Numerical simulation of high-pressure hydraulic systems with weak pipe strain

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Abstract

In the development phase of modern hydraulic tools the use of simulation of the complete system is of growing importance. The physical modelling of complex hydraulic flow in weak elastic systems and numerical solution of the appropriate conservation equations present a major challenge. The numerical solver has to cope shocks, phase transition, and the pipe strain provoked by pressure waves. The paper presents the modelling equations and system simulations with parameter variations for a hydraulic system consisting of a reservoir, a pipe, and a nozzle.

1 Introduction

High pressure hydraulic systems are employed in different industrial applications such as in aircraft for controlling the landing gear or vertical and horizontal stabilizer, in breaking systems for vehicles, and in injection systems for Diesel and gasoline engines. The pressure level in Diesel injection systems is in a range of 1 bar up to 1800 bar. Due to the time dependent pressure levels and the resulting local strain of the cross-sectional areas of hydraulic pipes, the wave motion can be influenced significantly. Many hydraulic components are controlled by valves, and the efficiency of the system depends on the dynamics of the wave motion which is influenced
by the strain of the pipe. Due to this fact, the fluid-structure interaction, i.e. the correlation of pressure waves and variation of the cross-sectional area of pipes has to be considered in the system simulation. In addition to this fact, the complete conservation laws, i.e. the continuity equation, the momentum equation, the energy equation have to be used for simulation of the fluid flow. On account of the increasing rail pressure in the injection systems, the energy equation has to be considered in order to compute the time- and space-depending temperature of the fluid.

The paper presents the study of a simplified high pressure injection system considering the complete conservation equations (Euler equations with friction forces) and two approaches for the strain of the pipe. The system consists of a high pressure tank, a pipe with constant cross-sectional area (for constant pressure) and a constrained controlled valve on the end of the pipe. Two models for the simulation of pipe strain are used:

1. A simple model based on the theory of membrane shells which modifies the speed of sound of the fluid,

2. A model based on the theory of shells which leads to a fourth order differential equation for the pipe strain.

An explicit first order method derived in [3] to compute the time-dependent fluid properties such as pressure, velocity and temperature is combined with a finite element method to compute the strain of the pipe. Several different parameter modifications for the coupled model are studied and discussed. Hereby the strain of the cross-sectional area is considered depending on the valve characteristics, the pressure level in the reservoir, the diameter, and the thickness of the pipe.

### 2 Flow equations

The flow in hydraulic systems is often modeled by the one-dimensional Euler-equations. In the past, the so-called p-system consisting of the continuity and the momentum equation was used, cf. [1, 4]. Due to the degreased injection pressure up to 1800 bar in modern injection systems, the energy equation has to be used in order to simulate the complex physical process with higher accuracy. The model equations for the flow are restricted to the pure liquid flow, i.e. cavitating flow is not considered here. The density of the fluid is denoted by \( \rho \), the velocity by \( v \), the specific inner energy by \( e \) and the diameter on the inside of the pipe by \( d \). The specific inner energy \( e \) and the specific enthalpy \( h \) are connected by \( h = e + p/\rho \). The cross-sectional area of the pipe is denoted by \( A \) which is a function of pipe length \( z \) and the time \( t \). The friction force is a function of the diameter \( d \) of the pipe, the velocity \( v \) and the unsteady pipe friction value \( \lambda_{in} \). With the heat flux \( q \), the local heating of the fluid in the pipe can be considered.
Following [2], the flow equations are

\[
\begin{align*}
\frac{\partial}{\partial t} \rho + \frac{\partial}{\partial z} (\rho v) &= -\frac{\rho}{A} \frac{dA}{dt} = -\frac{\rho}{A} \left( \frac{\partial A}{\partial t} + v \frac{\partial A}{\partial z} \right), \\
\frac{\partial}{\partial t} \rho v + \frac{\partial}{\partial z} (\rho v^2 + p) &= \rho F - \frac{\rho v}{A} \frac{dA}{dt}, \quad \text{with} \quad F = -\frac{\lambda_{in}}{2d} |v|v \\
\frac{\partial}{\partial t} \left[ \rho \left( e + \frac{v^2}{2} \right) \right] + \frac{\partial}{\partial z} \left[ \rho v (h + \frac{v^2}{2}) \right] &= \dot{q} - \frac{\rho}{A} \left( e + \frac{v^2}{2} \right) \frac{dA}{dt}.
\end{align*}
\] (2.1)

