Computation of the starting flow in a shock tunnel 2D nozzle

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Abstract

This paper summarizes the results obtained with a Reynolds Averaged Navier-Stokes solver applied to the case of the starting flow in the two-dimensional nozzle section of a shock tube and which constitutes a numerical test to assess the accuracy of the code when dealing with highly non-stationary internal flows with shock waves interactions. Since the shock-boundary layer phenomenon drives the flow separation on the nozzle flaps and turbulence plays an important role on the evolution of the structures which arise, the need of an adequate turbulence closure, as well as the temporal resolution of the flow by using a small enough time-step, makes the computations expensive. Within this framework and to drop the computing time down, the parallel implementation of the solver based on the MPI paradigm has been exploited to carry out a set of simulations with various eddy viscosity models.

1 Introduction

The patterns of shock waves and contact surfaces which arise in a two-dimensional round-inlet wedge nozzle after the impingement of a planar shock wave generated in a shock tunnel, has been investigated experimentally by Amann [1], [2], [3], who visualized the flowfield by means of shadowgraph and interferometry techniques. Besides the experiments, numerical simulations of the starting process have been conducted under different approaches: Saito & Takayama [4] computed the time dependent flowfield with a full Navier-Stokes solver; Saito et al.[5] compared the Navier-Stokes and Euler solutions for the same testcase. Igra et al. [6] performed Euler simulations for the sharp inlet wedge tested by Amann; Tokarcik-Polsky et al. [7], [8] and Prodromou &
Hillier [9] solved the transient for the round-inlet wedge nozzle case with a Navier-Stokes and Euler solver, respectively. In contrast to the previous numerical works, the present results include the effect of turbulence, modelled by various eddy viscosity closures.

2 Numerical modelling

2.1 Solver

The simulations were carried out with the computer code SPARC (Structured PArallel Research Code) [10] which solves the three-dimensional compressible Navier-Stokes equations in Reynolds averaged form (RANS, Favre averaging) on multiblock structured grids. The spatial discretization follows a semi-discrete cell-centered finite-volume formulation and uses a second order central difference stencil for the convective and diffusive transport terms. Explicit (standard) Runge-Kutta and also explicit dual time-stepping second-order temporal discretization schemes are available. Convergence is accelerated by means of local-time stepping, implicit averaging of the residuals and Full-MultiGrid (FMG) algorithms. Nevertheless, the computation of time-accurate solutions disregards the use of FMG because it is of no sense when explicit schemes are selected to solve the flowfield.

2.2 Artificial dissipation

In the presence of discontinuities, an amount of artificial dissipation is needed for the sake of numerical stability. The selected approach consists of a bend of adaptive second and fourth-order differences according to the Jameson, Schmidt and Turkel formulation [11], known as JST scheme, which ensures the elimination of the spurious wiggles and provides a sharp resolution of the shock waves. Thus, the total flux $F$ through the face $(i+1/2,j,k)$ of area $\Delta S$, namely $F\Delta S$, comprehends the sum of the convective flux $F_c$ and the artificial dissipation $D$. Abreviating $(i+1/2,j,k)$ as $i+1/2$ hereafter is

$$(F\Delta S)_{i+1/2} \approx F_c(U_{i+1/2}) \Delta S_{i+1/2} - D_{i+1/2}$$

with

$$D_{i+1/2} = R_{i+1/2} \{ e^{(2)}(U_{i+1} - U_j) e^{(4)}(U_{i+2} - 3U_{i+1} + 3U_i - U_{i-1}) \}$$

being $U$ the flow vector averaged as $U_{i+1/2} = 0.5(U_{i+1} + U_j)$, $R_{i+1/2}$ an artificial dissipation scale factor and $e^{(2)}$, $e^{(4)}$ solution dependent coefficients. Herewith, $e^{(2)} = \min\{k^{(2)}, \beta_1 \lambda_{i+1/2}\}$, $e^{(4)} = \max\{0, \beta_2 k^{(4)} - \beta_3 k^{(2)}\}$, $\lambda_{i+1/2} = \max\{v_i, v_j\}$, where $v_i = \|p_i - 2p_{i-1} + p_{i-1}\|/\|p_{i+1} - 2p_i + p_{i-1}\|$ is a shock sensor based on the pressure field, $p$, which controls the bend. The set of tuning parameters is given in table 1.
Table 1. JST parameters used in the present work, compared with [11]

