A super-cube interconnection network for parallel computers

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Abstract

In this paper, we consider some interconnection networks for parallel computers. Let d be the greatest distance in a network, and let l be the maximum number of circuits connected from optional processor. It is well known, for the hypercube interconnection network, $d = \log_2 n$, $l = \log_2 n$, where n is the number of processors in the network. In this paper, we present a super-cube interconnection network for parallel computers. For this new interconnection network, d and l both satisfies $d \leq 0.86 \log_2 n$ and $l \leq 0.86 \log_2 n$.

1 Introduction

In fields of modern computer architecture, SIMD type computers, MIMD type computers and pipeline control type computers etc. called parallel computers, are increasingly undergoing rapid development. There are many arithmetic units, processors and memory units known as terminals in these parallel computers. It is well known that the terminals are required to mutually connect at as high a speed as possible in a very close range. This interconnection network is an important factor when evaluating the performance of parallel computers[1-4]. As the static networks, we have got the perfect connection network, star connection, bus connection, ring network (loop connection)[5], the mesh connection, tree structure, layer structure,

hypercube network, super-cube circuit (CCC) etc. Particularly, for hypercube connection, we know that, the greatest distance in the network (the maximum number of exchanging times to forward datas between terminals), and the maximum number of circuits connected from optional terminal, reaches $\log_2 n$ separately. Where n is number of the terminals in the network[1-4].

In this paper, we'll present a super-cube interconnection network. For this connection form, the greatest distance in the network and the maximum number of circuits connected from optional terminal, will be decreased to $0.86 \log_2 n$ separately.

2 Hypercube Network

If there are $n = 2^k$ terminals in a network, then we know it can be connected by an k dimension hypercube form. When $k = 3$, it becomes Cube connection which is well known. When $k = 4$, it is composed by connecting corresponded terminals between two cubes, such as be shown in Figure 1. Generally, for a Hypercube network with $n = 2^k$ terminals, its greatest distance in the network is $\log_2 n$, and the maximum number of circuits connected from optional terminal is also $\log_2 n$.

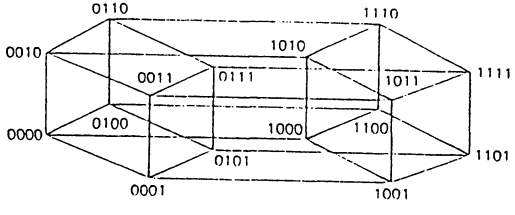

Figure 1. Hypercube network ($k = 4$)

3 A Super-Cube Interconnection Network

Suppose that there are $n = 5^k$ terminals in some network. Let us consider the following Super-Cube network with Five-Connection Form. Figure 2. and Figure 3. are showing this connection form for case $k = 1$ and $k = 2$, separately.

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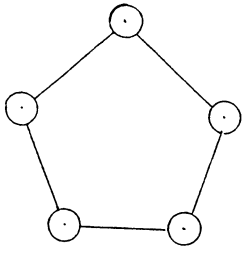

Figure 2. Five-Connection Form ($k = 1$)

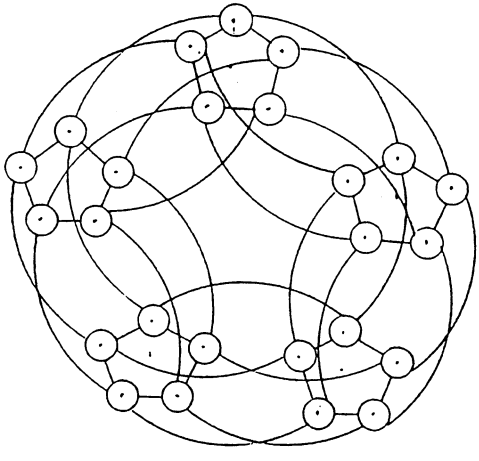

Figure 3. Five-Connection Form ($k = 2$)

In general, by an recursive structure with the fractal rule, $n = 5^k$ terminals always can be connected in Five-Connection form. It is easy to be known that the greatest distance in the network is $2k$, and the maximum number of circuits connected from optional terminals is also $2k$. From

$$2k = 2\log_5 n = \frac{2}{\log_2 5}\log_2 n \approx 0.86\log_2 n,$$

we can know that the Five-Connection Form is superior to usual Hypercube connection form. Because Hypercube used binary digits to express the address of terminals, so it is obvious that, for Five-Connection Form, we may express the address of terminals by quinary digits. From address code of quinary digits, we can decide the shortest path to forward data between two terminals.

