Axial leakage flow-induced vibration of a thin cylindrical shell

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Abstract

The stability of a thin cylindrical shell subjected to axial leakage flow is presented. The analytical model of a cylindrical shell is composed of the inner thin shell and the outer rigid one. We derived the added mass, added damping and added stiffness due to the unsteady fluid forces acting on the thin cylindrical shell subjected to axial leakage flow. The coupled equations between a shell and a fluid in narrow clearance are obtained by using the Donnell's shell theory and the Navier-Stokes equation. As the vibrational characteristics of a thin cylindrical shell, the axisymmetric (ring-mode) vibration, the lateral (beam-like) vibration and the circumferential (ovaling) vibration are classified. The effect of the axial leakage flow on the unstable phenomena of a thin cylindrical shell focusing on these vibrational characteristics is clarified by using the root locus (Argand diagram) due to the complex eigenvalue analysis. Then, we investigate physically on the divergence phenomenon and flutter one of a thin cylindrical shell subjected the axial leakage flow. Comparing the uncoupled vibration modes, that are dry modes, the coupled vibration modes between a fluid and a structure, that are wet modes, is clarified to be considerably changed before and after the unstable phenomenon of the coupled vibration. And, the numerical parameter studies are performed on the shell with the both simply supported ends taking the dimensions of the shell and fluid in narrow clearance. The effects of each dimension are made to be clear for the stabilization of flow-induced vibration.
1 Introduction

A fluid is often conducted as the energy transfer medium in many industrial machinery. On the other hand, many structures must be constructed and operated in flowing fluid.

The velocity of fluid becomes faster in order to increase the efficiency and make the size more compact. In this paper, the fluid-elastic instability which occur in a coaxial cylindrical shell structures subjected to an axial leakage flow is investigated. These structures are recognized in the combustors of gas turbines, the core internals of nuclear power plants, the annular structures in large diameter pipings and various large scale seals and values.

M. P. Paidoussis et al. [1] studied the dynamics and stability of coaxial cylindrical shells containing flowing fluid based on the Flugge's equation. However, the fluid forces are calculated by velocity potential theory in these reports, the influence of viscosity is not sufficiently considered. One of authors [2, 3] studied the instability phenomena of circular cylindrical shell due to annular leakage flow experimentally and analytically. However, the studies in this field can be said to be not necessarily sufficient.

So in this paper, the coupling vibration analysis is proposed, concerning axisymmetric, lateral and ovaling vibration of thin cylindrical shell subjected to axial leakage flow. The stability analysis, whether the thin cylindrical shell shows the unstable vibration due to velocity of the axial leakage flow, is also proposed. Finally, regarding dimensions of the thin cylindrical shell, velocity of the axial leakage flow as the parameters, the physical effect of parameters on the self-excited vibration is investigated.

2 Equation of motion

Let us consider the thin cylindrical shells subjected to the axial leakage flow shown in Fig.1. The inner shell is set as elastic body, on the other hand, the
outer shell is set as rigid one. The dimensions of the inner elastic shell are length: \( \ell \), radius: \( a \), thickness: \( h \), Young's modulus: \( E \), Poisson ratio: \( \nu \), density: \( \rho \). The fluid passes through the gap between two shells. The inner elastic shell vibrates due to the fluid force of the axial leakage flow. In this paper, the annular gap is considered to be sufficiently small compared with the radius of shell. On the other hand, even if the inner shell is rigid body and the outer shell is elastic one, our theory can be similarly analyzed by changing the acting direction of the fluid pressure.

The equations of motion of an incompressible fluid based on Navier-Stokes equations are given by equations of continuity and momentum:

\[
\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z_f} = 0
\]

\[
\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} + W \frac{\partial U}{\partial z_f} = -\frac{1}{\rho_f} \frac{\partial P}{\partial x} + \nu_f \frac{\partial^2 U}{\partial z_f^2}
\]

\[
\frac{\partial V}{\partial t} + U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} + W \frac{\partial V}{\partial z_f} = -\frac{1}{\rho_f} \frac{\partial P}{\partial y} + \nu_f \frac{\partial^2 V}{\partial z_f^2},
\]

where \( U, V, W \): flow velocity of the \( x-, \theta-, z_f- \) directions, respectively \((y = a\theta)\), \( P \): pressure, \( \rho_f \): density of fluid and \( \nu_f \): kinematic viscosity of the fluid as shown Fig.1.

