A wavelet-based method to simulate gust response of structures

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Abstract

Under winds having stationary stochastic processes, the standard deviation of the gust response displacements of a structure is generally obtained with the frequency domain analysis, and the maximum response amplitude is estimated using the standard deviation and the peak factor. On the contrary, in the case of wind having time-dependent characteristics, the frequency domain analysis is not practical and it is impossible to estimate the maximum response amplitude. In order to overcome this underlying problem, a wavelet-based method to simulate the time history of the gust response is suggested in this study. The time-scale characteristics in wind fluctuations are configured with the system of the discrete wavelet, and their effects are reflected in the time histories of the gust responses to be synthesized. The horizontal gust responses of a structure in various wind velocity fluctuations are simulated with the proposed method, and the accuracy of the predicted response time history is discussed.

Keywords: gust response, peak factor, structure, wavelet, wind fluctuation.

1 Introduction

The aerodynamic forces on structures are generally expressed as the sum of the buffeting forces and the motion-induced forces. These forces include the frequency-dependent characteristics, and the frequency domain analysis [1, 2] has been conventionally used to estimate the gust responses of structures; many studies were performed on incorporating these frequency-dependent characteristics in this analysis. However, the frequency domain analysis is applicable only under the assumption of winds having stationary stochastic processes. The time domain analysis is then required in order to reflect the time-dependent characteristics present in winds to the gust responses, in which case a
problem is how the frequency-dependent characteristics are configured. A
method to overcome this underlying problem is the application of the rational
function [3, 4], and in many studies [5, 6, 7] this function was applied to
represent the motion-induced aerodynamic forces of bridges. Chen and Kareem
[7] recently established an integrated method using the rational function for the
time domain aeroelastic analysis of bridges.

In this study, a simple method to simulate the time history of the gust
responses of structures is proposed. Although this method is based on the quasi-
steady approach, the time-scale characteristics present in wind fluctuations are
configured with the system of the discrete wavelet, and the effects of them are
reflected to the time histories of the gust responses to be synthesized. The
wavelets have been used as a tool to analyze measured data, and they have also
been applied to simulate time histories with time-dependent characteristics [8, 9,
10, 11]; the method proposed in this study is classified to one of these studies.
Omitting the aerodynamic admittance in the aerodynamic force for the purpose
of investigating the fundamental performances of the proposed method, the along
wind gust responses of a structure with single-degree-of-freedom (SDOF) are
simulated. The simulated gust response time histories are compared with those
obtained from the Newmark $\beta$ method, and in particular the accuracy of the peak
factors of the gust responses are discussed.

2 A wavelet-based simulation on gust response

According to the quasi-steady theory on the horizontal gust response of a
structure with SDOF, the fluctuating drag force due to the horizontal fluctuating
wind $u(t)$ is expressed as

$$F(t) = \frac{2P}{U} X_D \left( \frac{fd}{U} \right) u'(t) - \frac{2P}{U} \dot{x}$$  \hspace{1cm} (1)

where $U$ : mean wind speed of $u(t)$, $u'(t) = u(t) - U$, $\dot{x}$ : response velocity of a
structure, $d$ : width of a structure, $X_D$ : aerodynamic admittance, $f$ : frequency, $P$
$= 1/2 \rho d C_D U^2$, $\rho$ : air density and $C_D$ : drag coefficient. In this study, in order
to configure the time-dependent characteristics of $u(t)$ to the drag force, local wind
fluctuations decomposed by discrete wavelets $\psi(t)$ and the wavelet coefficients
$\alpha_{j,k}$:

$$u_{j,k}(t) = \alpha_{j,k} \psi_{j,k}(t)$$  \hspace{1cm} (2)

$$\alpha_{j,k} = \int_{-\infty}^{\infty} u(t) \psi^*_{j,k}(t) dt$$  \hspace{1cm} (3)

$$(\psi_{j,k}(t) = 2^{j/2} \psi(2^j t-k), \quad \ast : \text{complex conjugate}, \quad j, k \in \mathbb{Z})$$

are applied to eqn. (1). In this study, the Meyer orthonormal wavelet [12, 13]
shown in fig. 1 is used for $\psi(t)$, which has the compact support in frequency
domain. Substituting $u_{j,k}(t)$ to $u'(t)$ in eqn. (1), eqn. (1) is redefined as

