Nonlinear response analysis of a SeaStar offshore Tension Leg Platform in six degrees of freedom

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Abstract

Tension Leg Platforms (TLPs) are used for deep-water oil/gas exploration. Among the compliant platforms, TLPs are vertically moored structures with excess buoyancy. Therefore the tethers can be tensioned to such an extent that heave, roll and pitch motions of the platform induced by ocean waves are virtually eliminated. SeaStar is a new generation of mini-TLPs, which is similar to a spar and has the favorable response features of a TLP. In this paper, a computer program was developed to evaluate the dynamic response of a typical SeaStar TLP to regular wave forces. In this analysis, coupling between the degrees of freedom surge, sway, heave, roll, pitch and yaw have been considered. Wave forces on the elements of the platform are calculated numerically employing linear wave theory and Morison's equation. However the effect of diffraction has been ignored due to the small ratio of cylindrical elements diameter to wavelength. The nonlinear equations of motion are solved numerically by Newmark’s beta integration scheme in the time domain. This yields to motions of the platform in six degrees of freedom. The results have also been presented in the frequency domain and the response amplitude operator for each motion of the platform has been calculated. The results show that for this typical platform the movement of the TLP is considerable in some degrees of freedom. However, changing the dimensions of the platform can affect the values of these movements. This can give us a guideline for optimum design of this kind of platform.

Keywords: SeaStar TLP, response amplitude operator, Morison equation.
1 Introduction

A Tension Leg Platform is a kind of compliant type offshore platform that is generally used for deep-water oil exploration. It has a sea surface piercing vertical cylinder supporting the deck and accessories with the excess buoyancy offered by three fully submerged cylindrical pontoons well below the surface. Vertical tension legs are attached to these buoyancy chambers. The horizontal pontoons help to improve lightly damped heave motion of vertical cylinder that is problematic for spars.

A number of studies have been made on the dynamic behaviour of tension leg platforms under both regular and random waves (Taudin, [5], Tan and De Boom, [8], Denis and Heaf, [1], Bhattacharyya et al. [3]). These studies mostly consider the behaviour of the platform in two or three degrees of freedom. Furthermore the type of platform in the above-mentioned investigations was not a SeaStar one except for the last one. In this study, nonlinear response of a SeaStar TLP in six degrees of freedom under regular waves is presented developing a computer program in MATLAB.

2 Platform dynamics

The multiple degree of freedom equations of motion for a stationary floating body in gravity waves can be formulated by matrix equations in the six rigid body degrees of surge, sway, heave, roll, pitch and yaw using a column vector $X$.

$$
X = \begin{bmatrix}
\text{surge} \\
\text{sway} \\
\text{heave} \\
\text{roll} \\
\text{pitch} \\
\text{yaw}
\end{bmatrix}
$$

(1)

The force on a general floating body of a TLP can be described as [2]:

$$
M \ddot{X} = \sum M_A (\dot{\eta} - \dot{\bar{X}}) + \sum M_{FK} \ddot{\eta} + \sum B_v |\dot{\eta} - \dot{\bar{X}}| (\dot{\eta} - \dot{\bar{X}}) - KX - K_m X
$$

(2)

where $M$, $M_A$ and $M_{FK}$ are the $(6 \times 6)$ coefficient matrices quantifying structure physical mass, added mass and the Froude–Krylov added inertia respectively. $B_v$ is a $(6 \times 6)$ matrix representing the nonlinear drag force contribution, while $K$ and $K_m$ are $(6 \times 6)$ hydrostatic and stiffness matrices.
Rearranging the previous equation and applying some modifications the final form of this equation is used as:

\[(M + M_A)\ddot{X} + B_v\dot{X} + (K + K_m)X = F(t)\]  

(3)

in which \(F(t)\) is the external force vector as:

\[
F(t) = \begin{bmatrix}
F_x \\
F_y \\
F_z \\
M_x \\
M_y \\
M_z \\
\end{bmatrix}
\]

(4)

In fact \(F(t)\) is the sum of different force components in the marine environments as:

\[F(t) = F_E + F_{DL} + F_{DB} + F_{BV}\]  

(5)

where \(F_E\) is the inertia force. \(F_{DL}\) is the drag force. \(F_{DB}\) is the drag force at the bottom of vertical cylinders. \(F_{BV}\) is the buoyancy force induced by wave profile on surface piercing elements.

### 3 Hydrodynamic coefficients

Hydrodynamic coefficients in equation (3) are calculated as follows:

**Physical mass matrix**: physical mass matrix, \(M\), is diagonal with the structure total mass in the first three diagonal positions and the structure mass moments of inertia for the remaining three diagonal terms [2].