The equations of a thin cylindrical shell are given by Donnell's equations:

\[
\frac{\partial^2 u}{\partial t^2} - C \left\{ \frac{\partial^2 u}{\partial x^2} + \frac{1 - \nu}{2a^2} \frac{\partial^2 u}{\partial \theta^2} + \frac{1 + \nu}{2a} \frac{\partial^2 u}{\partial \theta \partial x} + \nu \frac{\partial w}{\partial x} \right\} = p_x
\]

\[
\frac{\partial^2 v}{\partial t^2} - C \left\{ \frac{1}{a^2} \frac{\partial^2 v}{\partial \theta^2} + \frac{1 - \nu}{2} \frac{\partial^2 v}{\partial x^2} + \frac{1 + \nu}{2a} \frac{\partial^2 v}{\partial \theta \partial x} + \frac{1}{a^2} \frac{\partial w}{\partial \theta} \right\} = p_\theta
\]

\[
\frac{\partial^2 w}{\partial t^2} + D \left\{ \frac{\partial^4 w}{\partial x^4} + \frac{2}{a^2} \frac{\partial^4 w}{\partial \theta^2 \partial x^2} + \frac{1}{a^4} \frac{\partial^4 w}{\partial \theta^4} \right\}
\]

\[
+ C \frac{1}{a^2} \left\{ \frac{\partial v}{\partial \theta} + \frac{w}{a} + \nu \frac{\partial u}{\partial x} \right\} = p_x + \frac{\partial m_x}{\partial x} + \frac{1}{a} \frac{\partial m_\theta}{\partial \theta},
\]

where \( \mu = \rho h \), and \( p_x, p_\theta, p_z \): external fluid forces acting on a shell, \( m_x, m_\theta \): moments due to fluid pressure acting on a shell, and the extensional rigidity and flexural rigidity \( C, D \) are \( C = Eh/(1-\nu^2), D = Eh^3/12(1-\nu^2) \).

### 3 Flow rate

Now, introducing the axial and circumferential flow rate \( Q_x = \int_0^H U dz_f, Q_y = \int_0^H V dz_f \), where \( H \) is the gap, and integrating the equations (1)-(3), the flow velocity \( W \) disappears by using boundary conditions between shell and fluid and the following equations which describe the relation among the flow rate, pressure and gap are obtained:

\[
\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + \frac{\partial H}{\partial t} = 0
\]
Fluid Structure Interaction

\[
\frac{1}{\rho} \frac{\partial P}{\partial x} = -\frac{1}{H} \left\{ \frac{\partial Q_x}{\partial t} + \frac{\partial}{\partial x} \left( \frac{Q_x^2}{H} \right) + \frac{\partial}{\partial y} \left( \frac{Q_x Q_y}{H} \right) + \frac{12\nu Q_x}{H^2} \right\} \tag{8}
\]

\[
\frac{1}{\rho} \frac{\partial P}{\partial y} = -\frac{1}{H} \left\{ \frac{\partial Q_y}{\partial t} + \frac{\partial}{\partial x} \left( \frac{Q_x Q_y}{H} \right) + \frac{\partial}{\partial y} \left( \frac{Q_y^2}{H} \right) + \frac{12\nu Q_y}{H^2} \right\}, \tag{9}
\]

The boundary conditions of the pressure at the inlet and outlet are

\[
P(0, y, t) = P_{in} - (1 + \zeta_{in}) \frac{\rho Q_x^2(0, y, t)}{2H^2(0, y, t)} \tag{10}
\]

\[
P(\ell, y, t) = P_{out} + \zeta_{out} \frac{\rho Q_x^2(\ell, y, t)}{2H^2(\ell, y, t)}, \tag{11}
\]

where \(P_{in}\) and \(P_{out}\) are the pressure just in front of the inlet and just behind the outlet in axial leakage flow, and \(\zeta_{in}, \zeta_{out}\) are pressure loss coefficients at the inlet and outlet.

