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Optimum design of submarine hulls

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Abstract

Vibration characteristics of high performance structure such as submarines are very critical. Submarine hulls are mainly constructed of cylindrical shells. Cylindrical shells are also used in many structural designs, such as offshore structures, liquid storage tanks, and airplane hulls. The vibration characteristics of cylindrical shells present many unique challenges in optimum design of submarine hulls for vibration control. In cylindrical shells, the lowest natural frequency does not necessarily correspond to the lowest wave index. In fact, the natural frequencies do not fall in ascending order of the wave index either. Eigen solutions of cylindrical shells also indicate repeated natural frequencies. These are referred to as double peak frequencies. Mode shapes associated with each natural frequency are combination of *(i)* Radial (flexural); *(ii)* Longitudinal (axial); and *(iii)* Circumferential (torsional) modes.

In this paper, uniqueness of modal spectrum, redundancy of modal constraints and non-uniqueness in optimum design of cylindrical shells for vibration requirements are presented. The implications of these characteristics on submarine design are highlighted. Related issues such as the new mode sequence, mode crossing, repeated natural frequencies and stationary modes are also discussed.

1 Introduction

Unlike that of simpler structures such as beams and plates, the modal spectrum of cylindrical shell exhibits very unique characteristics. In cylindrical shells, the lowest natural frequency does not necessarily correspond to the lowest wave index shown in Figure 1. In fact, the natural frequencies do not fall in ascending order of the wave index either as Table 1 indicates.

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Shell displacements of each mode shape are defined in three orthogonal directions that are associated with radial (flexural), longitudinal (axial), and circumferential (torsional) components.

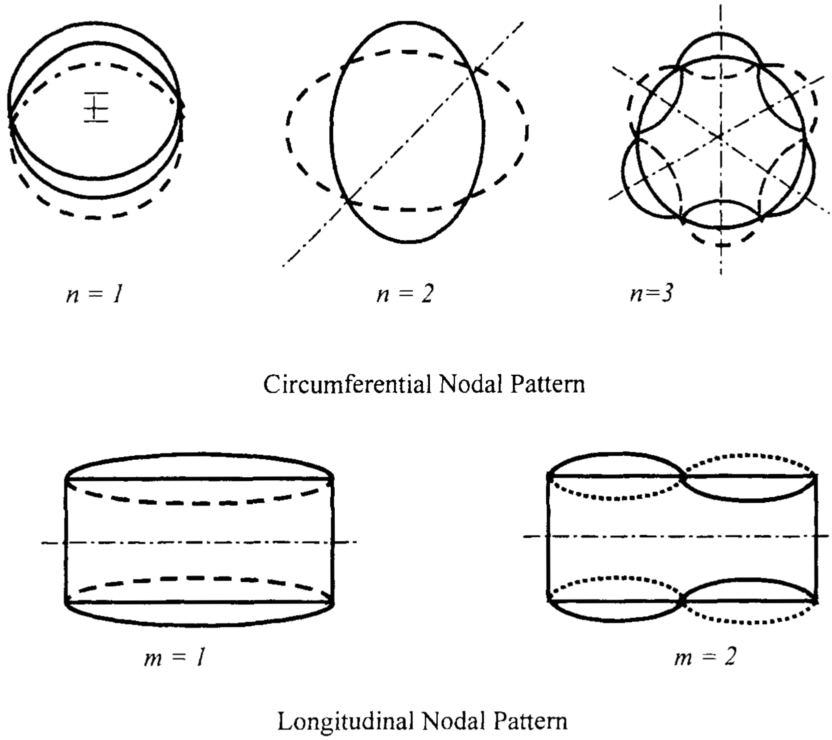

Fig. 1: Normal mode patterns for simply supported cylindrical shells

The coupling between the transverse and in-plane vibration is due to the shell curvature. Eigen solutions of cylindrical shells indicate multiple eigenvalues, i.e. repeated natural frequencies with similar mode shapes. These are referred to as double peak frequencies[1].

