\right\}$$

(2)

where one has denoted by:

- \(\{p\}\) the material parameter column that has to be identified
- \([c_c]\) the material parameter weighting matrix
- \([S]\) the sensitivity matrix
- \([R]^{\exp}\) the measured (experimental) output column
- \([R]^j\) the computed output column
- \([c_R]\) the output weighting matrix
- \(j\) the iteration index

The initial solution \(\{p\}^0\) can be either approximated by experience or by calculation.

According to the fact that for an accurate model and experimental data, the calculated parameters are independent of the values of \([c_c]\) and \([c_R]\) we will not use them in this stage of the permeability identification method. They will be considered to be equal to the identity matrix.

For the in – plane permeability identification eqn (2) reads:

$$\{K\}^{j+1} = \{K\}^j + \left(2[c_K] + [S]^T[c_T][S]\right)^{-1}\left\{[S]^T[c_T]\{T\}^{\exp} - \{T\}^j\right\}$$

(3)

where:

- \(\{K\}\) is the permeability column; \(\{K\} = \begin{bmatrix} K_x \\ K_y \end{bmatrix}\)
- \(\{T\}^{\exp}\) is the column containing the flow front arrival times for some specific points in the mold
- \(\{T\}^j\) is the column containing the calculated flow front arrival times for the same points as \(\{T\}^{\exp}\)
- \([c_K]\), \([c_T]\) is the weighting matrices for permeability and for flow front arrival times

$$[S]\) is the sensitivity matrix; \([S] = \begin{bmatrix} \frac{\partial T_1}{\partial K_x} & \frac{\partial T_1}{\partial K_y} \\ \vdots & \vdots \\ \frac{\partial T_n}{\partial K_x} & \frac{\partial T_n}{\partial K_y} \end{bmatrix}, i = 1,...,n$$
and \( n \) denotes the number of the activated sensors and \( T_i \) the arrival time of the sensor \( i \).

For the time being, the fabric permeability is considered to be deterministic, represented by a fixed value corresponding to a certain type of material. The large scatter for this parameter can also lead to the concept of permeability as stochastic values with a probability distribution, as in Hoes et al. [6]. Our method uses the deterministic approach, emphasised by eqn (3).

3 A new mixed numerical / experimental technique (MNET) for permeability identification

The new MNET for permeability tensor identification is summarised in the flow chart from Figure 1. Starting from an initial solution \( \{ k \}_\text{init} \), the flow front arrival times \( T_c \) are generated with an analytic model. The convergence criterion is checked by comparing \( T_c \) with the experimentally measured times \( T_e \).

The iterative technique makes corrections in each iteration by minimising a Cost function which incorporates a sensitivity matrix of the arrival times. An optimal choice of the initial solution minimises also the number of iterations. The most time consuming step of the iteration is the computation of \( T_c \).

3.1 The components of the permeability identification MNET

3.1.1 The experimental model
The flow front arrival times will be measured using a new sensor – based set – up, described by Hoes et al. [6] and equipped with 43 electric sensors, with a simple geometry, as shown in Figure 2.
Figure 2: Experimental set-up

Figure 3: The analytic flow front simulation
3.1.2 The analytic model

The iterative technique needs to generate $T_c$ at each step by using a mathematical model. In order to have a fast permeability identification procedure, the computational time required by the model has to be as low as possible. A standard FE simulation needs a large computation time even for a simple geometry so it is not advised to use a FE model. A simple analytic model that gives accurate flow front arrival times has been developed for the implementation in the MNET by Dinescu et al. [7].

The analytic model also uses an iterative procedure for the calculation of the flow front arrival times. The elliptical extent of the flow front given by the Adams and Rebenfeld model [1], [2] on X and Y directions will be increased until it will reach the value of each sensor position vector. A graphic interface (Figure 3) records successive positions of the flow front and registers the $T_c$ for each activated sensor.

This model has been compared with the finite element program LCMFlot. The small errors have demonstrated that the filling time prediction is accurate.

3.1.3 A numerical approach for the Cost function minimisation

The software that includes the experimental data and the analytic model has been written in Visual Basic. A user - friendly interface assures a fast data processing, depending on the available computing power and on the characteristics of the tested fabrics. The program can generate model arrival times using the analytic simulation or can read experimental times registered in text files.

The permeability calculation can be followed on two graphs, which show the iterative procedure step by step, from the start value until the final value (Figure 4). Eqn (3) gives an iterative procedure, which is stopped by the following criterion:

$$
\frac{\{K\}^j - \{K\}^j-1}{\{K\}^j} < 0.5\%.
$$

(4)

The experimental arrival times and the arrival times, which are calculated after each iteration are also displayed for each sensor. The user can compare them and can decide if it is necessary to continue the iterative calculation using a "step by step" calculation feature.

The most delicate problem is represented by the sensitivity matrix evaluation. The numerical approach proposed below is taking into account the fact that smaller reinforcement permeability will lead to longer flow front arrival times. For a sensor $i$ the shape of the function $T = T(K)$ (arrival time vs. permeability) is shown in Figure 5.

