Hydroelastic analysis of a floating carpet in waves

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Abstract

A concept of wave energy extraction using a large flexible low-draft floating structure called ‘wave carpet’ is analyzed. In determining the motion of the carpet, a simplified linear two-dimensional problem is formulated. The carpet is assumed to be a slender elastic beam with free ends. The theory is developed in two steps. First the dry eigenmodes of the carpet are determined. Then, the hydrodynamic pressures along the carpet in waves are computed taking into account the wave diffraction/radiation effect of the carpet. The dry eigenmodes are included in this step in computing the pressure distribution. The amplitudes of carpet deflection and available energy stored in the carpet are estimated. The amount of energy that can be extracted for a typical carpet size is determined. The simplified method shows that it represents physical displacements of such a configuration by comparing the deflection of the carpet with available model test data on such plates.

1 Introduction

A concept of a floating ‘wave carpet’ was developed for the extraction of usable energy from the ocean waves. The design requirements for this wave carpet were that it operates in deepwater, produces steady power output and is rapidly redeployable. Such a concept for deepwater power generation is extremely useful for the Navy for futuristic marine operations in monitoring the oceans round the clock. Conceptually, the wave carpet assumes a networked grid, of interconnected independent power take off units. Each of these power take-off mechanisms is small in size compared to the present-day wave energy devices. Hence the wave carpet will contain a large number of such power units.

The dimensions of wave carpet are envisioned of the order of 1 km in length by 1 km in width. Thus, one carpet is a power plant by itself in the 10 MW
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range (10kW/m * 1000m effective capture width). It is relatively thin and rubber is intended to be the outer skin. Consequently, the wave carpet is expected to take the shape of the wave as it passes through the carpet.

An interesting feature of this wave carpet design is its ability to be rapidly redeployed using a self-propulsion system, as required in Navy applications. The system as envisaged is dynamically positioned using its own power. The main technology components of this innovative design are flexible floating body hydrodynamics, distributed controls and embedded fault redundant electric power grid that supports energy storage augmented by fluid power storage. This paper addresses the large flexible floating body hydrodynamics problem.

A simplified two dimensional problem is formulated for the wave structure interaction of the carpet. In this formulation, the wave carpet is excited along the predominant wave direction in a head sea. The carpet undergoes vertical displacements along its length. The analysis closely follows the work of Yamashita and Harada [4].

The eigenvalue problem for the dry mode of the carpet is solved first, assuming it to be a slender elastic beam with free ends (Wu, et al. [2]). For the hydroelastic response, an elastic beam theory is used and a modal expansion method is applied. The solution for the plate is obtained for a uniform pressure distribution in the vertical direction and the displacements of the plate at the defined finite elements are determined. Linear wave theory is used. The diffraction force, as well as the added mass and damping forces corresponding to the bending modes are obtained from the linear diffraction/radiation theory and are included in terms of equivalent coefficients. A computer code called SWEEP (Sea Wave Energy Extraction from Plate) is developed to analyze the vertical motion of the carpet undergoing eigenmodes subjected to regular waves along its length. Comparisons of the present analysis are made with the measured deflections. Model test data are obtained from available experimental results of a floating plate in regular waves.

In order to include the wave energy extraction, the damping term is modified through an additional term, which accounts for the wave energy extraction. This is accomplished by an equivalent damping coefficient on the entire carpet in the vertical direction by assuming that the energy is extracted uniformly.

2 Hydroelastic analysis

The hydroelastic behavior of a floating plate is considered within the framework of linear theory. The plate vibration is caused by periodic external load of a given frequency and amplitude. The plate deflection is governed by the Euler beam equation. The end points of the beam are assumed free of stresses. In the two-dimensional method, the wave direction is limited to 0-deg incidence along the length of the carpet. The formula is based on a single degree of freedom of the carpet in the vertical direction.

2.1 Problem definition

The configuration of the floating carpet is described as an elastic thin plate (e.g., a rubber material of suitable stiffness) of large horizontal extends. Both the
thickness and draft of the plate are much smaller than its length and the water depth. Therefore, the carpet is expected to undergo a hydroelastic response when subjected to waves, such as, in the application of VLFS as floating airports (Mamidupudi and Webster [1]).