Modes shapes associated with membrane shell deformations require a lot of stain energy while mode shapes associated with bending deformation require less strain energy. Realizing that the total potential strain energy in a shell is the sum of both membrane and bending strain energy, the first mode shape corresponding to the lowest total energy may not necessarily correspond to the lowest wave index n.

The ratio of membrane strain energy to kinetic energy (or the total strain energy) is high for modes with simple modal patterns n and decrease toward zero as the number of nodal lines increases, while the ratio of the bending strain energy to

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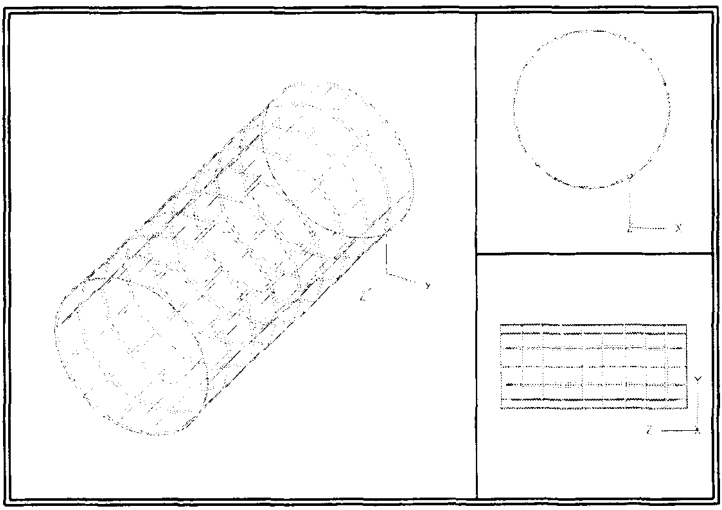
the to kinetic energy (or the total strain energy) is small for simple nodal patterns and increase with the increase of wave index n [2].

The natural frequencies that are controlled by the membrane strain energy are approximately independent of the shell thickness change, while the natural frequencies controlled by bending stain energy vary with shell thickness [3].

2 Modal dynamics of cylindrical shell

A finite element model of the baseline cylindrical shell shown in Figure 2 is analyzed using MSC/NASTRAN [4] to obtain its modal characteristics. The chosen cylindrical shell represents a segment of submarine hull. It has the following dimensions: length L = 594 in, radius R =198 in, thickness h = 2 in [5]. The dynamic characteristics of the cylindrical shell for shear diaphragm boundary conditions are summarized in Table 1.



Length L = 594 in, Radius R = 198 in, Thickness h = 2 in
$E = 30 \times 10^6$ psi, $\nu = 0.3$, $\rho = 7.324 \times 10^{-4}$ lb.sec^2/in^4

Fig. 2: Baseline finite element model of simply supported cylindrical shell

The results listed in Table 1shows the natural frequencies from the finite element analysis and the natural frequencies obtained from an analytical solution derived from references [6,7]. In developing the analytical solution, an energy approach is used. An energy functional which include both bending and membrane strain energies was formulated using the Donnell-Mushtari formulation of strain-displacement. Then Rayligh-Ritz procedure was employed leading to an eigenvalue problem [8].

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Table 1: Modal Characteristics of baseline cylindrical shell

Mode No.	Mode		Frequency (Hz)		Total Strain Energy	
	n	*m*	FEA	Analytic	FEA	Analytic
1	4	1	12.89	12.98	3337.6	3551.5
2	4	1	12.89	12.98	3361.3	3551.5
3	5	1	14.13	14.31	4042.6	4216.7
4	5	1	14.13	14.31	4042.6	4216.7
5	3	1	17.39	17.30	6031.3	6637.8
6	3	1	17.39	17.30	6031.3	6637.8
7	6	1	18.39	18.75	6666.1	7135.1
8	6	1	18.39	18.75	7062.8	7135.1
9	7	1	23.54	24.86	11272.	12450.
10	7	1	23.54	24.86	11272.	12450.
Length $L = 594$ in, Radius $R = 198$ in, Thickness $h = 2$ in $E = 30 \times 10^6$ psi, $\nu = 0.3$, $\rho = 7.324 \times 10^{-4}$ lb.sec^2/in^4						

3 Design optimization

A series of three design optimizations were performed using MSC/NASTRAN optimization module (SOL 200) [9]. In each run, a minimum weight optimization is performed for a single modal constraint. Shell thickness is used as a design variable. Nine design variables, each representing a segment thickness, are chosen as shown in Figure 3.

