Time-domain analysis of the hydroelastic response of cavitating propulsors

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Abstract

A 3-D boundary element method (BEM) is coupled with a 3-D finite element method (FEM) to model the hydroelastic response of cavitating propulsors. The BEM is applied to evaluate the moving cavity boundaries and hydrodynamic forces, as well as the added mass and hydrodynamic damping matrices. The FEM is applied to evaluate the dynamic blade stresses and deformations. The effects of fluid-structure interaction are included by superimposing the added mass and hydrodynamic damping matrices to the material mass and damping matrices in the equilibrium equation of motion. The problem is solved in the time-domain using an implicit time integration scheme. An overview of the formulation is presented along with numerical and experiment validation studies. The effects of fluid-structure interaction and cavitation on the propeller performance are discussed.

Keywords: fluid-structure interaction, hydroelastic, cavitation, propeller.

1 Introduction

Since the pioneering works of [1, 2, 3, 4], boundary element methods (BEMs) have been widely used for the analysis of complex flows around propellers. Most recently, a perturbation potential-based BEM developed by [2, 3, 5, 6, 7] has been further extended to predict unsteady cavitating and ventilated flows for conventional fully submerged propellers [8, 9], supercavitating propellers [10, 11, 12], and surface-piercing propellers [11, 13]. For cases without blade vibration, the developed BEM has been shown to predict forces and cavitation patterns that compared well with experimental measurements. However, hydroelastic effects become important for non-conventional propeller geometries (e.g. highly skewed
propellers, supercavitating propellers, and partially submerged propellers). Thus, more rigorous methods of determining the structural response and dynamic characteristics are needed in order to avoid material failure, excessive blade distortions, and/or resonant blade vibration. The objective of this work is to develop a simulation tool to predict the unsteady hydroelastic performance of cavitating propulsors.

In the past, most structural analysis models assumed the blades to be rigid, and applied the averaged pressure distribution on the blade surface obtained from steady lifting line or lifting surface methods. Thus, the dynamic propeller performance and the effects of fluid-structure interaction are ignored. To account for changes in pressure distribution due to blade deformation, an iterative procedure was developed by [14]. A lifting surface method was used to determine the fluid pressure, which was imposed on the blade surface to compute the change in blade geometry via a FEM, and the process was repeated until a stable operating condition was reached. The effect of cavitation was considered, but the study was limited to steady flows [14]. Later, a coupled approach was introduced by [15] for dynamic blade stress analysis. A potential-based method was used to determine the hydrodynamic blade loads, as well as the added mass and hydrodynamic damping associated with dynamic blade motion. However, the analysis was limited to fully wetted flows, and the effect of blade distortion on the flow field was ignored. Recently, a nonlinear coupled strategy for hydroelastic blade analysis using finite element method and lifting surface method was developed by [16]. The effects of geometric nonlinearity and blade distortions are considered, but the work was limited to steady, fully wetted flows. Recently, [17] presented a numerical model for the hydroelastic analysis of surface-piercing propellers. The effects of fluid inertia were considered by distributing point masses across the surface of the blade according to potential flow theory around a flat plate. The total damping ratio was taken to be 0.05. Due to the over simplification of the added mass, and the application of an assumed instead of evaluated hydrodynamic load, the method is not able to accurately predict the dynamic blade response. Thus, the objective of this work is to develop an accurate simulation tool for the time-domain analysis of the hydroelastic performance of cavitating propulsors.

2 Formulation

The fluid-structure interaction problem is solved by coupling a potential-based BEM with the commercial FEM software ABAQUS. The BEM is applied to determine the moving cavity boundaries and fluctuating pressures, and the FEM is applied to determine the dynamic blade response. Details of the formulation for both the BEM and FEM are presented in [18], and are summarized here for the sake of completeness.