2.2 Dry eigenmodes due to plate motions

The carpet is assumed to be a slender elastic beam with free ends. Assuming that the wave carpet experiences various modes of oscillation, displacement of the carpet $\xi$ is expanded in a series of eigenmodes.

$$\xi(x) = \sum_{j=0}^{\infty} X_j \mu_j(x)$$

in which the coordinate $x$ is the direction of wave, $X_j$ is the deflection amplitude in the j-th mode and $\mu_j$ is the mode function of unit amplitude. The mode functions are known for the continuous beam given by

$$u_0(x) = 1; \quad u_{2j}(x) = \frac{1}{2} \left( \frac{\cos \mu_{2j} q}{\cos \mu_{2j}} + \frac{\cosh \mu_{2j} q}{\cosh \mu_{2j}} \right)$$

$$u_1(x) = \frac{x}{a} = q; \quad u_{2j+1}(x) = \frac{1}{2} \left( \frac{\sin \mu_{2j+1} q}{\sin \mu_{2j+1}} + \frac{\sinh \mu_{2j+1} q}{\sinh \mu_{2j+1}} \right)$$

in which the quantity $a$ is the half-length of the carpet. The zeroth and first modes correspond to the rigid body heave and pitch modes respectively. The other sets of functions in Eqs. 2-3 represent the bending modes of a uniform beam with free ends. The quantities $\mu_j$ (for $j = 1,2,...$) are the positive real roots of the equation

$$(-1)^j \tan \mu_j + \tanh \mu_j = 0$$

Note that for indices 0 and 1, $\mu_0$ and $\mu_1$ are zero. Since the draft of the plate is small compared to its other dimensions, the horizontal motions of the plate in surge, sway and yaw are relatively unimportant.

The flexible carpet undergoes sinusoidal deflection in various modes. The normalized modal displacements for the first five bending modes based on the wave modal equation (Eqs. 2-3) are shown in Fig. 1. The x-axis is normalized by the carpet length and shown about the coordinate center taken at the mid-point of the carpet length. The resultant displacement of the carpet under wave load is a linear superposition of these modes. Each of these modes is related to its own eigenfrequency. Therefore, the carpet may respond to one or more of these eigenfrequencies based on the imposed wave period. In this case the deflection of the carpet will be substantially influenced by these modes compared to the others and resonance response may appear. The amplitude of motion of the carpet will be a function of damping present in the system.
2.3 Hydrodynamic potentials on carpet

In the second step, the hydrodynamic pressures are computed by the pressure distribution along the length of the carpet. Linear wave theory is used to compute pressures. The incident wave potential acting underneath the carpet length is calculated by the formula for the linear wave theory:

$$\phi_i = \frac{gH}{2\omega} \frac{\cosh k(z + d)}{\cosh kd} \exp[ikx] \exp(i\omega t)$$  \hspace{1cm} (5)

Figure 2 Definition of Carpet and Coordinate System
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in which \( g = \) gravitational acceleration, \( H = \) incident wave height, \( \omega = \) wave frequency \((=2\pi/T)\), \( T = \) wave period, \( d = \) water depth, \( x = \) horizontal coordinate, \( z = \) vertical coordinate from the still water level, \( k = \) wave number, and \( t = \) time. The coordinate system is defined in Fig. 2.

The wave number is given by

\[
\omega^2 = kg \tanh kd
\]  

The spatial part of pressure due to incident wave is computed from

\[
p_i(x,z) = \rho g \frac{H}{2} \frac{\cosh k(z+d)}{\cosh kd} \exp[ikx]
\]  

in which \( \rho = \) mass density of water. The vertical force on the carpet is computed using linear diffraction theory. According to this theory, the scattered or diffraction velocity potential \( \phi_D \) in heave is computed first which gives the diffraction pressure \( p_D \). Then the vertical force \( f \) on a small segment \( \Delta x \) of the plate at a given location \( x \) is computed from the formula

\[
f(x,t) = \{p_i(x) + p_D(x)\} \Delta x \exp(i\omega t)
\]  

### 2.4 Hydroelastic interaction of carpet

In the third step, wave induced motions including the dry eigenmodes of the structure are coupled with the hydrodynamic pressures. For the hydroelastic response, a modal expansion method coupled with the elastic beam theory is applied. This results in vertical displacements of the carpet along its length as a function of \( x \) and \( t \). Assume that the vertical displacement is given by

\[
v(x,t) = \text{Re}[\xi(x)\exp(i\omega t)]
\]  

where \( v \) is the displacement amplitude as a function of the horizontal distance \( x \) and time \( t \). Using elastic beam theory that accounts for the deformation of the plate, the structural deflection equation becomes

\[
EI \frac{d^4v(x,t)}{dx^4} + \frac{w(x)}{g} v(x) = f(x,t) + f_a(x,t) + f_d(x,t) + f_r(x,t)
\]  

in which \( EI \) is the wave carpet stiffness factor, and \( w \) is the distributed weight per unit breadth. The quantities on the right hand side are: \( f = \) total hydrodynamic force intensity including the incident, and diffracted wave on the carpet, \( f_a = \) added mass force intensity, \( f_d = \) radiation damping force intensity, and \( f_r = \) restoring force intensity, each having a time dependence of the form of Eq. 9.