$$F_{j,k}(t) = \frac{2P_{j,k}}{U_{j,k}} X_D \left( \frac{fd}{U_{j,k}} \right) u_{j,k}(t) - \frac{2P_{j,k}}{U_{j,k}} \dot{x}_{j,k}$$  \hspace{1cm} (4)
\( P_{j,k} = \frac{1}{2} \rho d C_D U_{j,k}^2 \) and \( \dot{x}_{j,k} \) is the response velocity induced by the \( j-k \) dependent drag force \( F_{j,k} \). The local mean wind speed \( U_{j,k} \) is calculated from the wind velocity data around the time where \( u_{j,k}(t) \) locates:

\[
U_{j,k} = \frac{1}{T} \int_{T/2}^{T} u(t) dt
\]

where \( T \) is the total time of \( u(t) \). Applying eqn. (4) to the equation of motion of SDOF, the \( j-k \) dependent response displacement in frequency domain, \( x_{j,k}(f) \), is

\[
x_{j,k}(f) = F_{j,k}^E(f) H_{j,k}(f) / K.
\]

\( F_{j,k}^E(f) \) in eqn. (6) is the first term in the right-hand side of eqn. (4) in frequency domain:

\[
F_{j,k}^E(f) = \frac{2P_{j,k}}{U_{j,k}} X_D \left( \frac{fd}{U_{j,k}} \right) u_{i,k}(f)
\]

where \( \Psi_{f,k} \) is the frequency domain expression of \( \psi_{j,k}(t) \), and \( H_{j,k}(f) \) is the mechanical admittance of a structure; \( K \) is the spring constant. The damping ratio \( \zeta \) included in \( H_{j,k}(f) \) is given as the summation of the structural damping ratio and the aerodynamic damping ratio based on \( U_{j,k} \):

\[
\zeta = \zeta_0 + \frac{\rho d C_D U_{j,k}}{4 \pi f_0 M}
\]

where \( f_0 \) is the natural frequency of a structure and \( M \) is the mass. Similarly to the inverse wavelet transform, the gust response displacement in frequency domain, \( x(f) \), is synthesized as the summation of \( x_{j,k}(f) \):

\[
x(f) = \sum_j \sum_k x_{j,k}(f)
\]

However, the static responses are not configured in \( x(f) \) obtained by eqn. (9) since \( \Psi_{f,k} \) does not cover the very low-frequency range including the static component. Adding the response near \( f = 0 \) to the right-hand term of eqn. (9), \( x(f) \) can be

\[
x(f) = \Theta(f) + \sum_j \sum_k x_{j,k}(f).
\]
The response near $f=0$, $\Theta(f)$, is approximately calculated as

$$\Theta(f) \approx \frac{\nu^d C_u U^2}{2K} \Phi(f)$$  \hspace{1cm} (11)

where $\Phi(f)$ is the scaling function. Applying the inverse Fourier transform to eqn. (10), the time history of the gust response displacement $x(t)$ is obtained.

### 3 Gust response simulation using wavelet-based method

#### 3.1 Characteristic of approaching wind

Wind velocity data to be used as the input of the gust response simulation in this study were measured with an ultrasonic anemometer at the top of a building of Nagoya University. It is conjectured that measured wind fluctuation data include the effects of turbulence due to various buildings around the site because the position of the anemometer was only 15m high from the ground level and there were some buildings higher than the measurement height. The measurements were carried out for several days in March and August 2003. Although three velocity components in along, vertical and horizontal to the approaching wind direction were sampled simultaneously, only data in approaching wind direction were used as $u(t)$ because the simulation performed in this study is on horizontal gust responses of SDOF. The sampling frequency was 20 Hz and 80 wind velocity records have been obtained.

The mean wind speed $U$, the turbulence intensity $I_u$ and the turbulence scale $L_u$ of measured wind velocity records are shown in figs. 2 (a), (b) and (c) respectively. However, these values are of which the time histories are assumed to be stationary. In fact, the time dependency was present in many of the records. Additionally, because the power spectra of $u(t)$ were almost similar to the Karman spectrum:

$$S_x(f) = \frac{4I_u^2 L_u U}{[1+70.78(fL_u/U)^2]^{7/6}}$$  \hspace{1cm} (12)

$L_u$ in fig. 2 (c) were obtained by fitting (12) to the power spectra of $u(t)$ with the least-square method. In fig. 2 (a), $U$ distributed within 4 – 11 m/s and are not high comparatively to those as observed at high altitude or over flat terrains under typhoon oncoming. $I_u$ shown in fig. 2 (b) were of 0.2 - 0.5, and are much
higher than the values of which the winds over the sea or flat terrains provide, because of the existence of buildings surrounding the measurement site. $L_u$ in fig. 2 (c) varied in the range of 50 – 160 m and this is also thought to be due to the buildings near the site. Additionally, the arrangement and the scale of buildings around the measurement site were not uniform, and the approaching wind direction was different depending on the time and the date when the measurements were carried out. Thus, each measured winds had different turbulence development processes, and this may result in the variety of $L_u$.

