Dynamic behavior of a fluid plain journal bearing with incompressible fluid considering nonlinear fluid forces

A. Shintani¹, K. Fujita¹, K. Yoshioka¹, K. Okuno¹, H. Tanaka¹ & Y. Asaida²

¹Mechanical Systems Engineering, Graduate School of Engineering, Osaka Prefecture University, Japan
²Matsushita Electric Industrial, Japan

Abstract

Recently, in many areas such as computers and information equipments etc., the fluid journal bearings are required to rotate at higher speed. To satisfy this requirement, the strictly stability analysis of the journal is indispensable. In this paper, we investigate the stability analysis of the dynamic behavior of the fluid plain journal bearing with incompressible fluid considering the nonlinear terms of fluid forces. The stability analysis is examined by the numerical simulations on each model of rigid rotor and flexible rotor. The stable regions are compared by nonlinear analysis with the regions by classical linear analysis. Performing nonlinear simulation analysis, it becomes clear that there is rather a stable region in which the amplitude does not increase abruptly, and this phenomenon can not only be pointed out, but also is judged to be unstable by linear stable analysis. Finally, the experiment using actual bearings is performed and compared with the numerical results.

1 Introduction

Recently, fluid journal bearings are in much demand because they are very silent and can bear large loads. Many researchers have done the studies of the incompressible fluid plain bearing that were conducted on dynamic characteristics and stability by linearized equation [1-6]. However recently, in many areas, such as computers and information equipments, etc., the journal bearings are required to rotate at higher speeds. To satisfy this requirement, the strictly
stability analysis of the journal is indispensable. In this paper, we investigate the stability analysis of the dynamic behavior of the fluid plain journal bearing with incompressible fluid considering the nonlinear components of fluid forces. The stability analysis is examined by numerical simulations on each model of rigid rotor and flexible rotor. And we investigate how the nonlinear components in the fluid force affect the stability of fluid journal bearing by comparing with the stability analysis based on the linearized model. Finally the experiment using actual bearings is performed and compared with the numerical results.

2 Theory

2.1 Numerical simulation of rigid rotor model

When the rigid rotor on journal bearing rotating with the angular velocity $\omega$ is eccentric with eccentricity ($\varepsilon$, $\theta$) and eccentric velocity ($\dot{\varepsilon}$, $\dot{\theta}$) from the center of bearing, the dynamic fluid forces acting on the journal based on the infinite width plain theory are obtained as follows (shown in Fig.1);

\begin{align}
F_\varepsilon &= A \left[ \frac{2\varepsilon^2(\omega - 2\dot{\theta})}{(2 + \varepsilon^2)(1 - \varepsilon^2)} + \frac{2\dot{\varepsilon}}{(1 - \varepsilon^2)^{3/2}} \left\{ \frac{\pi}{2} - \frac{8}{\pi(2 + \varepsilon^2)} \right\} \right] \\
F_\theta &= A \left[ \frac{\pi\varepsilon(\omega - 2\dot{\theta})}{(2 + \varepsilon^2)\sqrt{1 - \varepsilon^2}} + \frac{4\dot{\varepsilon}}{(2 + \varepsilon^2)(1 - \varepsilon^2)} \right],
\end{align}

where the parameter $A$ is $A = 6\mu (r/h_0)^2 rb$, $\mu$: coefficient of viscosity, $r$: radius of a journal, $h_0$: averaged radial gap, $b$: width of a journal.

We assume that the thickness of oil film is small enough, and the oil pressure does not vary in the direction of thickness of oil film, and the inertia of oil is neglected. And we assume Half-Sommerfeld boundary condition. We consider that the journal is subjected to stationary load $F_b$ with fluid forces. The nonlinear equations of motion of journal are given by

\begin{align}
mh_0(\ddot{\varepsilon} - \varepsilon\ddot{\theta}^2) + F_\varepsilon - F_b \sin \theta &= 0 \\

mh_0(\ddot{\varepsilon} + 2\dot{\varepsilon}\dot{\theta}) + F_\theta - F_b \cos \theta &= 0,
\end{align}

where $m$ is the mass of a shaft. The relationship between stationary load $F_b$, and equilibrium displacement ($\varepsilon_b$, $\theta_b$) and other parameters of fluid bearing is expressed by the following equation:

\begin{align}
\frac{(2 + \varepsilon_b^2)(1 - \varepsilon_b^2)}{\varepsilon_b \sqrt{\pi^2 - (\pi^2 - 4)\varepsilon_b^2}} &= 6\mu \left( \frac{r}{h_0} \right)^2 \frac{rb \omega}{F_b}.
\end{align}