4 Unsteady pressure

The gap, pressure and flow rate can be divided into steady and unsteady parts as \(H = \bar{H} + \Delta H(x, y, t)\), \(P = \bar{P}(x) + \Delta P(x, y, t)\), \(Q_x = \bar{Q}_x + \Delta Q_x(x, y, t)\), \(Q_y = \Delta Q_y(x, y, t)\), where the bar and \(\Delta\) imply the steady and unsteady values. The steady circumferential flow rate is neglected because it is regarded smaller than the axial flow rate. Moreover, \(\bar{H}\) and \(\bar{Q}_y\) are constant. By applying the perturbation method and neglecting higher order terms of the unsteady components, the equations (7)-(11) can be linearized. Eliminating the flow rates from these equations, we derive the relation between the unsteady pressure and unsteady gap

\[
\frac{1}{\rho_f} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \Delta P = \frac{1}{\bar{H}} \frac{\partial^2 \Delta H}{\partial t^2} + \frac{2\bar{Q}_x}{\bar{H}^2} \frac{\partial^2 \Delta H}{\partial x \partial t} + \frac{12\nu_f}{\bar{H}^3} \frac{\partial \Delta H}{\partial t} + \frac{\bar{Q}_y^2}{\bar{H}^3} \frac{\partial^2 \Delta H}{\partial x^2} + \frac{36\nu_f \bar{Q}_x}{\bar{H}^4} \frac{\partial \Delta H}{\partial x}. \tag{12}
\]

5 Vibrational displacements for the coupled vibration between a shell and an axial leakage fluid

The vibrational characteristics of a thin cylindrical shell can be classified roughly into three types. The first is an axisymmetric vibration called ring-mode, which is the vibration due to elongation and shrinkage in circumferential direction depending on the membrane stiffness of a cylindrical shell mainly. The second is lateral vibration, which is also called beam-like vibration. This is the bending vibration in the lateral direction of a cylindrical shell, similar to beam vibration. The third is vibration in circumferential direction or ovaling vibration, which is mainly due to bending in circumferential direction. By this vibration, the cross area of the cylindrical shell
becomes non-circle. In this paper, classifying vibrations into these three phenomena, the fluid coupling and the unstable behavior are considered.

The displacements of a shell subjected to the axial leakage flow are assumed to be approximated by superposing the eigenmode functions in a vacuum (dry modes) in this paper. Employing the eigenmodes in axial, circumferential and radial directions, \( \hat{u}_m(x) \), \( \hat{v}_m(x) \) and \( \hat{w}_m(x) \), respectively, the fluid-coupled displacements \( u(x, \theta, t) \), \( v(x, \theta, t) \) and \( w(x, \theta, t) \) are expanded as follows:

\[
\begin{pmatrix}
  u(x, \theta, t) \\
v(x, \theta, t) \\
w(x, \theta, t)
\end{pmatrix} = \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \begin{pmatrix}
  \hat{u}_m(x) \cos n\theta \\
  \hat{v}_m(x) \sin n\theta \\
  \hat{w}_m(x) \cos n\theta
\end{pmatrix} \xi_{mn}(t),
\]

where \( \xi_{mn}(t) \) is the function of time. In this paper, the numerical calculation is done under both end conditions which are simply supported in radial and circumferential directions, not supported in axial direction. The eigenmode functions in a vacuum are as follows: \( \hat{u}_m(x) = C_{1mn} \cos(m\pi x/\ell) \cos n\theta \), \( \hat{v}_m(x) = C_{2mn} \sin(m\pi x/\ell) \sin n\theta \), \( \hat{w}_m(x) = C_{3mn} \sin(m\pi x/\ell) \cos n\theta \). Here, these equations show the mode functions of the axisymmetric, lateral and circumferential vibration for \( n = 0 \), \( n = 1 \), \( n \geq 2 \), respectively.