4 Structure of Five-Connection Network

In this section, we'll show some theoretical results for Five-Connection Network.

Difinition 1. n dimension Five-Connection Network Q_n has node addresses from 0 to $5^n - 1$. Two nodes is next to each other if and only if quinary digits of the address for these two nodes is different with just one digit, and satisfy (let the two different digits be a_i and b_i):

$$w(a_i, b_i) \equiv min\{|k| : k \equiv |a_i - b_i|, (mod \quad 5)\} = 1.$$

Difinition 2. For n dimension Five-Connection Network Q_n, let $a_n a_{n-1} \cdots a_1$, and $b_n b_{n-1} \cdots b_1$ be quinary expressions of two node addresses a and b, we call

$$H(a, b) = \sum_{i=1}^{n} w(a_i, b_i),$$

is Hamilton distance of two nodes with address a and b separately.

Difinition 3. The addresses of Q_n are composed by the following recursive structure :

(a) Let the node addresses of Q_1 be $0, 1, 2, 3, 4$.

(b) Let some node address of Q_{n-1} be $a_{n-1}a_{n-2} \cdots a_1$. In Q_n, we replace the node addresses of five Q_{n-1} to

$$0a_{n-1}a_{n-2} \cdots a_1,$$

$$1a_{n-1}a_{n-2} \cdots a_1,$$

$$2a_{n-1}a_{n-2} \cdots a_1,$$

$$3a_{n-1}a_{n-2} \cdots a_1,$$

$$4a_{n-1}a_{n-2} \cdots a_1,$$

and connect these nodes with Five-Connection Form.

For example, Figure 4. showed the node addresses for Q_1 and Q_2.

Q_1 :

0	1	2	3	4

Q_2 :

00	10	20	30	40
01	11	21	31	41
02	12	22	32	42
03	13	23	33	43

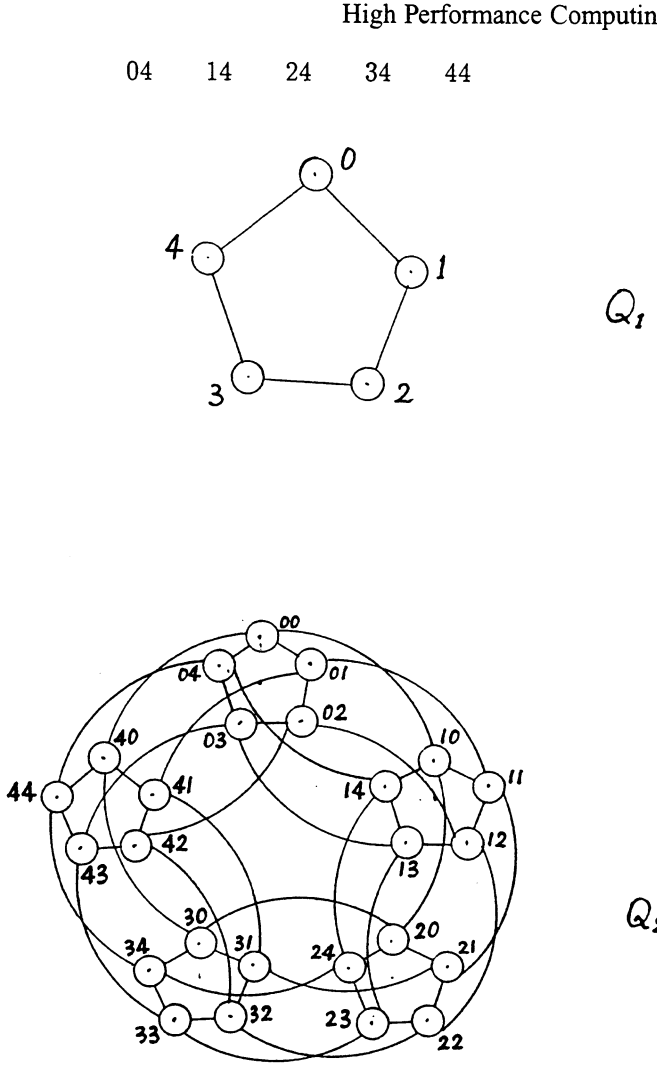

Figure 4. Node Addresses for Q_1 and Q_2

Theorem 1. For any two nodes A, B in Q_n, the greatest distance between A and B is $H(A, B)$.