Consider a flexible cavitating propeller subjected to a general effective inflow wake $\vec{q}_E$ ($\vec{q}_E$ is assumed to be the effective wake, i.e. it includes the interaction between the vorticity in the inflow and the propeller ([19] [20]).) in ship-fixed coordinates $(x_s, y_s, z_s)$. The inflow velocity, $\vec{q}_{in}$, with respect to the blade-fixed coordinates $\vec{x} = (x, y, z)$, can be expressed as the sum of the inflow wake velocity,
\( \vec{q} \), and the propeller’s angular velocity \( \vec{\Omega} \), at a given location \( \vec{x} \):

\[
\vec{q}_{in}(\vec{x}, t) = \vec{q}_E(x, r, \theta_B - \Omega t) + \vec{\Omega} \times \vec{x}
\]  

where \( r = \sqrt{y^2 + z^2} \) and \( \theta_B = \arctan(z/y) \). The resulting flow is assumed to be incompressible and inviscid. The total velocity, \( \vec{v} \), can be expressed in terms of \( \vec{q}_{in} \) and the perturbation potential \( \Phi \):

\[
\vec{v}(\vec{x}, t) = \vec{q}_{in}(\vec{x}, t) + \vec{\nabla}_\vec{x} \Phi(\vec{x}, t)
\]  

where \( \Phi \) satisfies the Laplace’s equation in the fluid domain (i.e. \( \nabla^2 \Phi = 0 \)).

Assuming linearity, \( \Phi \) can be decomposed into two parts:

\[
\Phi(\vec{x}, t) = \phi(\vec{x}, t) + \varphi(\vec{x}, t)
\]  

where \( \phi(\vec{x}, t) \) denotes the perturbation potential due to rigid blades rotating in non-uniform wake, and \( \varphi(\vec{x}, t) \) denotes the perturbation potential due to vibrating blades in uniform wake. Thus, the total velocity (Eqn. (2)) can be rewritten as:

\[
\vec{v}(\vec{x}, t) = \vec{q}(\vec{x}, t) + \vec{\nabla}_\vec{x} \varphi(\vec{x}, t)
\]  

where \( \vec{q}(\vec{x}, t) = \vec{q}_{in}(\vec{x}, t) + \nabla \phi(\vec{x}, t) \) is the fluid velocity due to rigid blades rotating in non-uniform wake; \( \nabla \varphi(\vec{x}, t) \) is the fluid velocity due to vibrating blades in uniform wake.

As shown in [18], the total pressure \( (P_t) \) acting normal to the blade surface can also be decomposed into two parts: \( \Delta P_t = \Delta P + \Delta P_v \), where

\[
\Delta P = \rho \left[ \frac{1}{2} |\vec{q}_{in}|^2 - g y_s - \frac{\partial \phi}{\partial t} - \frac{1}{2} \vec{q}^2 \right]
\]

\[
\Delta P_v = \rho \left[ -\frac{\partial \varphi}{\partial t} - \vec{q}_{in} \cdot \nabla \varphi \right]
\]

\( \Delta P \) is the rigid-blade hydrodynamic pressure, and \( \Delta P_v \) is the elastic blade hydrodynamic pressure distribution due to unsteady blade motion.

The dynamic equilibrium equation of motion can be written as follows:

\[
[M]\{\ddot{u}\} + [D]\{\dot{u}\} + [K]\{u\} = \{F_c\} + \{F\} + \{f\}
\]  

where \( [M] \), \( [D] \), and \( [K] \) are the structural mass, damping, and stiffness matrices, respectively. \( \{\ddot{u}\} \), \( \{\dot{u}\} \), and \( \{u\} \) are the nodal acceleration, velocity, and displacement vectors, respectively. The nodal force vectors on the right-hand-side of Eqn. (7) represent the contribution due to centrifugal force, hydrodynamic excitation associated with rigid blade rotating in non-uniform wake, and hydroelastic excitation associated with vibrating blade in uniform wake, respectively. The last two terms can be computed by integrating the corresponding pressures over the blade surface:

\[
\{F\} = -\int [N]^T \{\Delta P\} dS \quad \{f\} = -\int [N]^T \{\Delta P_v\} dS
\]  

where \( [N] \) is the displacement interpolation matrix.
Thus, the fluid-structure interaction problem can be decomposed into two parts: rigid blades rotating in non-uniform wake and flexible blades vibrating in uniform wake. Since both $\phi$ and $\varphi$ must satisfy the Laplace equation, they can be solved using the same potential-based BEM.