3.2 Time history of simulated gust response

The gust responses of SDOF system were simulated using the wavelet-based method. The structural property adopted in this study was of Akashi-Kaikyo Bridge in Japan: $d$ was given 35.5 m that is the width of the bridge deck section; $C_D$ was 0.386 and this value was obtained from a wind tunnel experiment; the logarithmic damping ratio $2\pi\zeta_0$ was given 0.03, which is the value of the experimental model. The natural frequency $f_0$ was set as 0.116 Hz and this is approximately 3 times as high as the first mode natural frequency of the bridge. This reason is to obtain many response wave numbers in the record time $T (=409.55 \text{ s})$. Also, as described in the introduction, the aerodynamic admittance $X_D$ in eqn. (7) was set to 1 for all frequencies.

At first, the gust response obtained by the wavelet-based method under $U_j,k = U$ is investigated. The response in the case of $U_j,k = U$ is equivalent to the response under the drag force of eqn. (1) with $X_D = 1$, because the summation of $u_j,k (t)$ corresponds to $u'(t)$ and the summation of $F_j,k (t)$ is then equal to $F (t)$ of eqn. (1). Fig. 3 is one of measured wind velocity records and local strong fluctuations were found around 280 s and 340 s; the mean wind speed after 120 s is somewhat higher than that before the time. The power spectrum of fig. 3 is shown in fig. 4, and that was almost consistent with the Karman spectrum of eqn. (12) under $U=7.5 \text{ m/s}$, $I_u=0.28$ and $L_u=100 \text{ m}$. Fig. 5 shows the response displacement under the wind velocity in fig. 3, which was simulated by the wavelet-based method with $U_j,k = U$. Also in fig. 5, the response displacement predicted with Newmark $\beta$ method is illustrated with a dotted line, in which the SDOF equation of motion using the drag force shown in eqn. (1) and the static component $P$ was solved in time domain. In this time domain analysis, in order to avoid the impulsive responses at $t = 0 \text{ s}$, the wind velocity data were smoothed in the duration of $0 \leq t \leq 10 \text{ s}$ such that the wind velocity became zero at $t = 0 \text{ s}$. The response displacements in the case of the time domain analysis were then rather small until $t = 100 \text{ s}$, and the displacements in this range were not used for the comparison. The gust response displacement simulated by the wavelet-based method with $U_j,k = U$ was almost the same as that by the time history analysis, and this consistency had been seen for the gust responses under the all wind records. On the other hand, in fig. 6, the response by the wavelet-based method under $U_j,k = U$ is compared with that by the time domain analysis using the drag force as

$$F(t) = \frac{1}{2} \rho d C_D (u(t) - \dot{x})^2 . \tag{13}$$
Figure 3: A measured wind velocity record.

Figure 4: Power spectrum of wind velocity fluctuation shown in fig.3.

Figure 5: Gust response displacements simulated with wavelet-base method with $U_{j,k} = U$ and with Newmark $\beta$ method with eqn. (1).

