A nonlinear numerical model for the interaction of surface and internal waves with very large floating or submerged flexible platforms

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Abstract

The interaction of both surface and internal water waves with floating/submerged platforms was studied by considering the nonlinear properties of the fluid motion and the flexibility of the oscillating plates. First a set of nonlinear governing equations for the interaction between a multi-layer fluid system and floating/submerged elastic thin plates was obtained. In each fluid layer, the Lagrangian was integrated vertically by taking the nonlinear boundary conditions on the interfaces into account. Then the variational principle was applied to yield a set of time-dependent, horizontally two-dimensional, fully nonlinear equations. Secondly, using this model, several numerical computations were carried out for the surface/internal long waves and the oscillations of horizontally very large platforms. The resonated pressure beneath the plate floating on the sea surface in the two-layer system was compared with that in the one-layer case. The surface/internal waves generated by the trembling of the floating/submerged elastic plates were also simulated.

1 Introduction

Research and development have made progress on large-scale floating-type offshore structures. Such structures are affected by the water with pressure and frictional forces and, in addition, influence the fluid motion through their oscillation. In coastal engineering, the interaction between floating
Flexible platforms and fluids has been treated for the problems of ice plates and sea water interaction (Squire et al. [1]). The results are applicable to designing floating flexible structures. For the interaction between a very large floating flexible structure and a fluid, various numerical models have been developed. Hermans [2] solved the interaction of thin-plate oscillation with fluid motion in a coexistence field of linear waves and a current by using BEM, and simulated the response of a plate to a moving weight, equivalent to an airplane on a floating airport. Takagi [3] adopted Boussinesq-type equations for surface waves and solved them using a finite-difference method to see the relation between bending momentum and flexural rigidity when a solitary wave propagates below a thin plate. Sakai et al. [4] studied the interaction between a solitary wave and a thin plate by coupling BEM and FEM without any assumption for the velocity potential, and found disintegrated waves to occur in front of the solitary wave as its nonlinearity becomes strong.

When a density stratification exists under these floating structures, internal waves can be generated, changing the water environment, e.g., temperature distributions. In this case, the fluid motion is different from that in a uni-density flow, resulting in different forces acting on the structure. In this study, the interaction of surface and internal waves with floating or submerged flexible structures is investigated by considering the nonlinearity of the fluid motion.

First a set of nonlinear governing equations for the interaction between thin plates and a multi-layer fluid system is derived. Structures are assumed to be horizontally large thin plates oscillating flexibly on/below the sea surface. Such floating structures on the sea surface are used for airports, large bridges, storage facilities, residential spaces, etc. Flexible structures in the sea are, for example, submerged elastic thin plates able to decay wave energy. On the other hand, for the fluid motion, a fully nonlinear model for a multi-layer system (Kakinuma [5]) is utilized. This model is applicable to strongly nonlinear and strongly dispersive surface and internal waves because it has been derived under no assumption on velocity potentials, by giving the nonlinear boundary conditions on the sea surface, the interfaces and the seabed. In this model, it is assumed that no fluids with different densities mix with each other. Since this leads to discontinuity of some observables or dynamic variables at interfaces, an elastic plate, which is also a continuum, can be put in between two different fluids. Then the fluid motion touching the elastic thin plates can be studied.

Secondly results of several numerical computations in the vertical 2-D are shown for the surface/internal long waves and the oscillations of horizontally very large platforms. In forced oscillation, the resonated pressure beneath the plate floating on the sea surface in the two-layer system is compared with that in the one-layer case. In free oscillation, internal waves or displacements of the floating/submerged elastic plates have been calculated when an initial profile is given on the sea surface. Furthermore, surface
waves generated by a trembling of the submerged elastic plate have also been simulated.

