Viscoelastic beam damping and piezoelectric control of deformations, probabilistic failures and survival times – analytical and massively parallel computational simulations

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Abstract

The effects of controlling viscoelastic beam creep deformations through applications of piezoelectric devices is investigated and solved by finite element methods. It is shown that delamination failure probabilities can be retarded and life times extended by the proper imposition of voltages. Computational performance evaluation studies for CPU times and speed ups are undertaken for distinct number of processors and evaluated. It is shown that the introduction of piezo devices and viscoelastic effects not only decrease temporal probabilities of failure, but also retard the structure’s survival times. These results are also of importance in inverse problems, where voltages may be input to generate controlled displacements. Thus, such voltages, either generated and dissipated through resistors or input from external sources, can be used to control viscoelastic deformations, failure probability envelopes and survival times of composite and elevated temperature metal structural elements. It also shown that the use of LS-DYNA3D™ significantly improves CPU times and speed ups compared to previous usage of ABAQUSTM.
1 Introduction

The subject of this study, as the title indicates, involves formulations of analyses to resolve problems of structural control of deformations and failures through the application viscoelastic material damping and the use of piezoelectric devices. Detailed viscoelastic constitutive relation material properties are extensively described in Lazan [1], Nashif et al. [2] and Jones [3]. On the other hand, available literature on viscoelastic deterministic and stochastic experimental failure properties is considerably more sparse and confined to uniaxial properties [3, 4]. Viscoelastic piezoelectric responses have been confirmed experimentally by Vinogradov and her associates [5 – 9].

The possibility of material damping due to viscoelastic material property degradation with time have been analyzed by Yi et al. [10]. Additional control of deformations and/or stresses by the introduction of elastic or viscoelastic piezo devices has been shown most advantageous in a number of vibration and aero-viscoelastic problems by Beldica et al. [11 – 14].

Piezoelectric effects are of increasing importance in the control and sensing of structural element deflections. Elastic piezoelectric theory has been extensively developed by Leibowitz & Vinson [15], Preumont [16] and Shrinivasan & McFarland [17]. Hilton et al. [14, 18] have presented an extended bibliography and discussed past contributions as well as formulated an analytical nonlinear theory of anisotropic piezoelectric-thermo-viscoelasticity.

Computational issues associated with viscoelastic formulations take on additional complications because of their time dependent constitutive relations resulting in differential-integral governing relations. Zak [19] and Taylor et al. [20] have evaluated these integrals in terms of incremental time steps while Yi et al. [21 – 25] based on Fourier series and recursion relations. The latter recursion relations depend on the last two time intervals as opposed to only on interval of [19, 20] and while requiring more storage space execute much faster because larger time intervals can be utilized.

Finally, Yi et al. [26] have extended the FE approach to viscoelastic piezo controlled structures. Beldica et al. [27 – 29] have used ANSYS™ to generate 3-D brick type finite elements to solve bending and torsion problems. Once realized, the mesh was transferred to ABAQUS™ for 2-D calculations of stresses and deformations using the native viscoelastic ABAQUS™ subroutines. Unfortunately, even with a mesh of 200,000 DOF, the commercial software was capable of supporting only sixteen processors. Such an inherent limitation does not utilize the full capabilities of modern supercomputers with a 1000 or more parallel CPUs.
The present pilot study is part of a series of systematic fundamental investigations into the combined effects of damping arising from piezoelectric-thermo-viscoelasticity used to control and minimize undesirable structural deformations contributions in flexible flight vehicles. A spanwise nonhomogeneous viscoelastic beam under spanwise piezoelectric structural control is considered (Fig. 1). The governing integral-differential relations are formulated, and these equations are reduced to spatial finite elements and temporal finite differences and solved through massively parallel computational approaches employing LS-DYNA3D™ software with spatial FE and temporal FD (Fig. 2), thus utilizing the massively parallel capabilities of NCSA’s supercomputers. Analytical and numerical solutions are presented for viscoelastic beams with piezoelectric devices. Probabilities of failure and survival times are calculated based bending and shear stresses on using the Hilton–Ariaratnam failure theory [30]. Examples of computational simulations based on this theory for predicting probabilities of failure and survival times can be found in [31–34].