2.1 Rigid blade problem

Details of the formulation and implementation for the rigid blade problem can be found in [8, 9, 12, 21]. In brief, the unknown values of $\frac{\partial \phi}{\partial n}$ on the cavity surfaces, and $\phi$ on the wetted blade and hub surfaces, are solved by applying Green’s third identity in the framework of a moving boundary-value problem. Only sheet cavities are considered. The known values of $\phi$ on the cavity surfaces are obtained by requiring the pressure to be constant and equal to the cavitation pressure. The known values of $\frac{\partial \phi}{\partial n}$ on the wetted blade and hub surfaces are obtained by applying the flow tangency condition. The blade, cavity, and wake surfaces are approximated with hyperboloidal panels [5] on which constant strength dipoles and sources are distributed. The wake surface is assumed to have zero thickness and the geometry is determined by satisfying the force-free wake condition. An iterative pressure Kutta condition [5] is applied to ensure equality of pressure at both sides of the blade trailing edge. The cavity heights on the blade and the wake surfaces are computed by applying the flow tangency condition. The correct cavity planform at each time step is obtained iteratively by applying a Newton-Raphson technique, which requires the cavity closure condition (zero thickness at the cavity trailing edge) to be satisfied. The cavity detachment locations are determined iteratively by applying the Villat-Brillouin smooth detachment condition [22, 23]. Once the values of $\phi$ are known everywhere at each time step, the rigid-blade hydrodynamic pressure, $\Delta P$, and the hydrodynamic force, $F$, are obtained by applying Eqn. (5) and (8), respectively.

2.2 Vibrating blade problem

The perturbation potential due to flexible blade motion, $\varphi$, is also determined using Green’s third identity, which can be expressed in matrix form as:

$$\{\varphi\} = [C] \left\{ \frac{\partial \varphi}{\partial n} \right\}$$  (9)

where $[C] = [A]^{-1}[B]$. $[A]$ and $[B]$ are the induced potential matrices due to unit strength dipoles and sources, respectively, and are taken directly from the rigid blade BEM analysis. Similarly, the spatial derivatives of $\varphi$ can be computed as:

$$\left\{ \frac{\partial \varphi}{\partial x_i} \right\} = [C_{x_i}] \left\{ \frac{\partial \varphi}{\partial n} \right\}$$  (10)

where $[C_{x_i}] = [A_{x_i}][C] - [B_{x_i}]$. $[A_{x_i}]$ and $[B_{x_i}]$ are the coefficient matrices representing the $i$-component of the induced velocities due to unit strength dipoles and sources, respectively.
Applying linear decomposition and ignoring convective terms, it can be shown that the kinematic boundary condition requires the perturbation velocity normal to the blade surface, $\frac{\partial \varphi}{\partial n}$, to be equal to the normal component of the solid body velocity $[24]$. Defining $[T]$ as the transformation matrix which relates the normal velocities at the element centroid to the element nodal velocities, the kinematic boundary condition can be written as follows:

$$\frac{\partial \varphi}{\partial n} = [T]\{\dot{u}\}$$  \hspace{1cm} (11)