The system (2.1) can be abbreviated as

\[
\frac{\partial}{\partial t} u + \frac{\partial}{\partial z} f(u) = g(u) 
\] (2.2)

with the vector of conservative variables \( u = (u_1, u_2, u_3)^T \), the flux function \( f \) and the source term \( g \). To close the hyperbolic differential system of equations, the equations of state for the fluid have to be considered, i.e. \( \rho = \rho(p, T) \) and \( e = e(p, T) \). The variation of the cross-sectional area \( A \) as a consequence of the pressure waves will be modeled separately, see Section 3. The initial conditions are given for \( t = 0 \). The boundary conditions are given by the reservoir and the time depending operating valve.

The system of eqn. (2.1) is solved numerically by a first order scheme in space and time using the p-v-T-formulation derived from (2.1), cf. [2].

### 3 Pipe strain models

Simple and highly complex pipe strain models are derived in the literature, cf. [7]. For general comprehension, the models used here are discussed shortly. Two pipe strain models are presented and compared in the following: a membrane shell model, and a shell model. The thickness of the pipe is denoted by \( s \). The normal stresses are denoted by \( \sigma_\phi \) and \( \sigma_z \). The shear stress is termed by \( \tau \). The elasticity module is denoted by \( E_s \), and the Poisson’s ratio by \( \nu_s \). The radius of the middle surface of the cross-sectional area of the pipe is denoted by \( R \), as illustrated in Figure 1.

#### 3.1 Membrane shell model

Let us start with the easiest model - the membrane shell model. Normal stresses are considered, but shear and bending stress are neglected as well as the inertia of the pipe. The forces can be computed by \( n_\phi = \sigma_\phi \cdot s \) and \( n_z = \sigma_z \cdot s \). The normal stress is assumed to be constant on the cross-sectional area \( A_p = R \pi s \). The equilibrium condition of forces in \( R \)-direction leads to

\[
p R \frac{d\varphi}{dz} = 2 n_\varphi \frac{dz}{d\varphi} \sin \left( \frac{d\varphi}{2} \right) = n_\varphi \frac{dz}{d\varphi}
\]
which is equivalent to \( n_\varphi = p \cdot R \). The equilibrium of forces in z-direction is

\[
\frac{dn_z}{dz} = 0 \quad \text{i.e.} \quad n_z = n_0 = \text{const}.
\]

The displacement in z-direction is denoted by \( u_0 \). The strain \( \varepsilon_z \) in z-direction is defined by

\[
\varepsilon_z = \frac{du_0}{dz}
\]

**Figure 1:** Membrane shell model and is assumed to be constant on the cross-sectional area \( A_p \). In radial direction, the strain is

\[
\varepsilon_\varphi = \frac{2\pi(R+w) - 2\pi R}{2\pi R} = \frac{w}{R},
\]

where the radial displacement of the middle surface is denoted by \( w \). Applying Hook’s law, which combines the stress and the deformation linearly, one gets for the strain

\[
\varepsilon_z = \frac{1}{E_s} (\sigma_z - \nu_s \sigma_\varphi) \quad \text{and} \quad \varepsilon_\varphi = \frac{1}{E_s} (\sigma_\varphi - \nu_s \sigma_z).
\]

Finally, the relation between the displacement \( w \) and the pressure \( p \) in the pipe can be derived as an explicit formula

\[
w = \frac{R^2}{E_s s} p - n_0 \nu_s \frac{R}{E_s s}.
\]

### 3.2 Bending of shell model

In contrast to the membrane shell model, normal stress as well as shear stress and bending stress have to be considered for the shell model. The inertia of the pipe is neglected yet. Following the previous section, the derivation of the functional relation of the strain as a function of pressure and additional physical values is in the same manner: equilibrium of forces, computation of the deformations and applying the law of Hook. As a consequence of this procedure, a fourth order differential equation can be derived [7]

\[
\frac{d^4w}{dz^4} + C_1 \frac{d^2w}{dz^2} + C_2 w = C_0 \quad \text{with} \quad C_1 = \frac{2\nu_s R^2}{1 - \frac{h^2}{3R^2}},
\]

\[
C_2 = \frac{1}{E_s s},
\]

\[
C_0 = \frac{R^2}{E_s s} p - n_0 \nu_s \frac{R}{E_s s}.
\]
\[ C_2 = \frac{1}{R^2} + \frac{3}{h^2} \left( 1 - \frac{\nu_s^2}{2} \right) \], \quad C_0 = \frac{1}{\left( 1 - \frac{h^2}{3R^2} \right) K} \left( p \left( 1 - \frac{h}{R} \right) - \frac{\nu_s n_0}{R} \right). \]