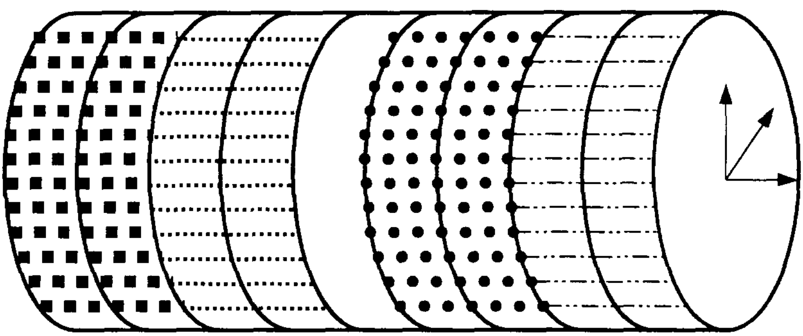

Fig. 3: Nine design variables shell model

In the first run, the modal constraint is imposed on the first natural frequency. The frequency $f_1=12.89$ Hz is constraint as follows $f_1'=14.92$ Hz. The modal characteristics of optimized design (Design *I*) are shown in Table 2.

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Table 2: Modal characteristics of Design *I*

Mode No.	Mode		Baseline Frequency (Hz)	Mode		Design *I* Frequency (Hz)
	n	*m*		*n*	*m*	
1	**4**	**1**	**<u>12.89</u>**	**4**	**1**	**<u>14.92</u>**
2	4	1	12.89	4	1	14.92
3	5	1	14.13	3	1	17.93
4	5	1	14.13	3	1	17.93
5	3	1	17.39	5	1	18.42
6	3	1	17.39	5	1	18.42
7	6	1	18.39	6	1	25.00
8	6	1	18.39	6	1	25.00
9	7	1	23.54	2	1	31.49
10	7	1	23.54	2	1	31.49

Examination of the results listed in Table 2 indicate mode crossing in which mode 3 (n=5, m=1) moved past mode 5 (n=5, m=1). Also the natural frequency of mode 3 (n=3, m=1) in the baseline design remained in close proximity to the original natural frequency.

In the second run, the modal constraint is imposed on the third natural frequency. The frequency $f_3=14.13$ Hz is constraint as follows $f_3'=18.66$ Hz. The modal characteristics of optimized Design *II* are shown in Table 3.

Table 3: Modal characteristics of Design *II*

Mode No.	Mode		Baseline Frequency (Hz)	Mode		Design *II* Frequency (Hz)
	n	*m*		*n*	*m*	
1	4	1	12.89	4	1	15.09
2	4	1	12.89	4	1	15.09
3	**5**	**1**	**<u>14.13</u>**	3	1	18.16
4	5	1	14.13	3	1	18.16
5	3	1	17.39	**5**	**1**	**<u>18.66</u>**
6	3	1	17.39	5	1	18.66
7	6	1	18.39	6	1	25.57
8	6	1	18.39	6	1	25.57
9	7	1	23.54	2	1	32.29
10	7	1	23.54	2	1	32.29

Examination of the results listed in Table 3 indicates the same mode crossing observed in Design *I*. Also the natural frequency of mode 3 (n=3, m=1) in the baseline design remained in close proximity (stationary) to the original natural

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frequency. In fact the modal characteristics of Design *II* are almost identical to the Design *I*.

In the third run, the modal constraint is imposed on the seventh natural frequency. The frequency f_7=24.92Hz is constraint as follows f_7'=24.92Hz. The modal characteristics of optimized Design *III* are shown in Table 4.