As shown in [18], the hydroelastic pressure, $\Delta P_v$, can be expressed as follows:

$$\{\Delta P_v\} = -\rho[C][T]\{\ddot{u}\} - \rho[QC][T]\{\dot{u}\}$$  \hspace{1cm} (12)

where $[QC] = \{q_{in1}\}^T[C_{x1}] + \{q_{in2}\}^T[C_{x2}] + \{q_{in3}\}^T[C_{x3}]$ and $(q_{in1}, q_{in2}, q_{in3})$ are the three components of the inflow velocity vector, $\vec{q}_{in}$. Thus, the dynamic equilibrium equation of motion, Eqn. (7), can be rewritten as follows:

$$([M] + [M^h]) \{\ddot{u}\} + ([D] + [D^h]) \{\dot{u}\} + [K]\{u\} = \{F_c\} + \{F\}$$  \hspace{1cm} (13)

where $[M^h]$ and $[D^h]$ denote the added mass and hydrodynamic damping matrices, respectively:

$$[M^h] = \rho \int [N]^T[C][T]dS \hspace{1cm} [D^h] = \rho \int [N]^T[QC][T]dS$$  \hspace{1cm} (14)

In general, the matrix of influence coefficients, $[C]$, is full and asymmetric. Thus, $[M^h]$ and $[D^h]$ will also be full and asymmetric. To reduce computational cost and storage requirement, an HRZ-like lumping technique is applied. Details of the lumping procedure are given in [18].

The fluid-structure interaction problem is solved by coupling the BEM with the FEM. The BEM is applied to determine the rigid-blade hydrodynamic pressures ($\{\Delta P(i)\}$), the lumped added mass matrix ($[M^h]$), and the lumped hydrodynamic damping matrix ($[D^h]$). The commercial FEM package, ABAQUS [25], is then employed to solve the equilibrium equation of motion, Eqn. (13). The Hilber-Hughes-Taylor implicit direct integration method in ABAQUS [25] is applied to calculate the dynamic blade response.

In the FEM model, one-layer of quadratic 3-D solid elements is used across the blade thickness. Quadratic elements are necessary to avoid hourglass and shear-lock failures associated with first-order elements [18]. The nodes at the root of the blade are assumed to be fixed in the structural model. The lumped added mass and hydrodynamic damping matrices are superimposed on to the structural added mass and damping matrices via the use of user-defined hydroelastic elements in ABAQUS. Each hydroelastic element has only one node, and each node has three degrees of freedom. The hydroelastic elements have no stiffness; thus only contributes to the total mass and damping of the system.
The resulting hydroelastic force due to blade vibration, \( \{ f(t) \} \), is computed as follows:

\[
\{ f(t) \} = -[M^h] \{ \ddot{u}(t) \} - [D^h] \{ \dot{u}(t) \}
\]

(15)

Finally, the total hydrodynamic blade load, \( \{ f_{tot}(t) \} \), is defined as:

\[
\{ f_{tot}(t) \} = \{ F(t) \} + \{ f(t) \}
\]

(16)

3 Experimental validation studies

3.1 Steady-state analysis

Extensive convergence and validation studies of the presented BEM for different types of propellers can be found in [6, 8, 9, 11, 12, 13]. For the sake of completeness, comparisons of the predicted and measured open water and cavitating performance of two propellers are shown in Figs. 1 and 8. The two propellers are part of a series of four 5-bladed skewed marine propellers tested at the David Taylor Naval Ship Research and Development Center (DTNSRDC) in the United States [26]. The objective of the tests was to determine the effect of skew on propeller performance. The propellers represent typical designs for container ships or single-screw destroyer-type ships. The parent propeller has a symmetric blade outline with zero skew, and the other three propellers have maximum skew angles (measured in the plane of the propeller disk) of 36, 72, and 108 degrees [26]. All four propellers have the same geometric configuration except for the skew, pitch, and camber. The propellers were designed to achieve equal open water performance (at \( J = 0.889 \))

![Figure 1: Predicted and measured thrust and torque coefficients, and cavitation patterns. Propeller 4381.](image1)