The stiffness \( K \) is defined by the modulus of elasticity \( E_s \), the Poisson ratio \( \nu_s \) and the thickness of the pipe \( s = 2h \)

\[ K = \frac{E_s s^3}{12(1 - \nu_s^2)}. \]

We concentrate on the rigid clamped pipe, i.e. the boundary conditions are \( w(0) = w(L) = w'(0) = w'(L) = 0 \). The fourth order differential equation (3.6) can be solved with a finite element approach, cf. [6]. The boundary conditions and the finite element approach define the functional spaces for the ansatz and test functions. Because the coefficients \( C_1 \) and \( C_2 \) are constant, the bilinear form can be derived as follows:

\[ a(u, w) := (u'', w'') - C_1(u', w') + C_2(u, w) = (C_0, u) \quad \forall u, w \in H_0^2(0, L). \]

Here, \((\cdot, \cdot)\) denotes the well-known \( L_2\)-scalar product on the interval \((0, L)\).

The properties of the values \( C_i \) guarantee the uniqueness and existence of the solution \( w \), see [6].

### 3.3 Flow equations and pipe strain

After deriving the flow equations and strain models, the next step is to combine these models. The simplest way of combination is the consideration of the strain in the speed of sound for the fluid. This is done for different pipe strain models in [5]. In general, the relative change of the cross-sectional area \( A \) is

\[ \frac{dA}{A} = \frac{\pi(R + dw)^2 - \pi R}{\pi R^2} \approx 2 \frac{dw}{R}. \quad (3.7) \]

For the rigid clamped pipe one gets

\[ \frac{1}{A} \frac{dA}{dt} = \frac{2R(1 - \nu_s^2)}{E_s s} \frac{dp}{dt} = F_1 \frac{dp}{dt}, \quad (3.8) \]

and in the case of adjustable pipe

\[ \frac{1}{A} \frac{dA}{dt} = \frac{2R}{E_s s} \frac{dp}{dt} = F_2 \frac{dp}{dt}. \quad (3.9) \]

The speed of sound \( c_s \) for the system with weakly variable cross-sectional area can be derived by the comparison of the continuity equations

\[ \frac{dp}{dt} + \frac{\rho c^2}{1 + \rho c^2 F_{1/2}} \frac{\partial v}{\partial z} = 0 \quad \text{and} \quad \frac{dp}{dt} + \rho c^2 \frac{\partial v}{\partial z} = 0. \quad (3.10) \]
The speed of sound \( c \) in eq. (3.10) is only a property of the fluid. For the system, the propagation speed \( c_s \) can be derived as:

\[
\frac{1}{\rho c_s^2} = \frac{1}{\rho c^2} + F_{1/2}.
\]  

(3.11)

For the rigid clamped pipe and the adjustable pipe it follows by eq. (3.8) and (3.9):

\[
\frac{1}{c_s^2} = \frac{1}{c^2} + \rho F_{1/2} \quad \text{resp.} \quad c_s = \frac{c}{\sqrt{1 + \frac{2R \rho c^2}{E_s}}} \quad \text{and} \quad c_s = \frac{c}{\sqrt{1 + \frac{2R \rho c^2}{E_s}}(1 - \nu_s^2)}.
\]

There are further approximative formulas for the speed of sound for very thick pipes in the literature, cf. [5]. The modification of the propagation speed using the properties of the flow and mechanical systems can be used directly for well-known numerical schemes such as the method of characteristics, [8]. For modern high order numerical schemes based on a conservative formulation, this method does not qualify. The flow equations and the mechanical system have to be solved separately and both systems are coupled by the source term \( g \).

