



New structure for rolling mapping

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Abstract

In this paper, we propose new architecture that has some PU(Processing Unit) arrays of Nearest Neighbor Mesh(NNM), Using some NNM PU arrays, it is faster than one PU array when calculate the rolling mapping for the mesh with various densities.

1 Introduction

Partial differential equations(PDEs) are required to be solved in many scientific problems for the analysis of physical region is divided into a nearest neighbor mesh(NNM).[5]

Although a dense mesh (a mesh with a large number of grid points) improves the accuracy of the solution, the computation time is stretched with a large amount of calculation. Thus, a mesh with partially various densities improves the accuracy locally without explosion of the computation cost. For example, a discrete physical region shown in Fig. 1 has a part which is the quadruple density of the surrounding region.

Here, we call this type of discrete region "mesh with various densities". [7]

For calculations of PDEs, parallel machines in which Processing Units(PUs) are connected with an NNM connection structure have been utilized because the required communication is usually matched with the connection structure. However, if a mesh with various densities is utilized, the PUassignment algorithm(mapping) becomes completes.

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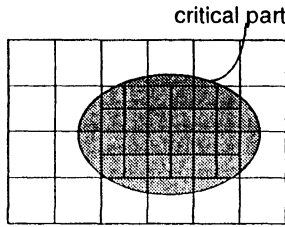


Figure 1: Physical region with quadruple density

A direct mapping, which is popularly utilized in a uniform mesh, causes a serve load imbalance for the mesh wish various densities. Using the modular mapping requires the cyclic connection structure and data example between far PUs on the boundary of density.

In this paper, we propose a new mapping algorithm called the rolling mapping, which neither requires the cyclic structure nor increases the data exchange. Next, we propose a structure of PU array for rolling mapping, and finally we propose 3-dimensional structure of PU array by folding up the 2-dimensional PU array.

2 Rolling mapping

2.1 The Modular mapping and its problem

The PDE is reduced into parallel tasks corresponding to the grid pints of the physical region. For parallel computations, these tasks are allocated into PUs. Here, we call this allocation "mapping" or "mapping between grid points and PUs".

The direct mapping[5][7] is the simplest mapping algorithm. But this method is unsuitable for mesh with various densities. It causes a severe load imbalance and performance degradation.

In the modular mapping[5][7], grid points are mapped into PUs as follows:

$$\begin{aligned} p &= \text{mod}(X, m) \\ q &= \text{mod}(Y, m) \end{aligned} \quad (1)$$

where the size of the PU array is $(m \times m)$.

Unlike the direct mapping, the same number of grids are assigned into a PU as shown in Fig.2.

Here, the direction of a PU array is defined. We call the PU array takes the direction 1, when the PU(0,0) is located at the left and top most as shown in Fig.3. The direction 2 is the vertical mirror image of direction 1.

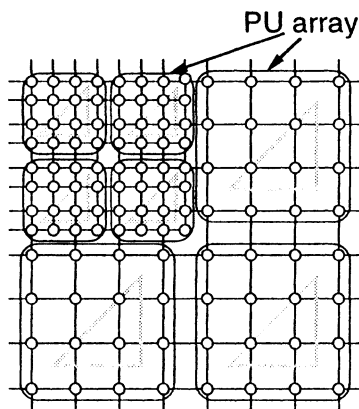


Figure 2: Modular mapping with critical part

The direction 3 and direction 4 are horizontal mirror image of the direction 1 and 2 respectively. We show the direction using the triangle mark in the PU array as shown in Fig. 3.

In the modular mapping, the PU array is mapped on the physical region so as to keeping its direction the same. Thus, the same triangle is marked for all mapped PU arrays in Fig.4.

If the following conditions are satisfied, the load of each PU becomes the same in the modular mapping:

1. The density only varies in the unit region size which is corresponding to the PU array size ($m \times m$).
2. The boundary of the normal part and denser part must form a nearest neighbor mesh.
3. The density of a denser part must be the quadruple of the neighboring part.

Fig.4 shows an example of the modular mapping in a uniform mesh, while Fig.6 shows the example of the mesh with various densities. Grid points in the region with high degree of density are distributed into 4 PUs.