<table>
<thead>
<tr>
<th>Parameters</th>
<th>(k^{(2)})</th>
<th>(k^{(4)})</th>
<th>(\beta_1)</th>
<th>(\beta_2)</th>
<th>(\beta_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>JST, present work</td>
<td>(1/2)</td>
<td>1/32</td>
<td>1</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>JST, [11]</td>
<td>(1/2)</td>
<td>1/128 to 1/256</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

2.3 Turbulence closure

Two-equation eddy viscosity models (EVMs) have been used in the computations, coupled with the RANS equations through the turbulent eddy viscosity \(\nu_t\). Although it is recognised EVMs fail to predict important aspects of the turbulence (as the influence of streamline curvature or the anisotropy which results from walls proximity), these closures provide a robust and cost-effective means to solve the flowfield. Compared to the zero-equation and algebraic models of the eddy viscosity class, with the two-equation closures it is expected to obtain improved predictions in wall-bounded domains with large regions of separated flow and significant anisotropy.

Within this framework, computations using the linear EVMs of Speziale et al. [12] and Launder & Sharma [13], as well as the nonlinear EVM of Craft et al. [14], all of them modified according to the \(k-\nu\) formulation (being \(\tau=\nu/\nu_t\) the turbulence timescale, defined as the turbulent kinetic energy \(k\) to the isotropic dissipation \(\nu_t\) ratio), have been accomplished. The nonlinear model extends the Bousinesq-type one-term tensor of the strain rate used in the linear EVMs to a higher order tensor representation, allowing to model the flow characteristics more accurately. Thus, the general expression which relates the Reynolds stresses \(\tau_{ij}\) with powers of the mean strain \(S\) and the rotation rate \(W\) tensors can be written as

\[
\tau_{ij} = \frac{2}{3} k \delta_{ij} + \sum_{n=1}^{N} a_n T^{(n)}_{ij}, \text{ with } T^{(1)} = S, T^{(2)} = SW - WS, \ldots
\]

where the sole contribution to the linear EVMs corresponds to \(n=1\) \((T^{(1)}=S\) and \(a_1=\nu/\nu_t\)). In contrast, the nonlinear EVM of Craft et al. incorporates a cubic expansion \((n=1, \ldots, 6,\) nonzero terms), where the nonlinearity makes the iteration sequence more time consuming and sensitive to diverge.

3 Computational domain and boundary conditions

3.1 Topology and mesh

The multiblock computational domain comprehends half planar physical domain, forcing the symmetry during the transient. The layout of the 15 block topology and connectivity is shown in Fig. 1b. The blocking dimensions are specified in table 2.
Table 2. Blocks and mesh size

<table>
<thead>
<tr>
<th>Block index</th>
<th>Dimensions</th>
<th>Number of cells</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>48 x 24</td>
<td>1152</td>
</tr>
<tr>
<td>2, 3, 14 &amp; 15</td>
<td>48 x 48</td>
<td>2304</td>
</tr>
<tr>
<td>4 &amp; 5</td>
<td>48 x 112</td>
<td>5376</td>
</tr>
<tr>
<td>6, 7, 8, 9, 10, 11, 12 &amp; 13</td>
<td>48 x 224</td>
<td>10752</td>
</tr>
<tr>
<td>Total number of cells:</td>
<td></td>
<td>107136</td>
</tr>
</tbody>
</table>

Moreover, a body-fitted refined grid with fairly uniform spacing near the axis and clustered to the flaps and to the shock tunnel end-wall has been generated, consisting of 107136 cells (Fig. 1c) and providing grid-independent results. The nearest cell neighboring the wall is placed to match the $y^+ \approx 1$ criterion closely during the time integration by means of successive $y^+$ adaptions.

Figure 1: a) Detail of the rounded-inlet 2D nozzle geometry set at the end of the shock tunnel; b) Multiblock grid topology plus blocking connectivity; c) Blanked mesh at 1/8 resolution with boundary conditions type.

3.2 Initial and boundary conditions

The flowfield initial conditions ($t=0s$) corresponds to the instant when a planar incident shock wave of Mach 3 reaches the shock tunnel end-wall ($x=0$). This shock separates two zones of gas, modelled here with constant properties as specified in table 3. The upstream conditions ($x<0$) have been calculated using the one dimensional shock jump equations and the air is taken as a perfect inert gas ($\gamma=1.4$). The extension of the computational domain upstream and downstream of the 2D nozzle is set according to the physics and time lasting of the phenomenon to minimize the size of the domain. Consequently, a reflective inlet boundary condition (with prescribed stagnation pressure and temperature) is located compliant with the distance travelled by the first reflected shock wave on the shock tunnel end-wall (Fig. 1c), being this distance calculated for a transient of $\sim160\mu s$. Downstream the nozzle the static pressure is specified.
Table 3. Gas conditions at initial time $t=0s$