6 Derivation of the fluid force

Here, in this section, the unsteady pressure \( \Delta P \) is derived and fluid force and moments are calculated more specifically. Since the unsteady gap width \( \Delta H \) coincides with the radial displacement of shell, the following relation can be obtained: \( \Delta H = w \). Using this relation, eq.(12) is rewritten as

\[
\frac{1}{\rho_f} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \Delta P(x, \theta, t) = \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \cos n\theta \left\{ A_m(x) \hat{\xi}_{mn}(t) \right. \\
+ B_m(x) \tilde{\xi}_{mn}(t) + C_m(x) \xi_{mn}(t) \left\} ,
\]

where the coefficients \( A_m(x), B_m(x), C_m(x) \) are the functions given by the modal function \( \hat{w}_m(x) \) and its derivatives. Since \( y \) is described as \( y = a\theta \), the left hand side of eq.(14) is the function of \( x, \theta \) and \( t \). Similarly to eq.(13), the unsteady pressure \( \Delta P \) is assumed to be written as \( \Delta P(x, \theta, t) = \Delta P_0(x, t) \cos n\theta \), where \( \Delta P_0 \) means the unsteady pressure at \( \theta = 0 \). Substituting this into eq.(14) and noting that \( \partial^2 \Delta P/\partial y^2 = -(n^2/a^2)\Delta P \), the ordinary differential equation is obtained. Solving the obtained ordinary differential equation under the linearized boundary conditions, the relation between the unsteady pressure \( \Delta P(x, \theta, t) \) and the modal displacement is obtained:

\[
\Delta P(x, \theta, t) = \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \cos n\theta \left\{ \bar{m}_{mn}(x) \hat{\xi}_{mn}(t) + \tilde{c}_{mn} \xi_{mn}(t) + \bar{k}_{mn} \xi_{mn}(t) \right\} .
\]

The functions \( \bar{m}_{mn}(x), \tilde{c}_{mn}(x) \) and \( \bar{k}_{mn}(x) \) are coefficient functions. Their concrete forms are omitted.
7 Derivation of fluid-shell coupled equation

The fluid forces acting on a shell are given by $p_x = -\mu \partial U/\partial z_f|_{z_f=0}$, $m_x = \mu \partial U/\partial z_f|_{z_f=0}(h/2)$, $p_\theta = -\mu \partial V/\partial z_f|_{z_f=0}$, $m_\theta = \mu \partial V/\partial z_f|_{z_f=0}(h/2)$, $p_z = \Delta P(x, \theta, t)|_{z_f=0} = \Delta P(x, \theta, t)$. Assuming the velocity distribution as $\Delta U = (1/2\mu) \partial \Delta P/\partial z_f(z_f - \bar{H})$, $\Delta V = (1/2\mu) \partial \Delta P/\partial \theta_f(z_f - \bar{H})$, fluid forces become $p_x = -(\bar{H}/2) \partial \Delta P/\partial x$, $p_\theta = (\bar{H}/2) \partial \Delta P/\partial \theta$, $p_z = \Delta P(x, \theta, t)$, $m_x = (h\bar{H}/4) \partial \Delta P/\partial x$, $m_\theta = (h\bar{H}/4) \partial \Delta P/\partial \theta$.

Substituting these and the equations of eigenmodes in a vacuum into the equations of a thin cylindrical shell (4)-(6) and utilizing the orthogonality of the mode of the axial and circumferential directions, the following fluid-shell coupled equation of motion is obtained, that is, multiplying eqs. (4)-(6) by the $p$-th eigenfunction in axial direction and the $q$-th eigenfunction in circumferential direction and integrating the resultant equation along the axial direction ($0$ to $\ell$) and circumferential direction ($0$ to $2\pi$); where $\omega_{pq}$ is the eigenfrequency of the shell in a vacuum. The left hand side of above equation is described by only the shell, so it can be understood that the left hand side is orthogonalized with $p$ and $q$ mode. On the other hand, the fluid forces in the right hand side in eq.(16) can not be orthogonalized. That is, the combination of the function of time with $m = 1, 2, \ldots, \infty$ is left. By this equation, even in the fluid-shell coupled vibration subjected to axial leakage flow, the vibration modes in axial direction and circumferential one can be transformed to the product of the function of each variable when coupled eigenvalue analysis is performed for eq.(16). And the orthogonality condition among the coupled mode can be applicable. Therefore, it is seen that similarly to the modes in vacuum, the axisymmetric, lateral and ovaling vibration are considered separately, even in fluid-coupling. Here the concrete form of the coefficient $\tilde{m}_{pq}$, $\tilde{c}_{pq}$, $\tilde{k}_{pq}$ are omitted. Truncating the number of mode as $M$ and employing vectors and matrix form, changing $q$ to $n$, eq.(16) can be rewritten as follows;