2 Analysis

2.1 Constitutive Relations

In a Cartesian system $x = x_i$ with $i = 1, 2, 3$, the linear viscoelastic materials at constant temperature the shear and bending stress constitutive relations are

$$
\sigma_{23}(x, t) = \int_0^t \phi_\sigma(x, t, s) \frac{\partial e_{23}(x, s)}{\partial s} \, ds
$$

(1)

$$
\sigma_{11} = \int_0^t \phi_\sigma(x, t, s) \frac{\partial e_{11}(x, s)}{\partial s} \, ds
$$

(2)

The governing relation for the bending deflection $w(x_1, t)$ for a prismatic, homogeneous and isotropic viscoelastic Euler-Bernoulli beam depicted in Fig. 1 then becomes

$$
\int_0^t [\phi_h(t - s) I_1^{eff} \frac{\partial^3 w(x_1, s)}{\partial x_1^2 \partial s}] \, ds = -M(x_1, t)
$$

(3)

where the applied moment $M$ is due to the beam loading and to piezo-viscoelastic stresses. Piezo device actions through applications of EMFs $\overline{E}_1(x_1, t)$ at $x_3 =$...
\[ +h = \pm(0.5h_0 + h_\alpha) \text{ with } h_\alpha \ll h_0, \text{ lead to stresses} \]

\[
\sigma_{11}(x_1, x_2, t) = \int_0^t E(t-s) \frac{\partial \bar{E}_1(x_1, s)}{\partial s} \, ds \quad \frac{h_b}{2} \leq x_2 \leq h
\] (4)

which yield additional applied moments equal to \(2h\sigma_{11}^P A_P\) with \(A_P\) the area of piezo device.

The term \([\phi J]_{eff}\) represents the effective torsional rigidity with relaxation functions \(\phi(t)\) given by

\[
\phi(t) = \phi_0 + \sum_{n=1}^{N} \phi_n \exp \left[ -\frac{t}{\tau_n} \right]
\] (5)

and where the \(N, \phi_n, \tau_n\) are material properties. As shown in Fig. 4, the beam, piezo device and its bonding agent may all have distinct viscoelastic relaxation moduli. Such material properties have been catalogued by Lazan [1], Nashif et al. [2] and Jones [3]. Finally, the initial conditions are prescribed by \(w(x_1, 0) = w^0(x_1)\).

Beam shear stresses arising from non-pure bending are given by

\[
\sigma_s(x_1, x_2, t) = \frac{1}{h^*(x_2)} \int_{x_2}^{h} \frac{\partial \sigma_b(x_1, x_2, t)}{\partial x_2} \, h^*(x_2) \, dx_2
\] (6)

where \(h^*(x_2)\) is the beam thickness.

### 2.2 Multiaxial Deterministic and Probabilistic Failures

Hilton & Ariaratnam [30] have formulated empirical invariant combined stress failure laws, which are applicable to both deterministic and stochastic conditions. These probabilistic formulations have been applied to column creep delamination buckling and to linear viscoelastic service and manufacturing delamination onset predictions (Hilton et al. [31–34]. Hiel et al. [5] have experimentally shown that uniaxial composite delamination failures obey Weibull distributions.

Consider applied random stress invariants defined by

\[
\bar{J}_1 = \bar{\sigma}_{ii}, \quad \bar{J}_2 = \bar{\sigma}_{ij} \bar{\sigma}_{ij}, \quad \bar{J}_3 = \bar{\sigma}_{ij} \bar{\sigma}_{ik} \bar{\sigma}_{jk}
\] (7)

with similar expressions for uniaxial random failure stresses \(\tilde{F}_{ij}\) and their means

\[
\bar{J}_1 = \bar{F}_{ii}, \quad \bar{J}_2 = \bar{F}_{ij} \bar{F}_{ij}, \quad \bar{J}_3 = \bar{F}_{ij} \bar{F}_{ik} \bar{F}_{jk}
\] (8)
Functions with a ~ indicate stochastic quantities, while those without are their corresponding mean values. Mean uniaxial viscoelastic delamination strengths are shown in Fig. 3 (Dillard & Brinson [4]).

