Analysing effect of residual stresses based on crack closure model for part through fatigue crack growth problems

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Abstract

The strip yield crack closure model has been extended for the fatigue crack growth in the residual stress field for the part through cracks. The analysis of residual stress includes not only the initial stress, but also its relaxation due to the crack growth and the fatigue load. The investigation shows that it is possible that the fatigue crack growth in the residual stress field can be reasonably analysed based on the elastic-plastic fracture mechanics method.

1 Introduction

The fatigue crack growth analysis is essential to guarantee a reliable welded joint design during its service life. An investigation of the fatigue crack growth analysis at the welded joint is presented in this paper. The investigation consists of both the residual stress analysis and the fatigue crack growth analysis. The residual stresses are measured using a neutron diffraction non-destructive technique, and the fatigue crack growth analysis is based on a strip yield crack closure analytical model and the weight function method for through as well as part-through crack problems.

2 Basic Solutions

For part-through crack growth problems, Elber’s crack closure relation gives
where \( \rho(\psi) \) is the distance of the crack front to the centre of the ellipse when the crack is approximated as at least one part of it. To defined a stress intensity factor \( K^u[\rho(\psi)] \) for a unit load, \( \Delta K_{eff}[\rho(\psi)] \) can be written as

\[
\Delta K_{eff}[\rho(\psi)] = [\sigma_{max} - \sigma_{op}(\psi)]K^u[\rho(\psi)].
\] (2)

The advantage of the crack closure model is that the plasticity induced crack growth behaviour can be accounted for so long as the solution for \( \sigma_{op}(\psi) \) can be found for the cycle-by-cycle evaluation of crack growth.

According to the strip yield assumption, the stress intensity factor at the boundary of the plastic zone should satisfy a condition of

\[
K_L(a + r_p) + K_{res}(a + r_p) + K_y(a + r_p) = 0
\] (3)

where \( K_L(a + r_p) \) is the stress intensity factor due to applied load, \( K_{res}(a + r_p) \) is the stress intensity factor due to the release of residual stress, and \( K_y(a + r_p) \) is the stress intensity factor due to the strip yield stress in the plastic zone. Eq.(3) can be used to compute the plastic zone size for through as well as part through crack problems.

An effective way to solve stress intensity factors under complex stress field is to use the weight function technique proposed by Bueckner [1] and Rice [2]. In the weight function technique, two systems of stress are considered for the same cracked body so that there is a correlation

\[
\int_{\Delta S_i} \frac{K^{(1)}(r)}{H} d(\Delta S_i) = \int_S \sigma^{(2)}(r) \delta u^{(1)}(r) dS = \int_S \sigma^{(1)}(r) \delta u^{(2)}(r) dS
\] (4)

In this relation, \( \Delta S_i \) is an arbitrary virtual crack front increment, \( \delta u^{(1)}(r) \) is the virtual displacement due to load system (1), and \( \delta u^{(2)}(r) \) is the virtual displacement due to load system (2). \( \sigma^{(1)}(r) \) is the stress for load system (1) and \( \sigma^{(2)}(r) \) is the stress for load system (2). \( H \) is a generalised elastic modulus. \( K^{(1)}(r) \) is the stress intensity factor for load system (1) and \( K^{(2)}(r) \) is the stress intensity factor for load system (2).

Based on Cruse and Besuner’s [3] approximation, a general approximate solution can be achieved as
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\[ K_{i}^{(2)} \approx \int_{S} \sigma^{(2)} \frac{H}{K_{i}^{(1)}} \frac{\partial u^{(1)}}{\partial (\Delta S_{i})} dS \]  \hspace{1cm} (5)

for one as well as two dimensional crack problems. In this relation, a root mean square stress intensity factor is defined

\[ K_{i} = \sqrt{\frac{H}{\Delta S_{i}} \int_{\Delta S_{i}} \frac{K^{2}}{H} d(\Delta S_{i})} \]  \hspace{1cm} (6)

which is the exact solution for one dimensional crack problems.