<table>
<thead>
<tr>
<th></th>
<th>Shock tunnel ($x&lt;0$)</th>
<th>2D Nozzle ($x&gt;0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure (Pa)</td>
<td>64700</td>
<td>6300</td>
</tr>
<tr>
<td>Temperature (°K)</td>
<td>785</td>
<td>293</td>
</tr>
<tr>
<td>Density (kg/m$^3$)</td>
<td>0.287</td>
<td>0.074</td>
</tr>
<tr>
<td>Velocity (m/s)</td>
<td>762.8</td>
<td>0</td>
</tr>
<tr>
<td>Sound speed (m/s)</td>
<td>562</td>
<td>343.3</td>
</tr>
</tbody>
</table>

A symmetry boundary condition is used at the centre line of the domain and at the shock tunnel upper-wall. For the rest of the walls a non-slip condition plus adiabaticity is set. The required turbulence quantities have been estimated from empirical correlations and specified using uniform profiles, corresponding to an inflow turbulence intensity of 3% and a free-stream turbulence of 0.1%. The rate of turbulence dissipation is based on a length-scale of 6mm.

4 Parallelization

The code, written in Fortran 90 and parallelized with the MPI library, relies on the domain decomposition technique to distribute the computational load on each processor. Following the graph of Fig. 2, a greater than 0.9 load balance is obtained when using domains of 2, 4, 5 & 10 CPUs. The mesh blocking causes a higher load imbalance for the 3, 8 & 9 CPUs domains, which induces an additional decay of the speed-up. Consequently and due to computing resources constraints, the computations were performed using a 5 CPUs domain of a SGI Origin 2000 (speed-up ~4.48; efficiency ~0.90) and a 10 CPUs domain of a Cray T3E (parallelization figures not estimated).

Table 4. Computing cost relative to the reference Speziale closure simulation (platform: SGI Origin 2000 / domain: 5 CPUs).

<table>
<thead>
<tr>
<th>Models and main parameters</th>
<th>Relative cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>N-S, k-ε Spz 5-step Runge-Kutta / CFL=2.2</td>
<td>1</td>
</tr>
<tr>
<td>N-S, k-ε Spz 5-step Runge-Kutta / CFL=1.2</td>
<td>2.04</td>
</tr>
<tr>
<td>N-S, k-ε LS 5-step Runge-Kutta / CFL=1.2</td>
<td>2.07</td>
</tr>
<tr>
<td>N-S, k-ε CLS 5-step Runge-Kutta / CFL=1.2</td>
<td>2.36</td>
</tr>
<tr>
<td>N-S, k-ε Spz Dual time-stepping / Δt=0.01μs</td>
<td>5.67</td>
</tr>
<tr>
<td>N-S, k-ε CLS Dual time-stepping / Δt=0.01μs</td>
<td>6.05</td>
</tr>
</tbody>
</table>
Table 4 shows the relative cost of the carried out RANS simulations for the linear EVM of Speziale (Spz) and Launder-Sharma (LS), as well as for the nonlinear EVM of Craft-Launder-Suga (CLS).

An increment of ~16% in the CPU time is associated to the extra terms of the nonlinear CLS closure compared to the linear Spz EVM, for the 5-step Runge-Kutta integration case with equal Courant number (CFL=1.2).

Figure 2: Load balance, speed-up (\(\phi\)) and efficiency (\(\eta\)) in the Origin 2000

5 Numerical results

5.1 Convergence and integration schemes

Both 5-step (standard) Runge-Kutta and explicit dual time-stepping (DTS) 2\textsuperscript{nd} order integration have been used with the Spz and CLS EVMs in the computations. With the DTS scheme, a small time-step is required to obtain an adequate residuals descent using a moderate number of R-K iterations per time-step (Fig. 3), but that makes the convergence expensive in terms of CPU time, compared to the standard explicit integration.

Figure 3: Residuals of density (\(\rho\)), x-momentum (\(\rho u\)), total energy (\(\rho E\)) and turbulent kinetic energy (\(\rho k\)), for the DTS & \(\Delta t=0.01\mu s\) integration.
5.2 Analysis of results

The experiments [1] reveal that various contact surfaces (emerging from the triple points of the Mach reflections upon the centerline and nozzle flaps) evolve growing crosswise meanwhile convecting downstream and the shock waves suffer new reflections. However, it should be noted that after 4 to 5 successive reflections, the shock damping is so high that makes the resulting shock fairly undiscernible, setting up this fact a further criterion to assess the accuracy of the computations.