Assuming the small vibration from the equilibrium displacement of a journal, the equations of motion of rigid rotor and the journal are linearized. Deriving
the characteristic equation of the linearized equation, and applying Routh-Hurwitz's method to the resultant characteristic equations, we obtain the following condition for stability of the rigid rotor as reported classically:

\[
\frac{F_b}{m\eta_0\omega^2} > f(\varepsilon_b). \tag{6}
\]

2.2 Numerical simulation of flexible rotor model

Similarly to the rigid rotor model, the equations of motion of rotor and journal are linearized, and linear stability analysis for the flexible rotor model is done using the stability discrimination of Routh-Hurwitz. The motion and stability of this system are determined uniquely by the values \( \alpha, \varepsilon, \frac{F_b}{m\eta_0\omega^2}, \frac{F_b}{m\eta_0\omega_1^2} \).

3 Nonlinear numerical simulation

3.1 Rigid rotor model

The dynamic behavior of the journal of rigid rotor model is described by numerical simulations based on the nonlinear equations (3), (4). The results are shown in Fig. 2. The dynamic behaviors of journal are classified into three cases as mentioned in the former reported paper[7]; one is stable behavior; where the journal converges to one point as time goes by (marked by \( \bigcirc \) here), another is unstable behavior; where the amplitude of whirl motion becomes larger (marked by \( \times \)), and the other is stationary behavior; where the amplitude is kept constant for long time without converging to the equilibrium point and without diverging (marked by \( \triangle \)). Fig.3 shows the rotational speed of whirl and amplitude of whirl for various rotational speed of journal on three cases of journal parameters A, B, C. From Fig.3, it is found that the dynamic behavior of the journal goes to unstable according to the increase of rotational speed of journal under the following processes;

1) When a rotational speed of journal reaches the critical speed of linearized analysis, the journal is whirling with small amplitude keeping always the center of bearing outside of the whirling area of journal center. This
Figure 2: Stability map marked by $\bigcirc$, $\triangle$, $\times$ of fluid journal bearing with rigid shaft in nonlinear simulation and stability boundary shown by solid line in linear analysis.

stationary, but unstable state is similar to the limit cycle of self-excited vibration ($\triangle$).

2) As the rotational speed of journal increases more, the radius of whirl motion becomes larger, and at last, the journal contacts to the inner surface of bearing. This unstable state is similar to 'oil whirl' ($\times$). The rotational speed of whirl becomes about half of the rotational speed of journal.

3) Considering the nonlinear components in the fluid forces, we can distinguish $\triangle$ and $\times$, in other words, we can tell whether the journal rotates keeping always the center of the bearing outside or inside of the whirling area of journal center, that is impossible by the linearized analysis.

3.2 Flexible rotor model

Similarly to the case of the rigid rotor model, we solved the equations of motion of the center of the journal and rotor numerically and obtained the dynamic behavior of the center of journal and the center of rotor by giving the initial variance from the equilibrium displacement.

The representative examples of the simulation results are shown in Figs.4-5. The upper two subfigures in each figure show the vibrational behavior of the journal, the lower subfigures show the vibrational behavior of the rotor. The dynamic behavior is a little different due to the initial condition. In these subfigures, the left ones show time history response, and the right ones show the loci of the behavior. As shown in Figs.4-5, the behavior of the journal and
Figure 3: Relation between rotational speed of journal and whirling of journal (rigid rotor model).

rotor are classified into following three patterns as well as rigid rotor model.
1) The case where the amplitude of whirling of both the journal and rotor disappear, and the journal and rotor converge to the equilibrium displacement with time going by (marked by ○ in Fig.6).
2) The case where the journal is rotating on the same loci such as like a limit cycle after some time goes by, and so is the rotor as shown in Fig.4 (marked by ∆).
3) The case where the amplitude of whirling of the journal increases until contacting the inner wall of the bearing, and amplitude of whirling of the rotor increases as time goes by as shown in Fig.5 (marked by ×).