Proof: Not lose generality, we suppose that the addresses of nodes A and B is different with just k digits in the head. Let address of A be $a_n a_{n-1} \cdots a_{n-k+1} a_{n-k} \cdots a_1$, and address of B be $b_n b_{n-1} \cdots b_{n-k+1} a_{n-k} \cdots a_1$. We may go forward from A to B by the following steps:

$$a_n a_{n-1} a_{n-2} \cdots a_{n-k+1} a_{n-k} \cdots a_1$$

Connection form	l	d	$m = ld$	$e = \frac{d}{l}$
Hypercube Network	$\log_2 n$	$\log_2 n$	$\log_2^2 n$	1
3-connection	$1.26 \log_2 n$	$0.63 \log_2 n$	$0.79 \log_2^2 n$	0.5
5-connection(ring)	$0.86 \log_2 n$	$0.86 \log_2 n$	$0.74 \log_2^2 n$	1
5-connection(complete)	$1.72 \log_2 n$	$0.43 \log_2 n$	$0.74 \log_2^2 n$	0.25
6-connection(ring)	$0.77 \log_2 n$	$1.16 \log_2 n$	$0.89 \log_2^2 n$	1.5
6-connection(incomplete)	$1.16 \log_2 n$	$0.77 \log_2 n$	$0.89 \log_2^2 n$	0.66

Table 1: Comparison of parameters for some connections

$$b_n a_{n-1} a_{n-2} \cdots a_{n-k+1} a_{n-k} \cdots a_1$$
$$b_n b_{n-1} a_{n-2} \cdots a_{n-k+1} a_{n-k} \cdots a_1$$
$$\cdots\cdots$$
$$b_n b_{n-1} b_{n-2} \cdots b_{n-k+1} a_{n-k} \cdots a_1$$

So this shortest distance is

$$\sum_{i=0}^{k-1} w(a_{n-i}, b_{n-i}) = H(A, B).$$

5 Some other Connection Networks

We can also consider some other connection networks excluding Five-Connection Form. In general, when design an network system, we put the point aimed at decreasing the number of circuits, or restraining the greatest distance in network. Connection form of the network will be decided by the above viewpoint.

Let d be the greatest distance in network, and let l be the maximum number of circuits connected from optional terminal. Define $m = l \cdot d$ is a parameter well-balanced for l and d, and $e = \frac{d}{l}$ shows the average greatest distance decreased per one circuit.

Table 1 showed a comparison of parameters for some connections. In Table 1, 5-Connection (Complete), 6-Connection (Ring) and 6-Connection (Incomplete) means an extend network by the fractal rule from the following one dimension case separately.

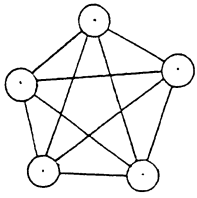

Figure 5. 5-Connection (Complete)

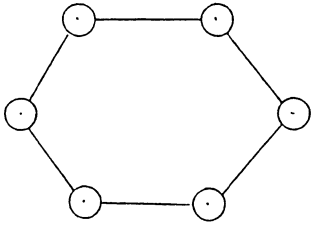

Figure 6. 6-Connection (Ring)

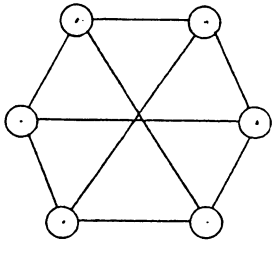

Figure 7. 6-Connection (Incomplete)

6 Conclusions

In this paper, we discussed connection forms of some super-cube interconnection network for parallel computers. In order to decrease the greatest distance in the network and the maximum number of circuits connected from optional terminal, we proposed a super-cube interconnection network

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with Five-Connection Form (Ring Form). The Five-Connection Form is superior to general Hypercube network if we just pay attention on the greatest distance of network and the maximum number of circuits connected from optional terminal.

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