2 A set of equations for oscillation of thin plates and surface/internal waves

2.1 Stratified fluids and thin plates

As illustrated in Figure 1, inviscid and incompressible fluids are named $i$ ($i = 1, 2, \cdots, I$) from the top to the bottom and the density $\rho_i$ is constant in the $i$-layer. None of the fluids mix together. The no-friction condition is assumed on every interface. No unstable phenomena, such as vortex generation or wave breaking, occur on any interface whose profile is assumed to be a single-valued function of $x$.

The $i$-layer is sandwiched between two elastic thin plates. The elevation of its lower interface is $z = \eta_{i,0}(x, t)$, and the pressure there is $p_{i,0}(x, t)$. The elevation of the upper interface is $z = \eta_{i,1}(x, t)$ where the pressure is $p_{i,1}(x, t)$. The pressure acting on any point over a plate surface is assumed equal to that of the fluid touching this point.

The thin plate touching the upper interface of the $i$-layer is called the $i$-plate. The density and the vertical width of the $i$-plate are $\rho_i$ and $\delta_i$, respectively, and both the values are constant for each plate. The case where $\rho_i = \delta_i = 0$ and the $i$-plate, with no flexural rigidity, does not resist the fluid motion corresponds to the case where two different fluids touch each other directly without any plate. In such a case where flexural rigidity is low, it is assumed that the density of the upper layer is less than that of the lower layer.

The structure is assumed to be horizontally very large and only the area near its center is dealt with. Thus the ends do not affect the calculation domain. Since no fluid goes into another layer around an edge, the still water depth will be $h_i(x)$ again after some motion as long as the fluid volume remains constant.

2.2 Equation of motion for an elastic plate

A dynamic field of a homogeneous, elastic body is formed in the $i$-plate. The horizontal length scale is much larger than the thickness of the plate. According to the classical theory, the state of motion of the $i$-plate is determined by eqn (1), which is a wave equation in a wide sense:

$$m_i \delta_i \frac{\partial^2 \eta_i}{\partial t^2} + B_i \nabla^2 \nabla^2 \eta_i + m_i g \delta_i + p_{i-1,0} - p_{i,1} = 0,$$  

where $B_i$ is the flexural rigidity of the $i$-plate and $g$ is the gravitational acceleration; $z = \eta_i$ is the profile of the neutral plane and $\nabla = (\partial/\partial x, \partial/\partial y)$ is a partial differential operator in the horizontal plane.
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Figure 1: Thin-plates and a multi-layer fluid system.

\[
\begin{align*}
\rho_i, \phi_i & , z = \eta_{i,1}, p = p_{i,1} \\
\delta_i & \\
\rho_{i+1}, \phi_{i+1}, z = \eta_{i,0}, p = p_{i,0}
\end{align*}
\]

\[
T\sqrt{g h/L} = 0.8\tau
\]

\[
T\sqrt{g h/L} = \tau
\]

\[
T\sqrt{g h/L} = 1.3\tau
\]

\[
\tau = 0.98
\]

\[
T\sqrt{g h/L} = 0.2\tau
\]

\[
T\sqrt{g h/L} = 0.5\tau
\]

\[
T\sqrt{g h/L} = 2\tau
\]

\[
T\sqrt{g h/L} = 5\tau
\]

Figure 3: Pressure on the plate. The interface is resonated by the forced oscillation of the structure on the sea surface \((L/h = 10, a/h = 0.2, \rho_2/\rho_1 = 1.5, x/L = 0.5)\).

(a) A thin-plate oscillating on the sea surface.

(b) Two thin-plates oscillating on and below the sea surface.

(c) A thin-plate oscillating below the sea surface.

Figure 4: Pressure on the plate. The interface is not resonated by the forced oscillation of the structure on the sea surface \((L/h = 10, a/h = 0.2, \rho_2/\rho_1 = 1.5, x/L = 0.5)\).