According to eq.(5), a load independent function can be defined as

\[ m_{i} = \frac{H}{K_{i}^{(1)}} \frac{\partial u^{(1)}}{\partial (\Delta S_{i})} \]  \hspace{1cm} (7)

so that stress intensity factor for any virtual crack increment under arbitrary stress \( \sigma^{(2)} \) can be explicitly solved in an integral of

\[ K_{i}^{(2)} \approx \int_{S} \sigma^{(2)} m_{i} dS. \]  \hspace{1cm} (8)

A reference stress intensity factor for a given geometry should be known at first. The corresponding crack surface displacement can be solved according to the approximate procedure proposed by Wang [4]. The weight function, eq.(7), can be determined. Then, the stress intensity factor for arbitrary crack surface pressure can be solved according to the integration of eq.(8). The superposition principle of linear elasticity can be used to computed the stress intensity for either residual stresses or applied stresses.

A specific virtual crack increment can be considered for \( \Delta a / \Delta c = a / c \),

\[ m(\rho) = \frac{H}{K^{(1)}} \frac{r}{\rho} \frac{\partial u^{(1)}}{\partial \rho}. \]  \hspace{1cm} (9)

This weight function can be used to compute the stress intensity factor due to the strip yielding,

\[ K_{i}(\rho) = \int_{0}^{\rho} \sigma_{i}(\rho) m(\rho, \rho) d\rho. \]  \hspace{1cm} (10)

According to the strip yield model [5], the plastic deformation ahead of the crack front as well as on the crack surface is divided into a system of strip rings with the stress on each ring approximated as a constant of,

\[ p_{i} = \frac{1}{r_{i+1} - r_{i}} \int_{r_{i}}^{r_{i+1}} \sigma_{i}(r) dr. \]  \hspace{1cm} (11)
The crack surface displacement for the stress on each ring can then be computed by

\[ p_{sg}(r, r, \rho) = \frac{1}{H} \int_{r}^{\rho} K_{p_{s}}(\rho) \frac{\rho}{r} \overline{m}(r, \rho) d\rho. \] (12)

In the same principle, the crack surface displacement due to the residual stress can be computed as

\[ u_{res}(x, a + r_{p}) = \frac{1}{H} \int_{r}^{\rho} K_{res}(\rho) \frac{\rho}{r} \overline{m}(r, \rho) d\rho \] (13)

and the crack surface displacement due to remote load can be computed as

\[ \sigma_{f}(x, a + r_{p}) = \frac{1}{H} \int_{r}^{\rho} K_{L}(\rho) \frac{\rho}{r} \overline{m}(r, \rho) d\rho. \] (14)

![Mode I stress on surface](image1.png) ![Through-thickness Mode I stress](image2.png)

Figure 1: Linear stress distribution on the specimen surface and through the thickness at the specimens.

The deformation on all the ring elements must satisfy a displacement compliance requirement of

\[ \delta_{p}(x, a + r_{p}) = \sigma_{f}(x, a + r_{p}) + u_{res}(x, a + r_{p}) - \sum_{i=1}^{n} p_{i} g(x_{i}, x, a + r_{p}) \] (15)

The plastic yielding condition for each ring element can be imposed as

\[ -\beta \sigma_{0} \leq p_{i} \leq \alpha \sigma_{0} \] (16)

for the ring element in the plastic zone and

\[ -\beta \sigma_{0} \leq p_{i} \leq 0 \] (17)
for the ring element on the crack surface. Here, $\alpha$ is a coefficient representing the three dimensional constraint ahead of crack front, and $\beta$ is a coefficient representing the Baushinger effect.

When this system of equations is solved, the plastic deformation ahead of the crack front as well as on the crack surface can be approximately solved. The crack opening stress can then be determined using a remote stress level at which the residual stress on the crack front ring element is equal to zero [6].

3 Stress and Stress Intensity Factors

Two types of specimens are used in the investigation for a cruciform non-load carrying welded joint specimen with and without TIG dressing at the root of the weldment. Finite element stress analyses were performed for each case. The computed stress results are shown in Fig.1. There is a rather high stress concentration for the as-welded specimen (AW). The stress concentration factor is as high as 6.5 on the surface. The stress concentration covers a large surface area while it reduces rapidly in depth.