Figure 4: Vibration behavior of journal and rotor in flexible rotor model ($\varepsilon_b = 0.3, F_b/(mh_0\omega_1^2) = 7, F_b/(mh_0\omega_1^2) = 10$) ($\triangle$).

Figure 5: Vibration behavior of journal and rotor in flexible rotor model ($\varepsilon_b = 0.4, F_b/(mh_0\omega_1^2) = 2, F_b/(mh_0\omega_1^2) = 10$) ($\times$).

For these patterns, in the state marked by $\triangle$, the journal whirls with small amplitude keeping the center of bearing always outside of the whirling area of journal center similar to the case of rigid rotor model. Compared with the journal, the amplitude of whirling of rotor is quite large, though the amplitude of whirling of journal does not become more than a constant amplitude. In the state marked by $\times$, the journal whirls with large amplitude keeping always the center of fluid journal bearing inside of the whirling area of journal center. That is, the journal whirls all over the clearance in the journal bearing and the amplitude of whirling of rotor continues increasing even after amplitude of the journal are saturated.

In case of $F_b/mh_0\omega_1^2 = 10$, we plotted these three states (marked by $\circ, \triangle, \times$) on the $\varepsilon_b-F_b/mh_0\omega_1^2$ diagram in Fig.6. For comparison, the stability boundary by the linear stability discrimination in case of $F_b/mh_0\omega_1^2 = 10$ is also shown together. From this figure, it is seen that the stability boundary by the linear analysis and the stability boundary between the states marked by $\circ$ and $\triangle$ in nonlinear simulation almost coincide. This is the same result as that in the case of the rigid rotor model. Considering the actual employment of the fluid journal bearing using the flexible rotor, the state marked by $\times$ is said to be very dangerous state because journal contacts the wall of journal bearing and the amplitude of whirling becomes quite large. In the state marked by $\triangle$, the amplitude of whirling of the journal is less than a constant amplitude.
Fluid Structure Interaction II

Figure 6: Stability map (marked by ○, △, ×) by nonlinear simulation (flexible rotor model, $F_b/(m\omega_1^2) = 10$).

However, the amplitude of whirling of the rotor becomes quite large, so such state should be considered to be avoided.

4 Experiment

4.1 Experimental apparatus and test specimen

The rotors of the fluid bearing motor used for laser printer and so on are employed as the test specimen. These rotors are considered to be rigid evaluating the vibrational characteristics of the shaft.

We fixed the test specimen on the well-grounded weighting table, and measured the vibration of the rotor using the laser sensor. Laser sensor is fixed on the x-y-z 3 axes moving stage of which the position can be tuned in exactly. The side surface of the circular top plate of the rotor was employed as the measured position. Measurement analysis is done by getting the reflected light. Sampling rate is up to 10kHz. The rotational speed is changed by the modulating the generating frequency of the pulse generator. The rotational speed is measured by the noncontacting type tachometer. As the stationary load $F_b$ becomes zero when the fluid journal bearing is operated keeping the purely vertical axis position, the journal becomes unstable according to the stability map shown in Fig. 2. Therefore, we inclined the well-grounded weighting table with angle $\theta_T$ to the horizontal plane for changing the values of the stationary load $F_b$. Thus, $F_b$ is simulated to be $F_b = mg \sin \theta_T$ ($g$: gravity acceleration). The main dimensions of the test specimen are the diameter of shaft: 1.5mm,
the width of shaft: 10mm. Six non-grooved plain fluid journal bearings with the radial gap of $h_0 = 3.25, 10.25, 30.25, 50.25, 70.25, 80.25\mu m$ are employed. The rotational speed of the plain journal bearings is adjustable between about 25-225Hz by using the oscillator. The angle $\theta_T$ of the table is adjustable between about 10-40deg.