Figure 2: Thin-plate structures and two-layer fluid systems.
2.3 A set of equations for surface and internal waves

2.3.1 Functional in the variational principle

The motion is assumed to be irrotational and the velocity potential $\phi_i$ is determined by

$$u_i = \nabla \phi_i, \quad w_i = \frac{\partial \phi_i}{\partial z},$$  \hspace{1cm} (2)

where $u_i$ and $w_i$ are the horizontal velocity vector and the vertical velocity component at each point in the $i$-layer, respectively.

The elevation of one of the two interfaces belonging to the $i$-layer, $z = \eta_{i,1-j}$ ($j = 0$ or 1), and the pressure on the opposite interface, $p_{i,j}$, are treated as known values. Thus the unknown variables are the velocity potential $\phi_i$ and the elevation of the interface $\eta_{i,j}$. In this case, the Lagrangian in the $i$-layer, $\mathcal{F}_i[\phi_i, \eta_{i,j}]$, is defined by eqn (3) modifying that of Luke [6]:

$$\mathcal{F}_i[\phi_i, \eta_{i,j}] = \int_{t_0}^{t_1} \int_A \int_{\eta_{i,0}}^{\eta_{i,1}} \left\{ \frac{\partial \phi_i}{\partial t} + \frac{1}{2} (\nabla \phi_i)^2 + \frac{1}{2} \left( \frac{\partial \phi_i}{\partial z} \right)^2 \right\} \rho_i \frac{p_{i,j} + P_i + W_i}{\rho_i} \, dz \, dA \, dt,$$ \hspace{1cm} (3)

where $(\nabla \phi_i)^2 \equiv |\nabla \phi_i|^2$; $P_i = \sum_{k=1}^{i-1} \{(\rho_i - \rho_k)gh_k\}$ and $W_i = \sum_{k=1}^{i} (-m_kg\delta_k)$ are constant in each layer.

2.3.2 Introduction of vertical distribution functions

The velocity potential $\phi_i$ is expressed as a sum of vertical distribution functions $Z_{i,\alpha}(z) = z^\alpha$ multiplied by their weights $f_{i,\alpha}$:

$$\phi_i(x, z, t) \equiv \sum_{\alpha=0}^{N-1} \{z^\alpha \cdot f_{i,\alpha}(x, t)\} = f_{i,0} \cdot 1 + f_{i,1} \cdot z + f_{i,2} \cdot z^2 + \cdots + f_{i,N-1} \cdot z^{N-1}.$$ \hspace{1cm} (4)

2.3.3 Euler-Lagrange equations in the variational principle

For each fluid layer, in the manner similar to that of Isobe [7], we substitute eqn (4) into eqn (3) and integrate the Lagrangian vertically. Then the variational principle is applied to yield a set of Euler-Lagrange equations of $f_{i,\alpha}$ and $\eta_{i,j}$ for surface and internal waves ($\alpha = 0, 1, 2, \cdots, N-1$):

$$\eta_{i,1}^\alpha \frac{\partial \eta_{i,1}}{\partial t} - \eta_{i,0}^\alpha \frac{\partial \eta_{i,0}}{\partial t} + \nabla(Q_i[\alpha + \beta] \nabla f_{i,\beta}) - R_i[\alpha, \beta]f_{i,\beta} = 0,$$ \hspace{1cm} (5)

$$\eta_{i,j}^\alpha \frac{\partial f_{i,\beta}}{\partial t} + \frac{1}{2} \eta_{i,j}^\gamma + \nabla f_{i,\gamma} \nabla f_{i,\beta} + \frac{1}{2} S_{i,j}[\gamma, \beta] f_{i,\gamma} f_{i,\beta} + g \eta_{i,j} + (p_{i,j} + P_i + W_i)/\rho_i = 0,$$ \hspace{1cm} (6)
where $Q_i[\alpha]$, $R_i[\alpha, \beta]$ and $S_{ij}[\alpha, \beta]$ are the functions of $\eta_i,e$ ($e = 0$ or $1$):

$$Q_i[\alpha] = \frac{1}{\alpha + 1}(\eta_i^{\alpha+1} - \eta_i^{\alpha+1})$$  (7)