The incident and diffracted wave pressures combine to provide the excitation force (Eq. 8). The diffraction is computed by the linear diffraction theory for the given wave. The radiated wave pressures are related to the various modes of motion of the carpet. The resulting added mass and damping terms are included in the left-hand side of Eq. 10. The displacement \( v(x,t) \) is replaced by the right hand side of Eq. 9. Both sides of Eq. 10 are multiplied by \( u_i \) and
integrated over the carpet length. Then the equation of motion may be written from Eq. 10 in a traditional format after eliminating time

$$\sum_{j=0}^{n} \left[ D_{ij} - \omega^2 (M_{ij} + m_{ij}) + i \omega N_{ij} + C_{ij} \right] X_j = F_i$$

(11)

The first term on the left hand side is the elastic stiffness term, the second term is the mass and added mass term, the third term is the damping term and fourth term is the restoring force term. The right hand side is the excitation force from the wave pressure. These terms for the various modes are obtained from the integration along the length of the wave carpet. The integral expressions for these terms are

$$D_{ij} = EI \int_{a}^{b} u_i(x) \frac{d^4}{dx^4} u_j(x) dx$$

(12)

$$M_{ij} = \frac{w(x)}{g} \int_{a}^{b} u_i(x)u_j(x) dx$$

(13)

$$m_{ij} - \frac{1}{i} \frac{N_{ij}}{\omega} = -\rho \int_{a}^{b} \phi_j(x)u_j(x) dx$$

(14)

$$C_{ij} = \rho g \int_{a}^{b} u_i(x)u_j(x) dx$$

(15)

$$F_i = \int_{a}^{b} \left\{ p_j(x) + p_d(x) \right\} u_j(x) dx$$

(16)

in which $\phi$ are the radiated velocity potential due to the motion of the carpet in various eigenfrequencies. The effect of diffraction is introduced in term of a diffraction coefficient. The added mass and damping terms are introduced in the equation in the form of added mass coefficients and linear damping factors obtained by the radiation theory.

The preceding equations may be simplified by making use of the following orthogonal relationship for all values of the subscripts $i$ and $j$:

$$\delta_{ij} = \frac{2}{a} \int_{a}^{b} u_i(x)u_j(x) dx$$

(17)

which when coupled with Eqs. 2-3 will yield

$$\delta_{00} = 4; \delta_{0j} = 0; \delta_{11} = 4/3; \delta_{ij} = 0;$$

(18)

$$\delta_{ij} = \begin{cases} 1 & (i = j) \\ 0 & (i \neq j) \end{cases} \quad i, j \geq 2$$

(19)

Then the integral equations reduce to the following simple forms
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\[ D_{ij} = EI \frac{1}{2\alpha^3} \mu^4 \delta_{ij} \]  
(20)

\[ M_{ij} = \frac{w a}{g} \delta_{ij} \]  
(21)

\[ m_{ij} = C_a \frac{w a}{g} \delta_{ij} \]  
(22)

\[ N_{ij} = 2\zeta \sqrt{(M_{ij} + m_{ij})(\rho g 2b)} \]  
(23)

\[ C_{ij} = \rho g 2b \frac{a}{2} \delta_{ij} \]  
(24)

\[ F_j = \int_a^b C_i p_j u_j(x) dx \]  
(25)

in which \( C_a \) and \( \zeta \) are the added mass coefficient and damping factor respectively. The force includes the diffraction coefficient \( C_a \) and incident wave pressure \( p_i \). The expressions from Eqs. 20-25 are included in the equations of motion in Eq. 11, which is solved for the amplitude \( X_j \) for the \( j \)th mode of motion. Note that \( \delta_{ij} \) is a diagonal matrix so that the expressions in Eq. 11 are uncoupled. Thus the modal deflection at each eigenfrequency may be obtained simply by dividing the right-hand side by the term within bracket on the left-hand side. The deflection along the wave carpet may then be derived from Eq. 1.

The modal displacement for each eigenmode may be separately determined. The results of the maximum displacement as a function of wave periods are shown in Fig. 3. The peaks and valleys of the deflection correspond to the carpet length for a given wave period becoming multiple of the wavelength.