However, the modular mapping increases the communication cost. Even for the mesh with the uniform density, it requires the torus structure in the target PU array. That is, cyclic(round-trip) links are required to connect between the ends of the PU array. Although the torus structure is also widely used, some recent parallel computers (for example Paragon[2], J-machine[3]) take the pure NNM structure in order to avoid the cost and performance overhead introduced by cyclic links which usually requires long cables.



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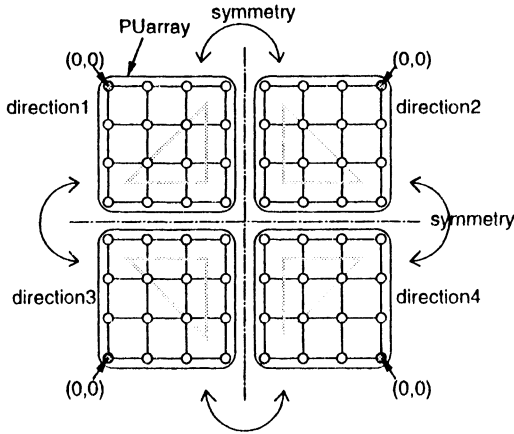


Figure 3: The direction of PUs

Even in the machine with the torus structure, cyclic links cannot be useful when the partitioning is introduced in the machine. When a large machine is shared with multiple users, the machine is divided into small PU arrays without cyclic links and assigned into each user. This technique is called the "machine partitioning", and widely used in a large scale multi-computer. In such a case, the PU array supported for each user does not provide cyclic links.

Moreover, the modular mapping also increases the communication between remote PUs when it is used for a mesh with various densities. As shown in Fig.5, the data exchange between grid points on the boundary is actually performed between remote PUs.

2.2 Rolling mapping

In order to cope with problems of the modular mapping, we propose a new mapping method called the rolling mapping. In this method, the PU array rolls on the physical region in the vertical and/or horizontal directions. The mapping method is shown as follows:

$$\begin{aligned}
 P &= \text{mod}(X, 2m) \\
 p &= P(P < m) \\
 p &= -P + 2m - 1(P \geq m) \\
 Q &= \text{mod}(Y, 2m) \\
 q &= Q(Q < m) \\
 q &= -Q + 2m - 1(Q \geq m)
 \end{aligned} \quad (2)$$

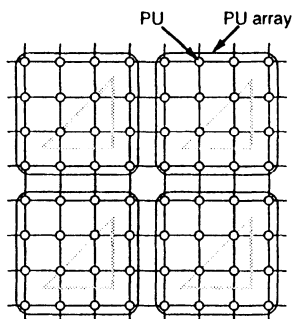


Figure 4: PU arrays on the grid points as Modular mapping

Fig.7 shows an example of the rolling mapping for a uniform mesh. In this figure, a physical region with 8×8 grid points is mapped into a PU array whose size is 4×4 . Unlike the modular mapping, the direction of the PU array is changed in the horizontal/vertical mirror image. For the uniform mesh, no cyclic links nor remote communication are required since grid points which requires cyclic links in the modular mapping are mapped into the same PU in the rolling mapping.

2.3 Rolling mapping on the physical region with differing densities

Like the modular mapping, the number of grid points which are assigned into a PU is equalized in the rolling mapping. The conditions to keep the load balance of a PU are almost the same as those of the modular mapping except the region size. When the unit region size of density is set to be the same as the size of PU array ($m \times m$), there may be an inconsistency in the direction of the array along the boundary as shown in Fig.8. This increases the communication between PUs, and the benefits of the rolling mapping are lost.

In order to avoid this inconsistency, the condition 1 for the modular mapping is modified for the rolling mapping as follows:

- 1' The density is changed in the unit region size which is corresponding to the quadruple size of the PU array ($2m \times 2m$).

As shown in Fig.10, the inconsistency never occur when this condition is satisfied. Fig.10 shows the boundary of the region with different densities. Although there are some PUs which require communication remote PUs, they are all allocated on the same side of the PU array.