$$R_i[\alpha, \beta] = \begin{cases} \frac{\alpha \beta}{\alpha + \beta - 1} (\eta_i^{\alpha+\beta-1} - \eta_i^{\alpha+\beta-1}) & (\alpha \beta \neq 0) \\ 0 & (\alpha \beta = 0) \end{cases}$$  (8)

$$S_{ij}[\alpha, \beta] = \begin{cases} \alpha \beta \eta_i^{\alpha+\beta-2} & (\alpha \beta \neq 0) \\ 0 & (\alpha \beta = 0) \end{cases}$$  (9)

### 2.3.4 Assumption for long waves

Every layer is assumed to be in a shallow water region. The following nondimensional variables are introduced:

$$x^* = \frac{x}{\ell}, \quad y^* = \frac{y}{\ell}, \quad z^* = \frac{z}{h}, \quad t^* = \frac{\sqrt{gh}}{\ell} t, \quad \nabla^* = \ell \nabla,$$

$$\frac{\partial}{\partial t^*} = \left(\frac{\partial}{\partial t}\right)^*, \quad \eta_i^* = \eta_i, \quad f_i^* = \frac{h^\alpha f_i,\alpha}{\sqrt{gh}} \left(\frac{h}{H}\right)^\alpha,$$

$$B_i^* = \frac{B_i}{\rho g \ell^4}, \quad p_i^* = \frac{p_{i,e}}{\rho gh}, \quad P_i^* = \frac{P_i}{\rho gh}, \quad m_i^* = \frac{m_i}{\rho}, \quad \delta_i^* = \frac{\delta_i}{h}, \quad W_i^* = \frac{W_i}{\rho gh}.$$  (10)

The order of the nondimensional values with the mark of ‘*’ is assumed to be $O(1)$. Substituting eqn (10) into eqns (1), (5) and (6), we obtain eqns (11) – (13):

$$\sigma^2 m_i^* \delta_i^* \frac{\partial^2 \eta_i^*}{\partial t^*^2} + B_i^* \nabla^{*2} \eta_i^* + m_i^* \delta_i^* + p_i^* - p_i^* = 0,$$  (11)

$$\eta_i^{\alpha,1} \frac{\partial \eta_i^{*1}}{\partial t^*} - \eta_i^{0,0} \frac{\partial \eta_i^{*0}}{\partial t^*} + \varepsilon^\beta \nabla^*(Q_i^*[\alpha + \beta] \nabla^* f_i^*),$$

$$-\frac{\varepsilon^\beta}{\sigma^2} R_i^*[\alpha, \beta] f_i^* = 0,$$  (12)

$$\varepsilon^\beta \eta_i^* \frac{\partial f_i^*}{\partial t^*} + \varepsilon^{\gamma+\beta} \frac{1}{2} \eta_i^* \gamma^{\gamma+\beta} \nabla^* f_i^* \gamma \nabla^* f_i^* + \frac{\varepsilon^{\gamma+\beta}}{\sigma^2} \frac{1}{2} S_{ij}[\gamma, \beta] f_i^* f_j^* f_i^* f_j^*$$

$$+ \eta_i^* + p_i^* + P_i^* + W_i^* = 0,$$  (13)

where $\varepsilon = H/h$ and $\sigma = h/\ell$ are representative values of the wave height to water depth ratio and the water depth to wavelength ratio, respectively. It is assumed that $O(\varepsilon) = 1$ and $O(\sigma^2) \ll 1$.