$$
\begin{align*}
\ddot{\xi}_{pq}(t) + \omega_{pq}^2 \xi_{pq}(t) = -\sum_{m=1}^{\infty} \left\{ \tilde{m}_{pmq} \dot{\xi}_{mq}(t) + \tilde{c}_{pmq} \xi_{mq}(t) + \tilde{k}_{pmq} \xi_{mq}(t) \right\},
\end{align*}
$$

where $\omega_{pq}$ is the eigenfrequency of the shell in a vacuum. The left hand side of above equation is described by only the shell, so it can be understood that the left hand side is orthogonalized with $p$ and $q$ mode. On the other hand, the fluid forces in the right hand side in eq.(16) can not be orthogonalized. That is, the combination of the function of time with $m = 1, 2, \ldots, \infty$ is left. By this equation, even in the fluid-shell coupled vibration subjected to axial leakage flow, the vibration modes in axial direction and circumferential one can be transformed to the product of the function of each variable when coupled eigenvalue analysis is performed for eq.(16). And the orthogonality condition among the coupled mode can be applicable. Therefore, it is seen that similarly to the modes in vacuum, the axisymmetric, lateral and ovaling vibration are considered separately, even in fluid-coupling. Here the concrete form of the coefficient $\tilde{m}_{pq}$, $\tilde{c}_{pq}$, $\tilde{k}_{pq}$ are omitted. Truncating the number of mode as $M$ and employing vectors and matrix form, changing $q$ to $n$, eq.(16) can be rewritten as follows;

$$
[M_{s,n} + M_{a,n}][\dot{\xi}(t)] + [C_{a,n}][\ddot{\xi}(t)] + [K_{s,n} + K_{a,n}][\xi(t)] = \{0\},
$$

where $\{\xi(t)\} = \{\xi_1(t), \xi_2(t), \ldots, \xi_N(t)\}^T$, $[M_{s,n}] = I_{M \times N}$, $[K_{s,n}] = \text{diag}\{\omega_1^2, \omega_2^2, \ldots, \omega_M^2\}$, $[M_{a,n}] = [\tilde{m}_{mpn}]_{1 \leq m, p, n \leq M}$, $[C_{a,n}] = [\tilde{c}_{mpn}]_{1 \leq m, p, n \leq M}$, $[K_{a,n}] = [\tilde{k}_{mpn}]_{1 \leq m, p, n \leq M}$. The matrices $[M_{s,n}]$, $[K_{s,n}]$ are the modal mass and stiffness of only the shell, and $[M_{a,n}]$, $[C_{a,n}]$, $[K_{a,n}]$ are the added mass, added damping, added stiffness matrices of fluid in the gap, and $n = 0, n = 1$ and $n \geq 2$ denote the axisymmetric, lateral and circumferential vibrations respectively. By calculating the complex eigenvalue $\lambda$ and plotting the root locus (Argand diagram), stability of the system can be evaluated.