**Added mass matrix**: to specify added mass, viscous damping and stiffness matrices for the whole platform first the local matrix for each element of platform was calculated. Then a global matrix for the whole platform was calculated assembling these local matrices. The general added mass matrix of circular cylinder with arbitrary end coordinates can be computed by assuming that only the components of acceleration normal to the cylinder axis are significant. Calculations can be readily adopted to rectangular elements leading to a symmetric matrix [2].

**Viscous damping matrix**: the fluid damping matrix, \(B_v\) for this platform is evaluated in a similar manner to the added mass matrix, as a sum of the contributions from each individual member.

The derivation of the generalized damping matrix for an arbitrary oriented circular cylinder depends on the assumption that only drag forces normal to the cylinder axis are significant. Unlike the added mass matrix, the damping matrix is not symmetric and all components should be calculated [2].
Stiffness matrix: the coefficients, $K_{ij}$, of stiffness matrix of the SeaStar TLP are derived as the reaction in degree of freedom $i$ due to unit displacement in $j$, keeping all other degrees of freedom restrained.

The coefficients of the stiffness matrix have nonlinear terms due to the cosine, sine, square root and squared terms of the displacements. Furthermore, the tether tension changes due to the motion of the TLP in different degrees of freedom. This leads to a response-dependent stiffness matrix. All the coefficients of the stiffness matrix for this SeaStar Tension Leg Platform were calculated using the initial pre-tension in the tether and its change due to an arbitrary displacement in the particular degree of freedom [4].

4 Calculation of forces

In addition to added mass force the Froude–Krylov force which is proportional to the acceleration of the surrounding fluid also acts on the body. This can be calculated as:

$$F_{FK} = C_M \int_{-\frac{l}{2}}^{\frac{l}{2}} M \dddot{\eta} dl$$

where $M$ is the $(6 \times 6)$ added mass coefficients matrix. $\dddot{\eta}$ is the water particles instantaneous acceleration vector in different parts of cylinder axis, and $C_M$ is the inertia coefficient.

Drag force about the vertical cylinder and three pontoons are calculated as:

$$F_{DL} = C_D \int_{-\frac{l}{2}}^{\frac{l}{2}} D \dddot{\eta} |\dddot{\eta}_{max}| dl$$

where $D$ is the $(6 \times 6)$ viscous drag coefficients matrix. $|\dddot{\eta}_{max}|$ is the water particles maximum velocity vector. $\dddot{\eta}$ is the water particles instantaneous velocity vector in different parts of cylinder axis, and $C_D$ is the drag coefficient about the cylinder. The drag force at bottom of cylinders can be obtained as:

$$F_{DB} = C_D T \int_S A_D \dddot{\eta} \frac{8}{3\pi} |\dddot{\eta}_{max}| ds$$

where

$$A_D = \frac{1}{8} \rho \pi D_c^2$$
$C_D$ is the disk drag coefficient. $T$ is the matrix to transform the force along the cylinder axis. $\eta$ is the instantaneous water particles velocity matrix in each time interval and in different parts of cylinder axis.

The drag force on bottom of vertical cylinder is calculated using the strip mesh generation. The wave dynamic pressure induces another force on the water surface piercing TLP elements as:

$$F_{BV} = T\rho g \int_{S} \eta_B K_p(z) ds$$  \hspace{1cm} (10)

where $\rho$ is the water density. $\eta_B$ is the wave profile height compared to still water level in each time interval. $g$ is the gravity acceleration, and $K_p(z)$ is pressure response factor for depth $z$.

5 Coordinate systems

Three coordinate systems have been used for modeling of this SeaStar TLP. A fixed or inertial coordinate system called $g$. The second coordinate system is a moving coordinate system called $b$. The third coordinate system is a fixed coordinate called $w$. Figure 1 shows these coordinate systems.

![Figure 1: Fixed and moving coordinate systems.](image)

6 Solution of the equations of motion in time domain

The Newmark’s $\beta$ method used to solve the equations of motion in time domain. In this method the displacement and velocity at the end of a time interval is related to the displacement, velocity and acceleration at the beginning of the time interval as [2]:

$$\dot{X}_{i+1} = \dot{X}_i + \frac{1}{2}(\Delta t)[\ddot{X}_i + \ddot{X}_{i+1}]$$  \hspace{1cm} (11)

$$X_{i+1} = X_i + (\Delta t)\dot{X}_i + (1-2\beta)(\Delta t)^2 \ddot{X}_i + \beta(\Delta t)^2 \ddot{X}_{i+1}$$  \hspace{1cm} (12)
The equations of motion in each time interval are solved as:

\[
\begin{align*}
[(M + M_A) + \frac{1}{2}(\Delta t)B_v + \beta(\Delta t)^2 K]X_{i+1} &= \\
(\Delta t)^2[\beta F_{i+1} + (1 - 2\beta)F_i + \beta F_{i-1}] + [2(M + M_A) - (\Delta t)^2 (1 - 2\beta)K]X_i \\
- [(M + M_A) - \frac{1}{2}(\Delta t)B_v + \beta(\Delta t)^2 K]X_{i-1}
\end{align*}
\]

where

\[
B_v = B_v\left|\dot{X}\right|
\]

In each time interval \( B_v \) must be modified for each iteration until the 6 errors converge simultaneously.