For simplicity, only one term is used for the vertical distribution function: $N = 1$; $Z_{i,0} \cdot f_i,\alpha = Z_{i,0} \cdot f_i,0 = 1 \cdot f_i,0 = f_i$. The terms of $O(1)$ are
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taken into account. In eqns (11) – (13), let \( j = 1 \) and eliminate \( p^*_1 \). Then the following equations are obtained in a dimensional form:

\[
\frac{\partial \eta_{i,1}}{\partial t} - \frac{\partial \eta_{i,0}}{\partial t} + \nabla \left\{ (\eta_{i,1} - \eta_{i,0}) \nabla f_i \right\} = 0, \tag{14}
\]

\[
\frac{\partial f_i}{\partial t} + \frac{1}{2} (\nabla f_i)^2 + g \eta_{i,1} + (B_i \nabla^2 \eta_{i,1} + p_{i-1,0} + P_i + W_{i-1})/\rho_i = 0, \tag{15}
\]

where the inertia term for the thin plate is relatively ignored. Takagi [3] assumed that \( m^* \delta^* = \varepsilon^{-1} m \delta/(\rho h) \) considering the orders up to \( O(\varepsilon) = O(\sigma^2) \ll 1 \), and in that case the inertia term for a thin plate is relatively ignored. Because of the plate width, the curvatures in the neutral plane \( z = \eta_i \) are different from that in the plate surface (Forbes [8]), but this difference is ignored in eqn (15), resulting in \( \nabla^2 \nabla^2 \eta_i \approx \nabla^2 \nabla^2 \eta_{i,1} \).

3 Numerical calculations

3.1 Forced oscillation

Figure 2 shows two-fluid systems with thin plates in the vertical 2-D. Let the elevations of the surface and the interface be \( z = \eta_{1,1} = \zeta \) and \( z = \eta_{1,0} = \eta \), respectively. Using a finite-difference method, eqn (14) and eqn (15) are solved on the nondimensional variables. Assuming that every floating structure is horizontally large and the same phenomenon appears periodically along the \( x \)-axis, one end of the domain is connected smoothly to the other. The air pressure is set at 0. In the following calculations, the initial value of the velocity potential \( \phi_i \) is equal to zero at any point.

A forced oscillation, which has a constant period and a constant wavelength, is given on the sea surface. In this case (see Figure 2 (a)), the oscillation of the structure floating on the sea surface gives rise to the fluid motion. In Figures 3 and 4, the pressure variation at the middle point \( (x,z) = (L/2,\zeta) \) on the structure, \( p_M \), is shown when a continuous oscillation of \( \zeta = a \sin(2\pi t/T) \cos(2\pi x/L) \) is given on the sea surface. The pressure \( p_M \) in the one-layer fluid for each corresponding case is represented by a broken line. For the cases shown in Figure 3, a resonance phenomenon has been generated. Thus the pressure acting on the floating structures becomes larger and larger with the increase in the amplitude of their displacement. The calculations were stopped when the internal waves touched the seabed on \( x = L/2 \) in the case where \( T\sqrt{gh}/L = 0.87 \) and \( \tau \), and touched the structure on \( x = 0 \) in the case where \( T\sqrt{gh}/L = 1.37 \). When \( T\sqrt{gh}/L = 0.87 \), the amplitude of the pressure decreases before increasing again. In Figure 4, the pressure does not increase so remarkably as in Figure 3. When the forced-oscillation period \( T \) is relatively short, the amplitude of the pressure becomes larger than that in the one-layer case. As the period is long enough, the amplitude of the pressure is almost equal to that in the one-layer fluid.
3.2 Free oscillation

The interaction between thin-plate structures and fluids is treated when a profile is given on the sea surface and then released at the initial time. Since, in the whole system, only the gravitation acts as an external force, a free oscillation is generated.