Here, there are $2M$ complex eigenmodes of eq.(17) for fixed $n$. $M$ pairs of eigenvalues exist. Each pair is a set of complex conjugate number or a
pair of two real number. Let us call the pair $1, 2, 3, \cdots$ th eigenvalues in lower order. Let consider only $M$ eigenvalues which are ones with positive imaginary part in $k$th complex conjugates or the eigenvalue with larger one in $k$th real number pair. Thus, the focused $M$ complex eigenvalues are written as $\{\lambda_{1,n}, \lambda_{2,n}, \cdots, \lambda_{M,n}\}$. Let the complex eigenvectors corresponding to $\lambda_{k,n}$ be $\{\Psi_{k,n}\}$, and $m$th element of $\{\Psi_{k,n}\}$ be $\Psi_{k,n,m}$. Moreover, let $\eta_{k,n}(t)$ be the coupled function of time corresponding to $\{\Psi_{k,n}\}$. Therefore, the focused $M$ complex eigenvalues are written as:

$$u_{k,n}^c(x, \theta) = \sum_{m=1}^{M} \tilde{u}_m(x) \cos(n\theta) \Psi_{k,n,m}$$

$$v_{k,n}^c(x, \theta) = \sum_{m=1}^{M} \tilde{v}_m(x) \sin(n\theta) \Psi_{k,n,m}$$

$$w_{k,n}^c(x, \theta) = \sum_{m=1}^{M} \tilde{w}_m(x) \cos(n\theta) \Psi_{k,n,m}$$

(18)

Here, note the coupled complex eigenvalues and eigenmodes vary when the flow velocity vary. By employing these equations, the displacement of the cylindrical shell can be expressed as follows:

$$\begin{pmatrix} u(x, \theta, t) \\ v(x, \theta, t) \\ w(x, \theta, t) \end{pmatrix} = \sum_{k=1}^{M} \sum_{n=1}^{N} \begin{pmatrix} u_{k,n}^c(x, \theta) \\ v_{k,n}^c(x, \theta) \\ w_{k,n}^c(x, \theta) \end{pmatrix} \eta_{k,n}(t)$$

(19)

The imaginary part of complex eigenvalue $s$, $Im(s) \neq 0$ and the real part $Re(s) > 0$ implies dynamically unstable (flutter), and $Im(s) = 0$ and $Re(s) > 0$ implies statically unstable (divergence). According to these procedures, the critical flow velocity and complex eigenmode shape for the leakage-flow induced vibration could be evaluated.

8 Simulation studies

As a numerical example, the cylindrical shell composed of the inner elastic shell and a outer rigid one are considered to be made of steel. Water is used as a fluid in this case. The shell is assumed to be regarded as simply supported at both ends in radial and circumferential directions as explained above. To examine conditions that the vibrations of axisymmetric, lateral and ovaling types become unstable, the dimensions of models are selected as length $\ell = 2[m]$, radius $a = 0.8[m]$, thickness $h = 0.001[m]$ annular gap $H = 0.003[m]$. And, parameter studies are performed based on these dimensions.

8.1 Eigenvalue analysis

Following this proposed method, the coupled eigenmodes subjected to axial leakage flow (wet modes) can be approximated by using the eigenmodes in
a vacuum (dry modes). Here, using the above-mentioned dimensions, the eigenvalue analyses are shown for dry and wet modes (at $\bar{U} = 0$[m/s]) in Fig.2. $m$ and $n$ show vibrational mode numbers in axial and circumferential directions, respectively. In the case of the calculations using these dimensions, the eigenvalue analyses of dry mode show the minimum at the mode of $(m, n) = (1, 9)$. On the other hand, those of wet mode show the minimum at the mode of $(m, n) = (1, 7)$. Comparing Figures 2(a) and 2(b), the eigenvalues of the wet mode are much smaller than those of dry mode because of the added mass effect of the fluid, especially for thin shell. As the result, we take the circumferential modes of $n = 7$ as the representative example for the ovaling type vibration. Thus, the calculation results on the axisymmetric vibration type ($n = 0$), lateral one ($n = 1$) and circumferential one ($n = 7$) are compared each other and considered on physical meanings.