![Figure 3 Maximum Deflection of the Carpet for the First Six Modes](image)

2.5 Energy extraction from carpet

The solution of Eq. 11 provides the displacement of the carpet in its free modes of oscillation before any energy extraction. Thus, it represents the maximum
oscillation of the carpet. When the energy is extracted from the carpet, the oscillation amplitude will be reduced depending on the amount of energy extracted. This effect may be represented by an additional linear damping coefficient in the equation of motion. In this case, the equation of motion becomes

\[
\sum_{j=0}^{\infty} \left\{ D_j - \omega^2 (M_j + m_j) + i \omega (N_j + N^e) + C_j \right\} X_j = F_i
\]  

(26)

in which the damping coefficient \( N^e \) corresponds to the damping due to energy extraction assuming that the energy is extracted uniformly. This damping term will reduce the motion of the carpet, which is directly equivalent to the amount of energy extracted. Then the total energy extracted from the carpet at a given wave height and period may be computed in the following way. The energy of the carpet due to its motion with radiation damping alone before the energy is extracted is derived first. Then the energy of the carpet in its motion with the additional damping term representing the energy extraction is computed. The difference of the two provides the amount of energy extracted for the given wave period and wave height. The formula for the energy in the carpet due to its motion at a modal frequency is given by

\[
E_j = \frac{1}{2} \rho g (2b) X_j^2 L_j
\]  

(27)

in which \( L_j \) is the modal oscillation length of the carpet and \( X_j \) is the oscillation amplitude at the \( j \)-th mode. Thus the total energy due to all modes may be computed by computing the left-hand side of Eq. 27 for all modes. The difference in the total energy between the two cases provides the extracted energy.

### 3 Numerical results

A few examples are chosen to demonstrate the capability of the SWEEP program and show the results for the wave carpet. For this purpose a carpet length typical of the possible prototype is chosen. The possible wave energy that may be extracted in ocean waves by the present concept is estimated.

#### 3.1 Displacement along length

The displacement amplitude in regular waves along the (normalized) length of the carpet is plotted in Fig. 4 for two different damping factors of 0.0 and 0.7 (radiation damping being 0.1 in both cases). These damping values are equivalent to the amount of power extraction from the carpet. The input values are shown in the figure for these cases. As shown, the motion amplitude of the carpet reduces to a small value in the large middle portion as the waves travel along its length. The amplitudes near the fore and aft ends result from the ends of the carpet being free. The overall reduction in the amplitude of motion with the power extraction (from 0.0 to 0.7 damping factor) is clearly evident in the
solution.

<table>
<thead>
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<th>Number of Meshes</th>
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<td>Number of Modes</td>
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<td>Carpet Length</td>
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<tr>
<td>Carpet Width</td>
<td>1000 m</td>
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<tr>
<td>Carpet Draft</td>
<td>2 m</td>
</tr>
<tr>
<td>Stiffness</td>
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</tr>
</tbody>
</table>

Figure 4 Normalized Displacement Amplitude along the Carpet

The displacement amplitudes at the fore end of the carpet versus wave period (i.e., transfer function) are shown in Fig. 5 for two different damping values. The zero damping means no power is extracted and the system only has radiation damping factor of 0.1. The reduction in the amplitude of motion as the power is extracted from the system is clear from the plot of displacement for an extraction equivalent damping factor of 0.3.

Figure 5 Displacement of the Fore End of the Carpet versus Wave Period

4 Comparison with available model test data

Model test data was obtained from Yamashita [3] on a flat plate of length 10m, breadth 2m and draft 0.017m. The model represents a floating plate at a scale of 1:30. The deflection of the plate was measured at 9 equidistant locations in regular waves. The comparisons of the present analysis are made with the measured deflections are shown for three different waves in Fig. 6. The solid circles in the figure indicate measured deflection amplitudes, which are normalized by the wave amplitude. The solid lines are obtained from the computer program SWEEP. The results are based 27 modes of the plate. The ratios of wavelength to plate length correspond of 0.2, 0.3 and 1.0 respectively. It is found that the correlation is fairly good in all cases except for a few stray points. The elastic deformation of the plate is evident in all cases.
5 Conclusions

A concept of a large flexible structure called 'wave carpet' floating at a small draft in the open ocean was considered for the extraction of wave energy. A simplified analysis of the wave carpet in head seas in waves was made. The analysis is based on a 2-D approach and the motions of the wave carpet in the vertical direction due to its eigenmodes were determined.

The computed displacement amplitudes of the carpet were compared with available experimental data on such structures with favorable results.

Energy is extracted in the wave carpet concept using a mechanical system. The extraction of energy reduces the overall motion of the carpet, which was represented by a linear damping factor. The amount of available energy was estimated based on an assumed extraction coefficient.

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References