The derivatives $\frac{\partial T}{\partial K_x}$, $\frac{\partial T}{\partial K_y}$ may be approximated using backward, central or forward differences. The lowest number of operations is performed using backward or forward differences because $T_c$ are generated only once in this way for $K-\Delta K$ or $K+\Delta K$. Backward differences will be used in this procedure.
Figure 4: The graphic interface of the iterative program

Figure 5: The time – permeability dependence
The derivatives $\frac{\partial T}{\partial Kx}, \frac{\partial T}{\partial Ky}$ for a sensor $i$ will be approximated as:

$$
\frac{\partial T}{\partial Kx} \approx \frac{\Delta T}{\Delta Kx} = \frac{T(Kx) - T(Kx - \Delta Kx)}{\Delta Kx}, \quad \frac{\partial T}{\partial Ky} \approx \frac{\Delta T}{\Delta Ky} = \frac{T(Ky) - T(Ky - \Delta Ky)}{\Delta Ky}. \quad (5)
$$

For the permeability values $Kx_i, Ky_i$, the sensitivity matrix will be calculated by running the analytic model of the simulation three times:

1. Input data $Kx - \Delta Kx; \quad Ky = \text{constant} \Rightarrow T(Kx - \Delta Kx)$ from eqn (5)
2. Input data $Ky - \Delta Ky; \quad Kx = \text{constant} \Rightarrow T(Ky - \Delta Ky)$ from eqn (5)
3. Input data $Kx; Ky \Rightarrow T(Kx); T(\Delta Ky)$ from eqn (5) and $\{T\}_i$ from eqn (3)

This calculation is not time consuming due to the rapid computation of the analytic model (a few seconds for one running). The third run will be used not only for the sensitivity calculation but also for the $\{T\}_i$ vector of the current iteration. Anyhow, an analytic solution for the sensitivity matrix computation will be searched in the future.

The fact that three simulations will be performed with three different sets of permeability data can lead to a different number of activated sensors per case: $i_1(1) \neq i_2(2) \neq i_3(3)$. An algorithm for sorting the minimum number of activated sensors on each direction has been used in order to avoid this problem. For each iteration, the sensitivity matrix will be adapted to have a number of elements equal with the smallest number of activated sensors from the three cases (1), (2), (3). The order of the other matrices and vectors from eqn (3) will be also adapted to match the order of the sensitivity matrix.

The software calibration is made by replacing the measured arrival times $\{T\}^\text{exp}$ by model arrival times $\{T\}^\text{model}$ generated with the analytic model for arbitrary input data $(Kx, Ky)$. The permeability values, which are estimated in this way, will match the input $Kx$ and $Ky$. The user can choose at the beginning one of these features: model or experimental arrival times.

The iterative process for the permeability must have a start value for $Kx$ and $Ky$. For a fast computation the number of iterations should be as small as possible, so the start value should be as close as possible to the final value. This is why a good approximation of the start value is a very important condition.

A good estimation of the permeability values for X and Y directions can be made using Darcy’s law:

$$
\bar{v} = \frac{1}{\mu} \bar{k} \nabla p. \quad (6)
$$

For one direction this can be written as:
The dynamic viscosity of the fluid is given by:

\[ \nu = \frac{R - R_0}{T} = -\frac{1}{\mu} \frac{p_f - p_0}{R - R_0}. \]

(7)

where: \( p_f - p_0 \) is the difference between the atmospheric pressure and the inlet pressure; \( R \) is the Cartesian coordinate of the sensor; \( R_0 \) is the inlet radius; \( \mu \) is the dynamic viscosity of the fluid.

If \( \Delta p = p_0 - p_f \) eqn (7) yields:

\[ K = \frac{\mu (R - R_0)^2}{T \cdot \Delta p}. \]

(8)

Some researchers have studied the dependence between an accurate permeability calculation and a minimum mold length, as Breard [8]. Consequently, we will apply eqn (8) for the last activated sensors on X and Y directions.

### 4 Comparison with a traditional method

The permeability values obtained with the MNET have been compared with those obtained using a traditional method (an Adams algorithm using the experimental times from the sensor – based set – up).

As an example, four different materials with different fibre volume fraction \( V_f \) have been tested. The main flow directions are the same as the X and Y directions of the mold, thus the elliptical flow front is not rotated with respect to the mold axes. The \( K_x \) and \( K_y \) permeabilities obtained with the two methods are presented in Table 1 for the next cases:

- <1> A non-crimp fabric with a nominal surface density of 530 g/m²
- <2> A plain weave with a nominal surface density of 420 g/m²
- <3> A non-crimp fabric with a nominal surface density of 1200 g/m²
- <4> A twill woven fabric with a nominal surface density of 380 g/m²

Table 1: Permeability values for the traditional method and for the MNET

<table>
<thead>
<tr>
<th>Material</th>
<th>Surface density</th>
<th>( V_f )</th>
<th>Traditional method</th>
<th>MNET</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>( k_x (m^2) )</td>
<td>( k_y (m^2) )</td>
</tr>
<tr>
<td>Non-crimp fabric</td>
<td>530 g/m²</td>
<td>0.315</td>
<td>2.55E-09</td>
<td>2.11E-09</td>
</tr>
<tr>
<td>Plain weave</td>
<td>420 g/m²</td>
<td>0.417</td>
<td>1.45E-10</td>
<td>2.08E-10</td>
</tr>
<tr>
<td>Non-crimp fabric</td>
<td>1200 g/m²</td>
<td>0.381</td>
<td>1.90E-09</td>
<td>3.36E-10</td>
</tr>
<tr>
<td>Twill fabric</td>
<td>380 g/m²</td>
<td>0.531</td>
<td>1.07E-11</td>
<td>2.01E-11</td>
</tr>
</tbody>
</table>
5 Conclusion

The proposed MNET allows a rapid and accurate permeability calculation using an iterative algorithm. Although the mathematical model is based on the Adams model and Darcy's Law, like the traditional method, the information that will lead to the final permeability value calculation is more complex, taking into account also the arrival times from the intermediate directions 22.5°, 45° and 67.5°. In this way the permeability can be calculated with higher precision.

References