The elevation of the interface at the middle of the calculation domain, \( z = \eta_M(L/2, t) \), is shown in Figures 5 – 9 when the sea surface is given as \( \zeta = -a_0 \cos(2\pi x/L) \) in the initial condition. There are several different modes in the surface and internal waves. In Figures 5 – 7, a structure is floating on the sea surface as shown in Figure 2 (a). As the flexural rigidity \( B_1^* = B_1/(\rho_1 g L^4) \) is lower, the amplitude of \( \eta_M \) is larger. In the cases where \( B_1^* = 1 \) and 10, as the wavelength of the initial profile is longer, the period of each mode is longer. In Figure 8, as shown in Figure 2 (b), there are two elastic plates, having the same flexural rigidity, on and below the sea surface. In Figure 9, as shown in Figure 2 (c), a submerged elastic plate oscillates in the sea. The initial conditions for the cases in Figures 8 and 9 are the same as those for the cases in Figure 6. Throughout Figures 5 – 9, when the flexural rigidity is low, disintegration occurs on \( z = \eta_M \). For example, when \( B_1^* = 0.1 \) in Figure 6, three waves are generated at \( t \sqrt{g h}/L \approx 20 \) and one of them disappears at \( t \sqrt{g h}/L \approx 70 \).

3.3 Surface waves generated by a submerged thin plate

Surface waves generated by a thin plate, which oscillates between two fluids as shown in Figure 2 (c), are numerically simulated.

Figure 10 shows the surface-wave profile \( z = \zeta \) and the structure configuration \( z = \eta \) for the case where the submerged elastic plate is released after an initial profile \( \eta = -a_0 \cos(2\pi x/L) - h_1 \) is given. Figure 10 includes two cases where the fluid-density ratios are different: \( \rho_1 < \rho_2 \) on the left-hand side and \( \rho_1 > \rho_2 \) on the right-hand side. Because of the flexural rigidity of the structure, the internal waves are also stable when \( \rho_1 > \rho_2 \). In both cases, several crests are generated on the sea surface. In the case where \( \rho_2/\rho_1 = 1/1.025 \), the oscillation period of the structure is shorter than that in the case where \( \rho_2/\rho_1 = 1.025 \) resulting in a phase shift between the two cases.

4 Conclusion

A set of nonlinear equations governing the interaction between floating thin plates and a multi-layer fluid system has been derived and several computations have been carried out for the two-layer fluids with the plates in the vertical 2-D. Under some conditions, the oscillation of a flexible structure floating on the sea surface makes the interface resonate. The interaction between thin-plate type structures and long surface/internal waves has been calculated when an initial profile is given on the sea surface or the platforms oscillating on/below the sea surface.
Figure 5: Interface elevation. The initial profile of the plate on the sea surface is given ($L/h = 6$, $a_0/h = 0.2$, $\rho_2/\rho_1 = 1.025$, $x/L = 0.5$).

Figure 8: Displacement of the 2-plate. The initial profile of the 1-plate is given ($L/h = 10$, $a_0/h = 0.2$, $\rho_2/\rho_1 = 1.025$, $x/L = 0.5$).

Figure 6: Interface elevation. The initial profile of the plate on the sea surface is given ($L/h = 10$, $a_0/h = 0.2$, $\rho_2/\rho_1 = 1.025$, $x/L = 0.5$).

Figure 9: Displacement of the 2-plate. The initial profile of the sea surface is given ($L/h = 10$, $a_0/h = 0.2$, $\rho_2/\rho_1 = 1.025$, $x/L = 0.5$).

Figure 7: Interface elevation. The initial profile of the plate on the sea surface is given ($L/h = 14$, $a_0/h = 0.2$, $\rho_2/\rho_1 = 1.025$, $x/L = 0.5$).

Figure 10: Surface waves generated by the plate oscillating in the sea. The initial profile of the plate is given ($h_1/h = 0.4$, $L/h = 4.5$, $a_0/h = 0.5$, $B_2/(\rho_1 g L^4) = 0.1$, $h = (h_1 + h_2)/2$).
A large-scale floating platform is supposed to influence the fluid-density structure outside the covered part of the sea. In an assessment of the environment, attention should be paid to the existence of interfaces. The oscillation of floating platforms, moreover, has the possibility to interact with the sea bottom if the seabed surface is composed of mud. In consideration of the influence by higher order terms that will play an important role when structure displacements are large or fluid layers are not in very shallow waters, further work is required.

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References