### 8.2 Stability on axisymmetric, lateral and circumferential vibrations

In this subsection, we compare three vibration types, that is the axisymmetric, lateral and circumferential vibrations of a cylindrical shell. The dimensions are the same as above-mentioned in this chapter. Argand diagrams of axisymmetric, lateral and circumferential vibrations are shown in Fig.3. Lower ten modes of uncoupled modes in axial direction are utilized here in the coupled calculation of three types (axisymmetric, lateral and circumferential vibrations) and the coupled vibration modes of $m = 1, 2$ are shown in Fig.3. It can be seen that all three type vibrations in axially first mode go into divergence phenomena increasing fluid velocity. The critical velocities about divergence for axisymmetric, lateral and circumferential vibrations are $\bar{U} = 26, 26, 13$[m/s], respectively. The root loci of 1st
axisymmetric and lateral vibrations are very similar. On the other hand, it can be seen that the critical velocity of circumferential vibration, that is ovaling vibration, is smaller than ones of other two vibrations and ovaling vibration is more unstable than axisymmetric and lateral vibrations. The tendencies of the axisymmetric and lateral vibration are not necessarily the same as in another dimensions. About the second modes, all three vibration types go into flutter phenomena increasing fluid velocity.

8.3 Comparison on beam-like vibrations of shell based on shell theory and modes based on beam theory

Letting the circumferential mode number be $n = 1$, the beam-like (lateral) vibration is obtained. Then, in this subsection, we compare the beam-like vibrations of shell based on shell theory and the vibration modes based on beam theory. The vibration analysis based on beam theory has been already performed in the former paper reported by authors [4]. Ten modes as uncou-
pled modes are utilized here in the coupled calculation of two models (beam and shell) as above-mentioned. The beam-like mode of the cylindrical shell are already has been shown in Figure 3(b). The root locus of the beam of the same dimensions as the above ones are shown in Fig.4(a). For comparison, the root locus of the lateral vibration of shell is also shown in Fig.4(a). Only the first mode in axial direction is shown in this figure. As the fluid velocity become high, 1st mode based on the beam theory and the 1st beam-like mode based on the shell theory approach to unstable area and go into divergence phenomena. Both models show the same tendency to increasing velocity. However, the beam-like mode based on the shell theory become unstable faster than than the vibration mode based on the beam theory. In the above dimensions, the shell effect considered to be larger comparatively because \(a/h=800\). Then, the further comparison between the both root locus of the mode based on the beam theory and beam-like mode based on the shell theory for the calculation model with the dimensions close to the beam (\(a/h=33.3\), \(\ell=1[m]\), \(a=0.1[m]\), \(h=0.003[m]\), \(H=0.002[m]\)) is shown in Fig.4(b) to certify the validity of our proposed method.

8.4 Effect of the gap width on the vibrational behavior

In the following, the effect of gap width on the stability of three types of shell vibration (axisymmetric, lateral and circumferential vibrations) is examined. For cases of \(\tilde{H} = 2 \sim 6 \times 10^{-3}[m]\), the critical velocities which cause the unstable phenomena for axisymmetric, lateral and circumferential vibrations are shown in Fig.5 by using the same dimensions (except the gap width) as shown above. It can be seen that as the gap width becomes larger and the critical velocities for all type vibrations become higher. Since the critical velocity for the circumferential vibration is smaller than ones for axisymmetric and lateral ones, circumferential vibration is more unstable.
than axisymmetric and lateral vibrations for these range of calculation dimensions.

8.5 Effect of the radius on the vibrational behavior

Let us consider the effect of the radius on stability keeping the other dimensions constant. The relations between the radius of a shell and the critical velocity for three vibrations are illustrated in Fig. 6 using the same dimensions (except the radius) as shown above. When the radius become larger, the behaviors of the axisymmetric vibration of the shell becomes more stable. On the other hand, the behaviors of the lateral and circumferential vibration of a shell become more unstable when the radius becomes larger. It can be considered that the coupled natural frequencies of lateral and circumferential vibrations become lower because the effect of the added mass become larger and the rigidity lower relatively when the radius become larger. This figure also implies that the circumferential vibration is more unstable than axisymmetric and lateral ones.
8.6 Effect of the thickness on the vibrational behavior

The relation between the thickness of a shell and the critical velocity are illustrated in Fig.7 for three type vibrations by using the same dimensions (except the thickness) as above ones. The critical velocities are almost constant for axisymmetric vibration because the eigenvalues of shell in a vacuum are same even if the thickness of a shell is changed. That is, the axisymmetric vibration of a shell is affected by only extensional rigidity, not by flexural rigidity. On the contrary, when the thickness becomes larger, the vibrational behaviors of the lateral and circumferential vibration of shell become more stable due to the increase of the bending stiffness of shell.