Table 4: Modal characteristics of Design *III*

Mode No.	Mode		Baseline Frequency (Hz)	Mode		Design *III* Frequency (Hz)
	n	*m*		*n*	*m*	
1	4	1	12.89	4	1	14.89
2	4	1	12.89	4	1	14.89
3	5	1	14.13	3	1	17.93
4	5	1	14.13	3	1	17.93
5	3	1	17.39	5	1	18.36
6	3	1	17.39	5	1	18.36
7	6	1	18.39	6	**1**	**24.92**
8	6	1	18.39	6	1	24.92
9	7	1	23.54	2	1	31.52
10	7	1	23.54	2	1	31.52

On further examination of the results listed in Table 4 one can observe the same modal characteristics of Design *I* and Design *II* despite the fact that both models were optimized for different modal constraints.

4 Uniqueness of modal spectrum

The modal characteristics of the three optimized designs indicate that the natural frequencies of a cylindrical shell are interlinked and uniquely determined based on one of the natural frequency. A summary of the modal characteristics of all optimized designs is listed in Table 5. One can also observe mode crossing and a stationary natural frequency represented in modes of wave index (n=3, m=1), and (n=2, m=1). Attempts to optimize the cylindrical shell for multiple modal constraints would yield no solution except for the case of compatible constraints, i.e. constraints consistent with the modal spectrum as obtained for single modal constraint [10].

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Table 5: Uniqueness of modal spectrum

Mode No.	Mode		Natural Frequency of Optimized Shells (Hz)		
	n	*m*	Design *I*	Design *II*	Design *III*
1	4	1	14.92	15.09	14.89
2	4	1	14.92	15.09	14.89
3	3	1	17.93	18.16	17.93
4	3	1	17.93	18.16	17.93
5	5	1	18.42	18.66	18.36
6	5	1	18.42	18.66	18.36
7	6	1	25.00	25.57	24.92
8	6	1	25.00	25.57	24.92
9	2	1	31.49	32.29	31.52
10	2	1	31.49	32.29	31.52

5 Non-uniqueness of optimum design

While the three optimized designs Design *I*, Design *II*, *and* Design *III* exhibited identical modal spectrum as shown in Table 5, the values of the design variables (segments' thickness) were not. The values of the design variables of each optimized design are listed in Table 6. This indicates non-uniqueness of optimal design points. Results in Table 6 show that the symmetry of the design was maintained even though it was not explicitly imposed.

Table 6: Non-uniqueness of optimal design

Thickness of Optimized Shells (in)		
Design *I*	Design *II*	Design *III*
$T_1 = 2.916$	$T_1 = 3.772$	$T_1 = 2.924$
$T_2 = 2.882$	$T_2 = 3.309$	$T_2 = 2.918$
$T_3 = 2.817$	$T_3 = 2.798$	$T_3 = 2.760$
$T_4 = 2.715$	$T_4 = 2.529$	$T_4 = 2.717$
$T_5 = 2.693$	$T_5 = 2.452$	$T_5 = 2.684$
$T_6 = 2.715$	$T_6 = 2.529$	$T_6 = 2.717$
$T_7 = 2.817$	$T_7 = 2.798$	$T_7 = 2.760$
$T_8 = 2.882$	$T_8 = 3.309$	$T_8 = 2.918$
$T_9 = 2.916$	$T_9 = 3.772$	$T_9 = 2.924$
Weight = 1514.0 lb	*Weight = 1628.4 lb*	*Weight = 1512.2 lb*

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6 Conclusions

Modal characteristics of cylindrical shells are very unique. The natural frequencies are often interlinked resulting in no optimum solution for optimization for incompatible multiple modal constraints. The uniqueness of the modal spectrum causes redundancy in modal constraints. However, there is non-uniqueness in optimum design of a cylindrical shell for vibration requirements resulting in different segments' thickness. The new modal spectrum shows a new frequency sequence, mode crossing, repeated natural frequencies, and stationary modes.

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