These failure criteria are cast in the forms of

\[
\frac{1}{3} \sum_{i=1}^{3} \left[ \frac{\tilde{J}_i(x, t)}{J_i(x, t)} \right]^{c_i} = \tilde{V}(x, t) \quad \frac{1}{3} \sum_{i=1}^{3} \left[ \frac{\tilde{J}_i(x, t)}{J_i(x, t)} \right]^{c_i} = \tilde{v}(x, t)
\] (9)

and local failure occurs whenever

\[
\tilde{u}(x, t) = \tilde{V}(x, t) - \tilde{v}(x, t) \geq 0
\] (10)

Uniaxial failure data by Hiel et al. [5] for epoxy/fiber composite delaminations indicate Weibull distributions, consequently for combined loads the local failure probability \( \tilde{P}_L \) and the failure probability \( \tilde{P} \) for any structural component then become

\[
\tilde{P}_L(x, t) = 1 - \exp \left\{ - \left[ \frac{\tilde{u}(x, t)}{\beta} \right]^\alpha \right\}
\] (11)

\[
\tilde{P}(t) = \max \left\{ \tilde{P}_L(x, t) \right\}
\] (12)

where \( \alpha \) and \( \beta \) are experimentally determined material parameters.

The stress invariants for this beam loading configuration are given by

\[
J_1 = \frac{1}{3} |\sigma_b| \quad J_2 = \frac{1}{3} (\sigma_b^2 + 2\sigma_s^2)
\]

\[
J_3 = \frac{1}{3} \left( |\sigma_b^2| + 3 |\sigma_s^2| \sigma_2^2 + 3 |\sigma_b| \sigma_s^2 + |\sigma_s^3| \right)
\] (13)

with similar expressions for the \( \tilde{J}_i \) and \( \tilde{J}_i \) stochastic invariants, and the mean value invariants \( J_i \).

3 Discussion and computational simulations

The viscoelastic material properties of the bar, piezo device and bonding agent may all have distinct relaxation curves, as shown in Fig. 4. In problems where geometry, stiffnesses and/or viscoelastic properties are functions of any coordinate exact analytical solutions are unachievable and one must resort to finite element (FE) and finite difference (FD) approaches.

An illustrative simulation example was undertaken consisting of a cantilever beam of length \( L \) (Fig. 2) with a concentrated step load at the end of 1 microsecond duration, i.e. \( P(L, t) = P_o[H(t) - H(t - 10^{-6})] \). Representative results
are illustrated in Figs. 5 – 7. It is seen that for this configuration both bending and shear stresses exhibit highly transient effects and generally start decreasing in magnitude due to viscoelastic damping and piezo control after $t > 10^{-3}$. Even though the uniaxial delamination (failure) stresses are also decreasing (Fig. 3), a peak in the failure probability takes place at $t \approx 10^{-3}$ with a corresponding survival time in the same neighborhood. Different combinations of viscoelastic beams, bonding agents, piezo devices and loads as well as control voltages will produce distinct results.

Previous similar studies based on the use of ABAQUS\textsuperscript{TM} [27 – 29] were limited to 16 processors by the software. Consequently, CPU times were greater and speed ups smaller than in the current study using LS-DYNA3D\textsuperscript{TM} as shown in Fig. 8. This figure shows that significant CPU time savings can be achieved ($\approx 1.5$ orders of magnitude) for 64 processors when the speed up is 0.48. At this number of CPUs computational times and speed ups are beginning to become asymptotic and additional computational efficiencies are not materializing.

**Conclusions**

The present study shows that the use of LS-DYNA3D\textsuperscript{TM} allows for significantly more parallel processors and considerably improves CPU times and speed ups when compared to ABAQUS\textsuperscript{TM}. The damping of viscoelastic beams is demonstrated through usage of material damping and the application of piezo electric voltages.

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**References**


Figure 1: Beam with piezo devices
Figure 2: Beam with finite elements

Figure 3: Uniaxial stresses for delamination onset

Figure 4: Component relaxation moduli

Figure 5: Bending stresses

Figure 6: Shear stresses

Figure 7: Probability of delamination onset and survival times

Figure 8: CPU time and speedup