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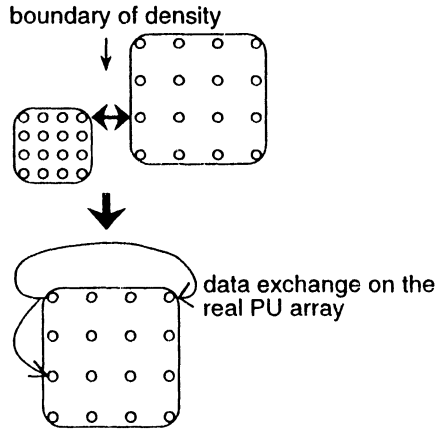


Figure 5: Data exchange on the boundary of different density

3 Rolling mapping with rolling mesh

3.1 The rolling mesh connection structure

Here, we introduce a connection structure called the rolling mesh dedicated for the mesh with various density. The rolling mesh is an extended NNM structure with remote links only on sides of the PU array. Using these links, there are no indirect communication between PUs even for the mesh with various densities.

Fig.11 shows the boundary of various densities when the rolling mapping is utilized. Here, we assume a PU array of 8×8 , and gives a serial number n and m for PUs on the both sides. In this case, the pairs of PUs shown in Fig.12 requires communication.

In general, for a PU array of $2N \times 2N$, PUs which require communication are shown as follows:

$$\begin{aligned}
 m &= f(n) = 2n + 0 & (0 \leq n \leq \lfloor (p+1)/2 \rfloor - 1) \\
 m &= f(n) = -2n + (4N - 1) & (\lfloor (p+1)/2 \rfloor \leq n \leq p - 1) \\
 m &= f(n) = -2n + (4N - 2) & (p - 1 \geq n \geq \lfloor p/2 \rfloor) \\
 m &= f(n) = 2n + 1 & (\lfloor p/2 \rfloor - 1 \geq n \geq 0) \quad (3)
 \end{aligned}$$

Therefore, if links are added between these PUs, every message can be transferred directly between PUs. Fig.13 shows 8×8 rolling mesh connection. 3 additional links are required other than 3 links to form the side of NNM structure. Therefore, the degree of the rolling mesh is 6.

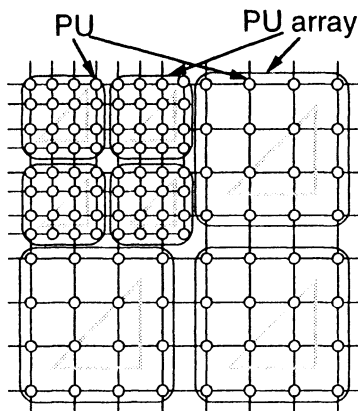


Figure 6: Modular mapping with critical part

3.2 3-D Structure of PU Array

When the density of unit region size is the quadruple of the PU array size, rolling mapping can assign the job so as to calculate mesh with various density effectively. With the advanced semiconductor technologies, it is possible to build a parallel computer with a large PU array. In this case, the quadruple of the PU array is sometime too large for the unit region size of grid density.

And, when the PU array size becomes large, 3-dimensional PU array structure is commonly used instead of the 2-dimensional structure. Here, we show a extended method of rolling mapping for 3D PU array and connection between PUs on the edge of PU array.

3.3 Folding PU array

Here, we suppose a PU array whose size is the quadruple the size which is used for the rolling mapping. Then, the modular mapping (not the rolling mapping) is applied to the mesh with various densities. In this case, since the modular mapping is used, the load is well distributed to each PUs' but cyclic links are required.

To avoid it, we fold the PU array twice as shown in Fig.14. So the size of folded PU array is the same as the unit region size. In this case, PUs on the edge of the PU array is located near from opposite PUs also on the edge of the folded PU array. That is, no long wires for cyclic links are required.

However, on this PU array, data interchange between PUs which are not on the folded lines but on the edges of PU array is required.