The stress concentration for the TIG dressed specimen is low, about 1.7. Its change on the surface as well as in depth is gradual. The finite element stresses are used to compute the stress intensity factor according to the weight function relation of eq.(5). The computed stress intensity factor will then be used in the strip yield model to evaluate the fatigue crack growth process.
4 Residual Stresses

In the welded joint, high tensile residual stress may exist at the joint. With the help of the neutron non-destructive diffraction measurement [7], we are able to measure the residual stress as well as its relaxation in the depth by tracing the same specimen before and after fatigue loading.

Fig. 2 shows the residual stress result for the as-welded specimen. A high tensile residual stress close to the yield stress is presented on the surface of the specimen. After the specimen was subjected to a half million load cycles of a load spectrum, the residual stress in the same location was measured again. The measured results show insignificant stress relaxation in the depth. A stress relaxation analysis has also been made according to the peak stress level, stress concentration, and the material constitutive equation. The analytical results also show that the residual stress relaxation in the depth of plate is minimal while most of the stress relaxation occurs on the surface, see Fig. 2.

![Residual stress relaxation for TIG dressed specimen](image)

**Figure 3**: Comparison of residual stresses before and after fatigue loading for the TIG-dressed specimen.

For the TIG-dressed specimen, the measured residual stresses through the thickness are shown in Fig.3. The TIG-dressing reduces significantly the residual stress on the surface. The maximum residual stress for TIG-dressed specimen occurs in the depth of plate, about 3 mm from the surface. After a half million of load cycles under a spectrum loading, the specimen is again measured for the relaxation of the residual stresses. The result of stress relaxation is shown in Fig.3. There is a noticeable stress relaxation at the highest residual stress location while the residual stress on the surface is basically unchanged. The analytical results are close to the measurement with a lower value. In the analytical method, no
three dimensional stress constraint is involved so that the final results are lower than the measured ones.

The residual stresses such as those given in Fig.2 and Fig.3 are used to compute the stress intensity factors according to the weight function solution of eq.(8) so that their effect on the residual strength as well as the fatigue life can be evaluated.

Figure 4: Comparison of experimental and analytical fatigue lives for as-welded specimens under constant amplitude loading.

Figure 5: Comparison of experimental and analytical fatigue lives for TIG-dressed specimens under constant amplitude loading.

5 Results and Discussions

The effect of residual stress on the fatigue life is mainly due to its effect on the stress ratio. A tensile type of residual stress at the crack location
will increase the stress ratio while a compressive type of residual stress will reduce the stress ratio at the fatigue crack location. The stress ratio effect on the fatigue life is, however, non-linear. The crack closure model based on the plastic deformation is in general capable of dealing with such an effect.

![Graph showing comparison between analytical and experimental fatigue lives for as-welded DOMEX 590 specimens under SP2 spectrum loading.](image)

**Figure 6:** Comparison of experimental and analytical fatigue lives for as-welded specimens under SP2 spectrum loading.

Three types of load are considered; a constant amplitude loading with a stress ratio of $R = 0$, a spectrum (SP2) with a gross stress ratio of $R = 0$, and a symmetrical spectrum (SP3). The mechanical properties of the material for the specimens, DOMEX 590, are given in [7]. Based on the statistical inspections, an average initial crack size of $a_0 = 0.1$ mm with an aspect ratio of $a/c = 1$ is used in all the analytical evaluations. Three analyses have been performed; one without the residual stress, one with the initial residual stress, and one with the stress relaxation.

Fig. 4 shows the comparison between analytical results and experimental results for the as-welded specimens under the constant amplitude loading. When the residual stress is omitted, the analytical fatigue life are more than four times longer than the experimental fatigue life, indicating the necessity to consider the effect of residual stress under this loading and specimen combination.

There is no substantial different between the analytical fatigue lives when either the initial residual stress or the stress relaxation is considered. This phenomenon may be explained according to the stress concentration shown in Fig.1 and the residual stress distribution shown in Fig.2. The stress near the surface is high. As a result, the part of fatigue life for the
crack size less than 1 mm is small. In addition, the crack closure is generally low for small crack sizes so that the effect of residual stress is also minimal. These factors lead to an insignificant difference in the analytical fatigue lives whether or not the residual stress relaxation is considered.