The snapshots of Fig. 4 (computed with Spz EVM) focus on the beginning of the transient and the main shock running downstream followed by the shock reflections which arise, prior to the separation of the boundary layer. The computed shock waves evolution stresses the good agreement with the experimental t=20μs shadowgraph included in the sequence. However, the tangential discontinuities and small vortices, clearly visible in the experiments, are smeared hear (identified by the concentration of isomach lines). This lack of sharpness in their resolution gets more and more pronounced as the time integration advances and the contact surfaces move to grid zones of larger cells and it is linked to the fact that they produce small density jumps, making difficult to have a crisp capture of them or even disappearing from the images.
At $t=34\mu s$ (Fig. 5) the presence of two strong shocks characterizes the flow: a primary bow-shaped shock (namely, PS) followed by a secondary shock pattern (SS) which consists of a Mach stem that grows crosswise until spanning the entire cross section during the next time instants. The computations have resolved some details of the contact surfaces trapped within the region bounded by the PS and SS, and the comparison of numerical-experimental shadowgraphs (Fig. 6) exhibits a good agreement. However, the sharpness of the contact surfaces appears degraded in the numerical images due to the larger cells of this zone and the small density gradients they induce (wiggles at the crests of the density profiles in Fig. 8).

Figure 4: Isomach plots (step: $\Delta M=0.05$) corresponding to the time instants of 12, 14, 16, 19 & 25$\mu$s and experimental shadowgraph at 20$\mu$s.

Figure 5: Side-by-side comparison of numerical (computed with the Spz EVM) and experimental shadowgraphs (reprint from [1]) at $t=34\mu$s.
Figure 6: Side-by-side comparison of numerical (upper half) and experimental (lower half) shadowgraphs for various time instants. (Closure: Speziale et al. / PS: primary shock; SS: secondary shock pattern; CS: contact surface; ST: shocklet; SL: shear layer; V: vortex; EW: expansion waves).
Computations have confirmed that shock-induced separation occurs at $\sim 48\mu s$ (for both linear and nonlinear EVMs), which agrees well with the experimental evidence. This good prediction of the separation station remains during the next $\sim 5$ to $8\mu s$, but the progressive divergence is clear from the graph given in Fig. 10. This divergence can be attributed to the complex interaction between the regions of accelerated and decelerated flow upstream the PS, which seems to depend on the turbulence modelling and influences the later SS pattern development. At $t=56\mu s$ (Fig. 6), the SS is a planar wave with an incipient vortex visible ahead its edge, in the vicinity of the flap. The small size of the separated flow region explains the low sensitivity of the results to the EVM.
As the shock pattern moves downstream and the shear layer spreads, the SS evolves towards a $\lambda$-type bifurcation, more sensitive to the eddy-viscosity closure. The set of numerical shadowgraphs given in Fig. 6 (corresponding to the Spz EVM) demonstrates the inaccuracy in the detection of the rear branch of the $\lambda$-shock structure. Thus, the examination of the $t=75\mu s$ snapshot in Fig. 6 permits to identify the rear branch as a weak shock wave that emanates from the vicinity of the separation point and reaches the Mach stem (a quite similar behaviour is obtained for the LS EVM computations).

On the contrary, the plots corresponding to the simulations performed with the nonlinear $k-\tau$ CLS closure, present a sharp resolution of the rear branch, as it is observed in the $t=75\mu s$ (Fig. 7) and $130\mu s$ (Fig. 9) isomach plots. Not only an improvement in the $\lambda$-shock resolution is obtained by means of the CLS closure, but besides the time-dependent station of the SS Mach stem on the centerline fits better the experimental data, as it is seen in Fig. 10, where the secondary shock Mach stem computed with the Spz EVM runs slightly ahead of the measured position. Nevertheless, it is stressed that though a crisp capture of the $\lambda$-structure has been attained with the CLS EVM, this closure has been neither capable to correctly predict the evolution of the separation point (Fig. 10), with an even additional lack of accuracy compared to the linear closure.

6 Final Remarks

RANS turbulent simulations of the starting flow in a shock-tunnel 2D nozzle have been performed and the comparison with the experimental data exhibits a general good agreement for the entire transient, though it is noticeable that exists a progressive lost of accuracy in the prediction of the time-dependent position of the separation and, consequently, in the arising shock structures.

This discrepancy seems to be linked to the inherent limitations of the EVMs, which fail in the capture of the complex physics involved in the strong secondary shock pattern ↔ shear layer interaction. The analysis of the results points out the
improvement obtained with the nonlinear CLS EVM, though it also fails to predict the time-dependent separation.

Acknowledgments

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References