![Figure 2: Predicted and measured thrust and torque coefficients, and cavitation patterns. Propeller 4383.](image2)
by varying the blade pitch and camber. The propeller geometries are given in [26, 27]. Comparison of the predicted and measured performance of the 0° (propeller 4381) and 72° (propeller 4383) skew propellers as functions of the advanced ratio (J = V/nD) and cavitation number (σ_v = (P_o - P_c)/0.5ρV^2) are shown in Figs. 1 and 8, respectively. The variables K_T = T/ρn^2D^4 and K_Q = Q/ρn^2D^5 denote the thrust (T) and torque (Q) coefficients, respectively; V, n, and D denote the propeller advance speed, angular frequency, and diameter, respectively. The BEM representation of the propeller geometries and predicted cavitation patterns are also shown in Figs. 1 and 8. The numerical predictions compared well with experimental measurements for both fully wetted and cavitating conditions, and the effect of thrust breakdown due to cavitation was accurately captured by the current BEM.

To evaluate the effect of skew on the structural characteristics, stress and displacement measurements for the same set of skewed propellers were conducted at DTNSRDC by [28, 29], respectively. The blades were subjected to a uniform static air pressure of 6895 Pa (1 psi) on the face (pressure) side, and the tests were conducted on 0.3048 m (1 ft) models (1/23-scale) constructed of 2014-T4 aluminum in specially designed pressure chambers. Examples of the comparison between predicted and measured principal stresses at different radial (r/R) locations for the 0° and 72° skew propellers are shown in Figs. 3 and 4, respectively. The material properties were taken as follows: ρ (fluid density) = 1000 kg/m^3, ρ_s (solid density) = 2800 kg/m^3, E (Young’s modulus) = 75 GPa, and ν (Poisson’s ratio) = 0.33. As shown in the figures, the numerical predictions compared well with experimental measurements. It should be noted that systematic convergence studies, and additional experimental validation studies are given in [18].
Figure 5: Observed (left) and predicted (right) ventilation patterns. Propeller M841B. $J = 1.2$. $Fr = 6$.

Figure 6: Predicted and measured (per blade) force coefficients and natural frequencies. Propeller M841B.

3.2 Dynamic analysis

To validate the presented BEM and FEM models for dynamic blade analysis, numerical predictions for surface-piercing propeller model 841-B are compared with experimental measurements given in [30]. The four-bladed high-speed partially submerged propeller, shown in Fig. 6, was designed based on sea trials on board a 13 m twin screw planning test craft [30]. The propeller diameter was 250 mm, and the blade tip immersion ratio was 33%. Comparison of predicted and observed ventilation patterns at $J = 1.2$ and $Fr = V/\sqrt{gD} = 6$ are shown in Fig. 5. The predicted and measured time-averaged thrust ($\bar{K}_T$), torque ($\bar{K}_Q$), and efficiency ($\eta = (K_T/K_Q)(J/2\pi)$) in ship-fixed coordinates are shown in the top right figure in Fig. 6. The lines and symbols in Fig. 6 represent the numerical predictions and experimental measurements, respectively. For each advance coefficient, multiple experimental data points are shown to depict the effect of Froude number and scale ratio. Additional experimental and numerical validation studies for propeller 841-B can be found in [9, 13]. Comparisons of the predicted and measured blade frequency in air (as a function of mode number) and in water (as a function of blade angle) are also shown in Fig. 6. The blade frequencies in water are normalized by the fundamental blade frequency in air. As shown in the figures, the predicted blade performance and natural frequencies compared well with experimental measurements.
4 Results