For TIG-dressed specimens, the comparison of experimental and analytical fatigue lives is shown in Fig. 5. As for the as-welded specimens, the analytical fatigue life is longer when the residual stress is omitted especially for the low stress level, about five times longer at 100 MPa. The discrepancy reduces to about two times for high stress levels.

![Figure 7: Comparison of experimental and analytical fatigue lives for TIG-dressed specimens under SP2 spectrum loading.](image)

The relaxation of residual stress seems to have negligible effect on the analytical fatigue lives. The reason for this case is, however, different from those of as-welded specimens. For TIG-dressed specimens, the main difference in the residual stress before and after relaxation is within the depth of the specimen. The fatigue life for the TIG-dressed specimen is longer than that of the as-welded specimen. The crack growth in the small size range contributes to a large part of the fatigue life. In the small crack range, the change of residual stress after relaxation is minimal so that its effect on the fatigue life is insignificant. This is a different mechanism which contributes to the same phenomenon.

For SP2 spectrum loading, Fig. 6 shows the comparison of experimental and analytical fatigue lives. There is no substantial difference between the analytical results whether or not the residual stress is considered. This
is quite different from what we observed in the fatigue lives for the constant amplitude loading even though the gross stress ratio is the same.

During the process of crack growth, the build-up of the crack closure is determined mainly by the peak load in the spectrum. Even though the number of peak load in the spectrum is small, their contribution to the crack closure may have a sustained effect on the crack closure. In the case of SP2 spectrum, the crack opening level during the most part of the crack growth process is around a load factor of 0.3 when the residual stress is not considered. Therefore, most of the load cycles are above the crack closure level so that their full range will contribute to the crack growth. When the tensile residual stress is introduced, their effect is to reduce the crack opening level. Since not many load cycles are covered by reducing the crack opening level for SP2 spectrum, the residual stress will have no substantial effect on the total fatigue life. This seems to be the major reason why the residual stress doesn’t contribute very much to reduce the fatigue life under SP2 spectrum.

For TIG-dressed specimen under SP2 spectrum loading, the comparison between analytical and experimental fatigue lives are shown in Fig.7. The same as for the as-welded specimen, the residual stress seems to have no effect on the analytical fatigue life even though the fatigue live for the TIG-dressed specimens are longer than that of the as-welded specimens.

![Figure 8: Comparison of experimental and analytical fatigue lives for as-welded specimens under SP3 spectrum loading.](image)

For the symmetrical spectrum loading SP3, Fig.8 shows a comparison between the experimental and analytical fatigue lives for the as-welded specimens. Very different from those of the SP2 spectrum loading, the analytical fatigue lives without residual stress are significantly longer than those when residual stress is considered. The analytical fatigue lives
are in general more than an order of magnitude longer when the effect of residual stress is omitted. There is a noticeable difference between the analytical fatigue lives when the relaxation of residual stress is considered. The initial residual stress leads to a shorter fatigue life. As expected, the analytical fatigue lives agree better with the experimental results when the residual stress relaxation is considered. For this specific load spectrum and specimen, neither residual stress nor its relaxation should be omitted in the fatigue life analyses in order to get acceptable estimations.

For TIG-dressed specimens, the comparison are shown in Fig. 9 between the experimental and analytical fatigue lives. In this case, the fatigue lives are in general longer than those of as-welded specimens. The discrepancy up to two order of magnitude can be found when the effect of residual stress is omitted especially for low load region. In this case, the stress relaxation doesn’t affect the fatigue life due to the same reason illustrated for the results shown in Fig. 5.

![Figure 9: Comparison of experimental and analytical fatigue lives for TIG-dressed specimens under SP3 spectrum loading.](image)

6 Conclusions

The proposed analytical procedure is capable of evaluating the crack growth and fatigue life for the welded joints under general loading cases when the residual stress is involved. The investigation indicates that the stress as well as residual stress and its relaxation at the welded joints should be carefully considered in order to make realistic evaluation of the crack growth. In general, no omission of residual stress and its relaxation after fatigue loading is allowed in the analysis of fatigue life except for
some extreme cases. Otherwise, serious non-conservative evaluation of fatigue life may be made.

References