![Figure 7: Effect of the Thickness on Critical Velocity of 1st Mode (Divergence)](image)

8.7 Complex eigenmode

In this subsection, we focus on the lateral vibration in three ones as an example and consider the complex eigenmode of shell in a fluid here. As the beam-like mode of shell, that is the lateral vibration of shell, is considered, the complex eigenmodes in radial direction are shown in Figs.8 and 9. The shapes of the complex eigenmodes during one period are shown in the two figures. As the complex eigenmode in a fluid is expressed by linear combination of the eigenmodes in a vacuum, the ratio of the eigenmodes in a vacuum (uncoupled modes, that is dry modes) contained in complex ones in a fluid (coupled modes, that is wet modes) are considered. The ratio is calculated based on eqs.(17), (18). The ratios of the eigenmode are generally expressed as complex numbers, however, the absolute values of the complex number ratios are calculated and normalized so that the sum of the ratios of ten modes become 1 to make the ratios more understandably. The resultant ratios of the eigenmodes in a vacuum contained in the 1st and 2nd complex eigenmodes in a fluid are expressed in Table 1. The 1st dry mode is illustrated in Fig.8(a). The first coupled complex mode for axial flow velocity of \( \bar{U} = 0 \) is almost the same as one in a vacuum. Then, it is
omitted, here. The 1st mode shape at $\bar{U} = 25.6$[m/s] (divergence) is a little different from one in a vacuum as shown in Table 1.

Let us consider the 2nd coupled complex eigenmode. The second dry mode is illustrated in Fig.9(a). Similarly to the first mode, since the coupled mode for axial flow velocity of $\bar{U} = 0$ is almost the same as one in a vacuum. The 2nd mode shape at $\bar{U} = 29.5$[m/s] is shown in Fig.9(b) after the flutter phenomenon occur. This shape is different from one in a vacuum from Table 1. As shown in Table 1, the ratios of modes in a vacuum except one of the second mode contained in second complex mode is about 70 %.

Table 1: Ratios of Eigenmodes in a Vaccum in 1st and 2nd Complex Eigenmodes.

<table>
<thead>
<tr>
<th>Eigenmode in a vacuum</th>
<th>1st complex eigenmodes</th>
<th>2nd complex eigenmodes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>axial flow velocity $\bar{U}$</td>
<td>axial flow velocity $\bar{U}$</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>25.0</td>
</tr>
<tr>
<td>1</td>
<td>1.0000</td>
<td>0.8852</td>
</tr>
<tr>
<td>2</td>
<td>0.0000</td>
<td>0.0969</td>
</tr>
<tr>
<td>3</td>
<td>0.0000</td>
<td>0.0070</td>
</tr>
<tr>
<td>4</td>
<td>0.0000</td>
<td>0.0065</td>
</tr>
<tr>
<td>5</td>
<td>0.0000</td>
<td>0.0013</td>
</tr>
</tbody>
</table>
Conclusions

In this paper, coupling vibrational analysis of the cylindrical shell subjected to axial leakage flow is proposed by using the Donnell theory of the shell and axial flow-approximated Navier-Stokes theory. By complex eigenvalue analysis, stability criterion method for a cylindrical shell subjected to axial leakage flow is presented. The numerical simulation being performed, the physical consideration on the dynamic stability is done for axisymmetric, lateral and circumferential, that is, ovaling vibrations of shell with simply supported at both ends in radial direction and free end in axial direction. The comparison between the stability analysis based on the beam theory reported in the former paper and the one on the shell theory for the lateral (beam-like) vibration is performed. The tendency of the stability characteristics between two types shows a good coincidence. Performing the parameter studies by changing the dimensions, the directional countermeasures for vibration proof and design are obtained.

References