If the bending of shell model is considered, the fourth order differential eq. (3.6) has to be solved. In this case, the boundary conditions have to be defined precisely and a declaration of the speed of propagation for the system is not possible anymore. In one time step, two systems of equations - the flow equations and the equation for the displacement of the pipe (i.e. a linear system of equations) - have to be solved separately.

4 Numerical simulations

The test experiment consists of a high pressure reservoir (rail) with constant pressure of 1250 bar, a pipe of \( L = 1.5 \) meter length with constant cross-sectional area and a time controlled valve acting for an injection nozzle, see Figure 2. The pressure in the injection camber is chosen by 78.5 bar. The needle motion in the nozzle is split into three parts - an opening procedure, a constant opened nozzle, and a closing procedure. A model fluid with the properties of water is chosen, see [9]. The viscosity which is needed to compute the friction forces is chosen.
by values of a Diesel fluid. The maximal effective cross-sectional area of
dual injection valve is denoted by $A_{\text{eff}}$. First, let us consider a linear press-
include in the pipe and the according pipe deformation $w$, see

Figure 3. Second, a simulation with a rigid pipe and an elastic pipe (shell
tory) is done with the initial and boundary conditions given in Table 1.
The time-dependent pressure values in the nozzle chamber are plotted in

Figure 4. During the opening procedure of the needle, the computed pres-

sure distributions in both cases are equivalent. Afterwards, the influence of
the pipe strain becomes apparent in the phasing and the amplitude of the

pressure waves. The resulting different pressure distributions in the pipe
are the initial conditions for the next injection procedure which is certainly

not considered here. A comparison of the computed pressure values for the

membrane model and the shell model was done for an adjustable injection

pipe.

**Table 1:** Valve characteristic and initial conditions

<table>
<thead>
<tr>
<th>Injection valve</th>
<th>Pipe</th>
<th>Initial cond.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{\text{eff}}$ $[m^2]$</td>
<td>$R$ $[mm]$</td>
<td>$3$</td>
</tr>
<tr>
<td>$t_1$ $[ms]$</td>
<td>$s$ $[mm]$</td>
<td>$4$</td>
</tr>
<tr>
<td>$t_2$ $[ms]$</td>
<td>$E_s$ $[Pa]$</td>
<td>$2 \cdot 10^{11}$</td>
</tr>
<tr>
<td>$t_3$ $[ms]$</td>
<td>$l$ $[m]$</td>
<td>$1.5$</td>
</tr>
</tbody>
</table>

The agreement of the time dependent pressure values computed by both

models is very good with respect to the phasing and the amplitude. Cer-

tainly, the deformation $w$ computed by the membrane model is overesti-
mated by a factor of 1.45 in comparison with the shell model. As a con-
sequence of these investigations, both models can be used on principle to

simulate the wave motion in an injection system.
In the following, we will concentrate on parameter studies and will analyze the results. For this, the diameter $d$, the thickness $s$ of the pipe, and the valve characteristic are modified in order to study the influence to the pressure variation directly on the nozzle. The initial and boundary conditions are given on the left side of Table 2. The maximal pressure value in the nozzle chamber is denoted by $p_{\text{max}}$ and the maximal displacement of the pipe for one injection procedure is $w_{\text{max}}$. The complete injection time $t_3$ is for all test cases the same. There are only differences in the opening and closing times of the needle.

**Table 2:** Conditions of simulation examples and chosen results

<table>
<thead>
<tr>
<th>var</th>
<th>$t_1$ [ms]</th>
<th>$t_2$ [ms]</th>
<th>$t_3$ [ms]</th>
<th>$d$</th>
<th>$s$</th>
<th>$p_{\text{max}}$ [Pa]</th>
<th>$m_\alpha$ [g]</th>
<th>$w_{\text{max}}$ [um]</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>8.333</td>
<td>9.333</td>
<td>6</td>
<td>4</td>
<td>$2475 \cdot 10^5$</td>
<td>16.39</td>
<td>1.26</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>7.666</td>
<td>9.333</td>
<td>6</td>
<td>4</td>
<td>$2431 \cdot 10^5$</td>
<td>15.05</td>
<td>1.26</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>6.333</td>
<td>9.333</td>
<td>6</td>
<td>4</td>
<td>$1846 \cdot 10^5$</td>
<td>13.05</td>
<td>1.26</td>
</tr>
<tr>
<td>4</td>
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<td>8.333</td>
<td>9.333</td>
<td>6</td>
<td>3</td>
<td>$2480 \cdot 10^5$</td>
<td>16.39</td>
<td>1.77</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>7.666</td>
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<td>6</td>
<td>3</td>
<td>$2432 \cdot 10^5$</td>
<td>15.04</td>
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<td>6</td>
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<td>9.333</td>
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<td>3</td>
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<td>9.333</td>
<td>4</td>
<td>2</td>
<td>$2385 \cdot 10^5$</td>
<td>8.57</td>
<td>1.18</td>
</tr>
</tbody>
</table>