7 Platform geometry

Geometry of two typical SeaStar platforms and characteristics of a wave sample were fed as input to the model. The model first check if the ratio of platform members’ diameter to wavelength is less than 0.2 to make sure diffraction effects can be eliminated. Geometry of main components of the typical SeaStar TLP (see fig 2) for each case is as follows:

![SeaStar Tension Leg Platform geometry.](image)

<table>
<thead>
<tr>
<th>Model 1</th>
<th>Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D1 = 20m )</td>
<td>( D1 = 10m ) (diameter of vertical cylinder)</td>
</tr>
<tr>
<td>( D = 40m )</td>
<td>( D = 20m ) (platform draft)</td>
</tr>
<tr>
<td>( H1 = 50m )</td>
<td>( H1 = 25m ) (height of vertical cylinder)</td>
</tr>
<tr>
<td>( L1 = 23m )</td>
<td>( L1 = 11.5m ) (length of horizontal pontoons)</td>
</tr>
<tr>
<td>( aa = 6m )</td>
<td>( aa = 3m ) (breadth of horizontal pontoons)</td>
</tr>
<tr>
<td>( bb = 8m )</td>
<td>( bb = 4m ) (height of horizontal pontoons)</td>
</tr>
<tr>
<td>( WW = 11460\text{ton} )</td>
<td>( WW = 1432.5\text{ton} ) (weight of hull)</td>
</tr>
</tbody>
</table>
$W_t = 360\text{ton}$ \hspace{1cm} $W_t = 246.66\text{ton}$ \hspace{1cm} (weight of three tendons)

$D_t = 1.11m$ \hspace{1cm} $D_t = 0.55m$ \hspace{1cm} (tether diameter)

$d = 1000m$ \hspace{1cm} $d = 500m$ \hspace{1cm} (water depth)

$L_t = 960m$ \hspace{1cm} $L_t = 480m$ \hspace{1cm} (length of tenders)

$\rho_t = 7850 \frac{kg}{m^3}$ \hspace{1cm} $\rho_t = 7850 \frac{kg}{m^3}$ \hspace{1cm} (density of steel tether)

$T_{tether} = 4000\text{ton}$ \hspace{1cm} $T_{tether} = 500\text{ton}$ \hspace{1cm} (tether pretension)

$\Delta = 15460\text{ton}$ \hspace{1cm} $\Delta = 1932.5\text{ton}$ \hspace{1cm} (platform displacement)

$VCG = 15.5m$ \hspace{1cm} $VCG = 7.75m$ \hspace{1cm} (vertical center of gravity)

$VCB = 17.1m$ \hspace{1cm} $VCB = 8.55m$ \hspace{1cm} (vertical center of buoyancy)

8 Results

Employing the above theories a numerical model was developed to calculate the movements of two typical rigid SeaStar platforms in six degrees of freedom due to regular waves. The input wave had a height of 2 m and a period of 20 sec. It should be noted that in each time interval the hydrodynamic coefficient matrices and force vector should be modified due to the change of platform element coordinates respect to inertial coordinate system.

Some of the results are presented in figures 3 to 11 as follows. It can be seen that frequency of motions in all degrees of freedom for model 1 and 2 are 7 HZ and 6 HZ respectively.

![Figure 3: Surge motion analysis for model 1 in time domain.](image1)

![Figure 4: Pitch motion analysis for model 1 in time domain.](image2)
Figure 5: Surge motion analysis for model 2 in time domain.

Figure 6: Pitch motion analysis of model 2 in time domain.

Figure 7: Surge spectral density RAO for model 1.

Figure 8: Pitch spectral density RAO for model 1.
Summary and conclusions

The motions of a SeaStar tension leg platform in the time and frequency domains were analyzed. The exciting force was a linear wave with a wave height of 2 m and a period of 20 sec. Hydrodynamic coefficients such as added mass, stiffness and damping have been estimated using proposed relations in the literature [2,6].

The results showed that for this typical platform the movement of the TLP in surge and pitch directions are considerable for small-dimensioned SeaStar. However changing the dimensions of the platform can dramatically affect on the values of these movements. This can be seen clearly comparing values of motion amplitudes in figures 3 and 4 with figures 5 and 6. This sensitivity to dimensions of the platform can give us a guideline for optimum design of this kind of platforms.
References