Figure 6: Gust response displacements simulated with wavelet-base method with $U_{j,k} = U$ and with Newmark $\beta$ method with eqn. (13).
The products of small quantities in eqn. (13) are assumed to be negligible, and eqn. (13) is linearized into eqn. (1). The differences of the response became remarkable specifically around $t = 280$ s and at $t > 340$ s, and the amplitudes by the time domain analysis using eqn. (13) became larger than those by the wavelet-based method with $U_{j,k} = U$ at these specific times. As shown in fig. 3, the extreme-looking wind velocity fluctuations appeared around 280 s and 340 s, at which the effects of the nonlinear terms included in the right-hand side of eqn. (13) on the responses are thought not to be negligible. Meanwhile, fig. 7 shows the gust response simulated using the wavelet-based method with variable $U_{j,k}$ shown in eqn. (5). The result obtained with the time domain analysis using eqn. (13) is also illustrated with a dotted line in fig. 7, which is the same as the dotted line in fig. 6. Comparing with the case of $U_{j,k} = U$ (fig. 6), the local large displacements around 280 s by the wavelet-based method with variable $U_{j,k}$ well agreed with those by the time domain analysis. In fig. 8, the displacement difference between the case of the wavelet-based method with variable $U_{j,k}$ and the case of the time domain analysis using eqn. (13) is plotted with a solid line. Additionally, in fig. 8, that between the case using $U_{j,k} = U$ and the case of the time domain analysis is illustrated with a dotted line. The response differences in the case of variable $U_{j,k}$ were less than those of $U_{j,k} = U$ totally, and in particular around 280 s and 340 s, the displacement agreement in the case of variable $U_{j,k}$ was much better than the case of $U_{j,k} = U$. This implies that the use of variable
$U_{j,k}$ in the wavelet-based method results in reflecting the effects of the nonlinear terms in the drag force to the gust responses. Successively, fig. 9 shows the ratio of the nonlinear component in the drag force for the linear component, $r_{nl} = \frac{(u'(t)^2 + \dot{x}^2 - 2u'(t) \dot{x})}{(2U u'(t) - 2U \dot{x})}$. As observed in figs. 8 and 9, when the nonlinear component became large, such as around 280 s and 340 s, the displacement agreement in the case of variable $U_{j,k}$ was better than the case of $U_{j,k} = U$.

![Figure 9: Ratio of nonlinear component included in eqn. (13).](image)

3.3 Standard deviation and peak factor of simulated gust response

The standard deviations and the peak factors of the gust responses simulated for all measured wind records are investigated. In fig. 10, the standard deviations of the response displacements by the wavelet-based simulation, $\sigma_w$, are compared with those by the time domain analysis using eqn. (13), $\sigma_t$. Because the response amplitude simulated with $U_{j,k} = U$ were totally smaller than that by the time domain analysis as depicted in fig.6, $\sigma_w$ in the case of $U_{j,k} = U$ became somewhat smaller than $\sigma_t$. In the case of variable $U_{j,k}$, while inconsistency remained in a few cases (e.g. $\sigma_w \cong 0.3$, $\sigma_t \cong 0.33$), they almost agreed each other. Successively, the peak factors by the wavelet-based simulation, $g_w$, and those by the time
domain analysis, $g_t$, are shown in fig. 11. In the case of $U_{j,k} = U$, the correlation between $g_w$ and $g_t$ was low, and $g_w$ tended to be smaller than $g_t$. On the other hand, using variable $U_{j,k}$, $g_w$ and $g_t$ were correlated higher than the case of $U_{j,k} = U$. In particular, when the peak factors were less than 4.5, $g_w$ were almost consistent with $g_t$. This result interprets the good agreements of the local large response amplitudes as presented around $t = 280$ in fig. 7. However, when they exceeded 4.5, the difference between $g_w$ and $g_t$ became large. Although the cause of these discrepancies at high peak factors is not identifiable, it would indicate that even $F_{j,k}$ with variable $U_{j,k}$ did not approximate the nonlinear components of the drag force in eqn. (13) sufficiently when the response amplitude and the wind fluctuation were extremely large.

![Figure 11: Effect of $U_{j,k}$ on peak factor of simulated gust response.](image)

**4 Conclusions**

A simple method using orthonormal discrete wavelets to simulate the gust response of structures was proposed. By way of the time-scale decomposition of the wind velocity data, this wavelet-based method can configure the time dependent characteristics present in the wind such as local extreme-looking wind fluctuations. Using measured wind records, SDOF gust responses along approaching wind direction were simulated with the wavelet-based method, in which the aerodynamic admittance was omitted aiming at the tests on the fundamental performances of the method. The gust responses simulated by the wavelet-based method were compared with those by the time domain analysis, and the results showed good agreement between these two methods. The results also indicated that local mean wind velocities introduced in the wavelet-based method enabled to reflect the nonlinear aerodynamic forces induced by strong wind fluctuations to the gust responses. This method would be then useful to predict the gust responses in which fluctuating components of approaching winds are large relatively to the mean wind velocities and are not negligible in
the aerodynamic forces. While the characteristics of this method without the aerodynamic admittance were investigated in this study, it would be of interest to verify the accuracy of the simulation by this method with the aerodynamic admittances. Moreover, in addition to extending this method to the multi-degrees-of-freedom systems, the spatial correlations of wind fluctuations are required to be configured.

References