In Figure 5 and 6, the pressure $p$ and temperature $T_e$ are shown directly on the nozzle. If otherwise not mentioned, the simulations are done with the shell model. The first simulation was done without heat flux. The second simulation was done with heat flux on the last 0.5 meter length of the pipe. The high pressure values up to 2200 bar are the result of the throttle influence of the nozzle during the closing procedure and the properties of the fluid. In addition, the change of temperature due to the compression and relieve of the fluid are plotted in Fig. 5 and 6 too. The middle value of the temperature increases as a consequence of the friction in the pipe and the heating flux of the nozzle. The results presented on the right side of Table 2 will be commented now. First, the simulations of case 1 and 3 are compared. The time dependent pressure values in the nozzle chamber are plotted in Fig. 7. The maximum of the pressure $p_{\text{max}}$ in the nozzle chamber appears directly after the closing of the needle. A fast closing needle in the nozzle (closing time 1 ms) leads to large maximal pressure values ($2475$ up to $2557$ bar). A slower closing needle (3 ms) leads to lower pressure values ($1846$ up to $2385$ bar) in the nozzle chamber. An increase of the pressure in the injection nozzle should be avoided because the spill procedure is delayed. A smaller diameter $d$ of the pipe leads to higher pressure values in
the injection nozzle for fixed effective nozzle orifice. The injected mass $m_a$ directly depends on $A_{eff}$, opening and closing procedure of the needle and the cross-sectional area of the injection pipe. As presented in Table 2, the $A_{eff}$ is fixed for all simulations. Thus, the differences of injected mass $m_a$ only depends on the different opening and closing procedure and the cross-sectional area of the injection pipe. The time-dependent pressure values in the injection nozzle vary in dependence of opening and closing procedures, cf. Fig. 7 and Table 2. This fact has a significant influence to the injection process, e.g. for the spray and the mixture formation.

![Figure 5: Pressure $p$ and temperature $T_e$ in the nozzle chamber, $q = 0 \text{ kgm/s}^3$](image1.png)

![Figure 6: Pressure $p$ and temperature $T_e$ in the nozzle chamber, $q = 1.4 \cdot 10^4 \text{ kgm/s}^3$](image2.png)

The reservoir capability with respect to the fluid in the pipe is reduced due to a reduction of the diameter of the pipe from $d = 6 \text{ mm}$ to $d = 4 \text{ mm}$. Consequently, the pressure in the nozzle reduces significantly during the opening procedure as shown in Fig. 8. During the injection procedure, the pressure in the nozzle is approximately 250 bar, which does not guarantee an effective mixture formation. The injected fluid mass is $m_a = 9.88 \text{ g}$ for var. 10 in contrast to $m_a = 16.39 \text{ g}$ in var. 1. As a consequence of the smaller pipe diameter, the damping of the pressure wave is larger than in case 1. For the chosen injection system, the smaller pipe diameter is unsuitable.
Conclusion

The coupling of the flow equations and the pipe-strain models allow simulation of the dynamics of high-pressure hydraulic systems with high accuracy. The derived models have to validate an ongoing injection system in the next step. The parameter variations show that several geometrical properties as well as material properties have an influence on the wave motion in the pipe, which influences the injected mass and consequently the mixture formation. The presented CFD-code can support the development of hydraulic systems and the projecting period of basic experiments. Further improvements of the pipe-strain models are the consideration of the inertia of the pipe and the use of a higher-order numerical scheme and the simulation of shock waves in cavitating fluids.

Figure 7: Time dependent pressure in the nozzle chamber for var. 1 and 3

Figure 8: Time dependent pressure in the nozzle chamber for var. 1 and 10
References