4.2 Experimental results

The FFT analyses were performed for six non-grooved plain fluid journal bearings in the experiment. It is discriminated whether the behavior of the plain journal bearing is stable or unstable by whether the component of half rotational speed is larger or not. That is, the behavior is unstable if the component of half rotational speed is seen severely in the spectrum, and it is stable if not so. When the power spectrum component of half rotational speed becomes large abruptly, the rotational speed is considered to be regarded as the critical speed. For example, the dynamic behavior of the plain journal bearing with $h_0=30.25\mu m$ for the rotational speed over about 126Hz as shown in Fig.7(a) can be considered to be unstable. Moreover, Fig.7(b) also shows the dynamic behavior of the plain journal bearing with $h_0=70.25\mu m$. The peak components of the half rotational speed are found to be seen over the rotational speed about 40Hz. Here, Fig.7(a) have been performed at first run test, and Fig.7(b) at second test, which have some intervals in time from the first run. Besides, the case of $h_0 = 3.25, 50.25\mu m$ are always stable for all rotational speeds. These stable or unstable experimental data are plotted on the nonlinear stability map as shown in Fig.8. When the experimental results are compared with the nonlinear results as shown in Fig.8, the boundary between the unstable and stationary (A) regions defined by the nonlinear simulation analysis for the lower value areas of the equilibrium displacement $\varepsilon_b$ of the stability map, which correspond to the cases with smaller clearances

![Figure 7: Power spectrum of displacement of non-grooved rotor.](image-url)
Figure 8: Stability analysis by experiment on the stability map of rigid rotor model.

$h_0$ and lighter stationary loads $F_b$. However, for the higher value areas of the equilibrium displacement of the stability map, which correspond to the cases with larger clearances $h_0$ and heavier stationary load $F_b$, the critical boundary by experiment can be considered to approach to the critical boundary defined by linear simulation analysis, not by nonlinear simulation analysis. These differences between the experiment and the numerical simulation are considered to be due to the difficulties of changing the stationary load $F_b$ in order to vary the values of $F_b/mh_0\omega^2$ and $\varepsilon_b$. In this experiment, as the stationary load is changed by inclining the fixed table, there might be the influence of the moment force on the plain journal bearing. Furthermore, there might be cavitation phenomena so on which affect on the assumption of the Half-Sommerfeld boundary condition. And it is also very difficult to check them due to too small clearance between a bearing and a journal. Though this kind of experiment is considered to be much difficult, we will challenge again for a more precise experiment.

**Conclusion**

From the above mentioned results, it can be concluded as follows:

1) The stability simulation analysis for the fluid plain journal bearings considering the nonlinear components in the fluid forces is proposed on each model of a rigid rotor and a flexible rotor.

2) It is understood by performing the nonlinear stability simulations that there are three kinds of regions, one is the stable state in which the journal converges to the static equilibrium position immediately (marked by ○),
another is the unstable, but stationary state in which the amplitude of
a journal is kept constant similar to a limit cycle for a long time neither
converging to the equilibrium position nor diverging (marked by \( \Delta \)) and
the other is the perfectly unstable state in which the amplitude of whirling
becomes large abruptly and the outer surface of a journal contacts the
inner surface of bearing (marked by \( \times \)).

3) Comparing the proposed nonlinear stability simulation analysis with the
classical linear stability analysis, it is found the stability boundary between
the stable state and the unstable, but stationary state in nonlinear sta-
bility analysis shows a good agreement with the classical linear stability
analysis. However, it is difficult to discriminate the stationary, but unsta-
ble state (marked by \( \Delta \)) and the perfectly unstable state (marked by \( \times \))
by the classical linear stability analysis.

4) Performing the rotating vibration experiment in which several actual
sized rigid rotor models consisted of a plain journal and bearing with
different clearances are used, the experimental results are confirmed to
be more stable than the critical boundary between the unstable region
(shown by \( \times \)) and the stationary region (shown by \( \Delta \)) defined by the non-
linear simulation analysis for the plain journal and bearing with rather
smaller clearances and lighter stationary loads. On the other hand, they
can be considered to approach to the critical boundary defined by the
linear simulation analysis for the plain journal and bearing with larger
clearances and heavier stationary loads. It is found that the differences
leave room for the more precise investigation in experiment.

References

893-918, 1996.
[7] Fujita, K., Shintani, A., Yoshioka, K., Proc. of the 7th International Confer-
[8] Yoshioka, K., Fujita, K., Shintani, A., Asaida, Y., JSME Dynamics and
[9] Fujita, K., Shintani, A., Yoshioka, K., Okuno, K., Okumoto, R., Asaida,
Y., Proc. of 6th Biennial Conference on Engineering Systems Design and