The coupled BEM/FEM method is applied to investigate the dynamic performance of the 72° skew propeller (4383) in unsteady cavitating flow. To simulate the unsteady hydroelastic response of the full-scale (23-ft diameter) Nickel-Aluminum-Bronze propeller in non-axisymmetric flow, a four-cycle wake (shown in Fig. 7) is applied. The design condition was applied: \( n \) (propeller angular frequency) = 1.75 Hz and \( J = 0.889 \). The cavitation number (\( \sigma_v = (P_o - P_c) / 0.5 \rho V^2 \)) and Froude number (\( F_{nD} = n^2 D / g \)) were set equal to 1.5 and 9, respectively. The assigned material properties are as follows: \( \rho \) (fluid density) = 1025 kg/m\(^3\), \( \rho_s \) (solid density) = 7524 kg/m\(^3\), \( E \) (Young’s modulus) = 124 GPa, and \( \nu \) (Poisson’s ratio) = 0.32. Comparison of the predicted modal frequencies in air and in water are shown in Fig. 7. Plots of the first four mode shapes in water are shown in Fig. 8. The modal frequencies decreased significantly in water due to the effect of added mass.

Comparisons of the predicted hydroelastic and hydrodynamic key-blade cavitating load coefficients in blade-fixed coordinates \((x, y, z)\) are shown in Fig. 9, along with the fully wetted load coefficients. In general, there is a slight reduction in the axial thrust \( (K_{Fx}) \) and torque \( (K_{Mx}) \) coefficients due to the effect of cavitation. In addition, fluctuations of the cavitating load coefficients are amplified when hydroelastic effects are considered. The harmonic decomposition of the key-blade load coefficients is shown in Fig. 10. As expected, the seventh harmonic is amplified due to resonance with respect to the first mode in water. The predicted shaft harmonics (integration of loads from all five blades in ship-fixed coordinates \((x_s, y_s, z_s)\)) are shown in Fig. 11. Although much of the fluctuations are cancelled out due to the phase difference and position of the blades, amplifications due to resonant blade vibration can still be observed in the shaft torque harmonics. The predicted unsteady cavitation patterns and pressure distribution \((-C_p = (P_o - P) / 0.5 \rho n^2 D^2)\) on the suction (back) side are shown in Fig. 12.
Figure 9: Predicted per-blade hydroelastic and hydrodynamic force and moment coefficients in local blade coordinates \((x, y, z)\).

Figure 10: Predicted per-blade hydroelastic and hydrodynamic force and moment coefficients in local blade coordinates \((x, y, z)\).

Figure 11: Predicted shaft hydroelastic and hydrodynamic force and moment coefficients in ship-fixed coordinates \((x_s, y_s, z_s)\).

Figure 12: Predicted cavitation patterns (top left), pressure distribution (top right), and Von Mises stress distribution (bottom).
Majority of predicted cavities are very thin and detach aft of the blade leading edge, and the cavitation pattern seems very unstable. This implies that $J = 0.889$ and $\sigma_v = 1.5$ is near the transition point been back sheet cavitation, back bubble cavitation, and no cavitation, which agrees with the experimental cavitation inception diagram shown in [26]. The cavitation pattern changes with blade angle due to the non-axisymmetric inflow. The predicted unsteady Von Mises stress distribution on the suction side and pressure side are also shown in Fig. 12. As expected, the peak stress is concentrated in a local area near the trailing edge at the root of the blade due to the high bending moment caused by the high skew angle. The maximum tensile stress is 86 ksi, which is very close to the yield stress of Nickel-Aluminum-Bronze.

5 Conclusion

A coupled BEM/FEM method is presented for the time-domain analysis of the hydroelastic response of cavitating propulsors. An overview of the formulation for both the BEM and FEM are shown along with examples of experimental validation studies. The predicted hydrodynamic forces and cavitation patterns compared well with experimental measurements and observations. The predicted hydroelastic response of a highly skewed propeller in unsteady, cavitating flow seemed reasonable. The results indicated that the blade’s natural frequencies decreased substantially in water due to the effect of added mass, which can lead to resonant blade vibration at lower excitation frequencies. Blade vibration, in turn, can lead to amplified hydrodynamic force fluctuations, as well as increase in maximum stress levels and displacements. The effect of resonant blade vibration can be particularly damaging for highly skewed propellers due to coupling of amplifications of hydrodynamic loads and stress concentration near the trailing edge at the roots of the blade.

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References