Here, we suppose four PU arrays to assign from the first layer to the

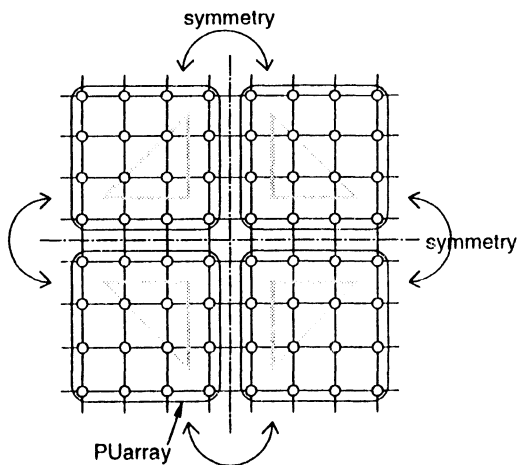


Figure 7: Rolling mapping

4th layer in Fig.14.

Here, we assign the increasing layer number into four PU arrays from the top PU array(Fig.15). On the edge A in Fig.15, the first layer is connected with the forth layer, and the second layer is connected with the third layer. On the other hand, on the edge B in Fig.15, the first layer is connected with the second, and the third is connected with the forth, respectively. In this case, two rolling mesh connections are requires for both A and B edges.

3.4 Two rolling meshes

Here, if shared vertical connection is added on the edge, the rolling mesh connection is required only one of four layers.

Since only limited number of PUs (in this case, four PUs) are connected with the vertical direction, a shared bus or crossbar can be used as shown in Fig.16.

3.5 Extension of the method

The above mentioned folding method can be also applied to the twice or four times of unit array. Fig.17 shows 4 sets of arrays each of consisting 4 unit arrays. This structure enables to parallel computation for dense and sparse part with 16 PU arrays.

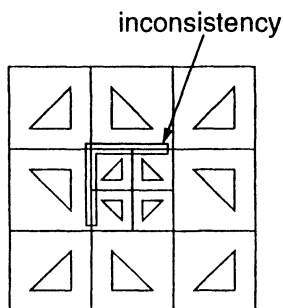


Figure 8: Inconsistency of Rolling mapping

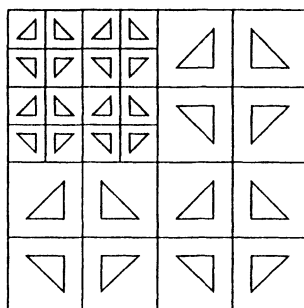


Figure 9: The direction of the PU array

4 Conclusion

In this paper, we proposed a mapping method called the rolling mapping for assigning tasks of the PDE into multiprocessor with the NNM structure. Using this method, the mesh with various densities are mapped into PU array without cyclic links nor increasing the data exchange between remote PUs. The implementation method for 3D connection structure is also shown.

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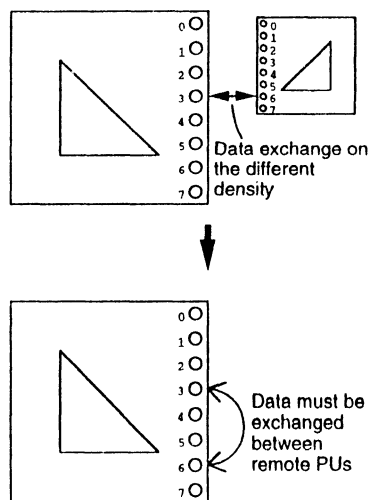


Figure 10: Data exchange on the boundary of different density

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boundary of density

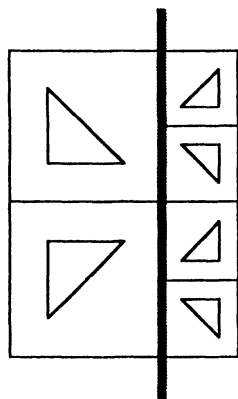


Figure 11: Unit of the boundary of density

n	m	
0	0	
1	1	
1	2	
2	3	
2	4	
3	5	
3	6	
4	7	
4	7	
5	6	
5	5	
6	4	
6	3	
7	2	
7	1	
7	0	
7	0	
6	1	
6	2	
5	3	
5	4	
4	5	
4	6	
3	7	
3	7	
2	6	
2	5	
1	4	
1	3	
0	2	
0	1	
0	0	

single

quadruple

$m=2n$

$m=-2n+15$

$m=-2n+14$

$m=2n+1$

Figure 12: PU numbers of the boundary of density