Plastic flow percolation across a microstructure with pores

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Abstract

A method to estimate the yield surface of porous metals, by identifying evolution of local yield connectivity and the onset of local yield percolation across the microstructure from finite element analyses, is investigated. The finite element analyses are used to model the types of pore morphologies that have been observed in porous metals. Linear superposition of basis deformation cases is used to generate arbitrary applied elastic biaxial deformation states used to estimate the yield surface, in place of more costly and time consuming nonlinear analyses.

Introduction

Macroscopic plastic flow is a well known and important characteristic of a metal alloy. It is commonly described by its yield surface, representing the onset of nonlinear deformation over possible multiaxial deformation states. The prediction of the yield surface from a knowledge of microstructural features is not a trivial matter, particularly when the material is a closed cell porous metal. The yield surface can be determined experimentally, but in view of the effort involved to both produce and test the materials, some knowledge of the yield surface is desirable before committing significant effort. As material processing methods for producing porous metals continue to improve (e.g. foamed metallurgy, entrapped gas powdered metallurgy and gasar casting methods), and new applications are found for these materials, the ability to translate
knowledge of the pore morphology and matrix alloy to the yield surface of the porous metal becomes more important.

The analysis of complex porous microstructures by finite element methods has progressed to the point where mesoscale models contain many features in a representative volume element (RVE) of a complex porous microstructure. Simulations of deformation and yield for a single loading state are both possible in two and three dimensions. The combination of novel pore morphology and yield strength has been investigated using a two dimensional image based model of gasar copper pore microstructure (Schapovalov[1], Kee & Matic[2]) for classical one dimensional and equal biaxial tension/compression combinations (Kee, Matic & Everett[3]). This has provided insight into the yield behavior for specific loading cases, but is not sufficient to construct the entire yield surface.

The yield behavior associated with a deformation applied to the porous microstructure can, in principle, be determined and related to evolution of local yield connectivity percolation of local yielding across the microstructure (Fig. 1). The time and cost of constructing a yield surface by a series of individual simulations, each imposing different multiaxial deformation states on the RVE model, remains prohibitive.

![Figure 1: Schematic of (a) microstructure, (b) microyield, (c) plastic percolation across x₁ and (d) plastic percolation across x₂. Plasticity is represented by the shaded areas.](image)

One approach to gain insight about the yield surface, without conducting a full set of physical experiments or a full set of nonlinear computational analyses, is to estimate when percolation of local matrix yielding occurs across the microstructure from linear superposition of fundamental elastic loading cases. A limited number of fully nonlinear
analyses could be conducted to benchmark, or calibrate, the estimated yield surface as necessary based on the features of the estimated yield surface.

This investigation explores a method of yield surface estimation involving six steps: (i) generation of the two dimensional finite element model, (ii) linear elastic analyses of the model for basis deformation cases, (iii) post processing of the basis deformation results to create elastic stress state databases, (iv) linear combination of the basis deformation stress state databases for a full set of biaxial deformation combinations, (v) determination of local yield percolation threshold for each biaxial deformation, and (vi) construction of the estimated yield surface. Aligned elongated pores with different pore aspect ratios, of the type physically observed in gasar castings (Kee & Matic[4]), are the focus of this study for their known effects on narrowing the strain yield surface in the direction parallel to the long axes of the pores.

**Finite element simulation**

ABAQUS/Explicit (Version 5.6) (HKS[5]) finite element models were generated from a 50 x 50 element RVE grid. Pores were created using model preprocessor programs by removing elements from the grid according to statistical models of the pore morphology. These programs generated the aligned pore patterns based on the required mean pore aspect ratio and a global pore volume fraction of 15 percent. Three different models with mean pore aspect ratios $r$ of 1.0, 2.0 and 3.0 were created. Type CPS4R four noded constant strain elements were used in the model. The models contain 2125 elements and 2601 nodes. A representative portion of each model (20 elements x 20 elements) which originated at the bottom left corner of the full model is shown in Fig. 2 and was used to create result databases.

![Three images of microstructural models with porosity and pore aspect ratios](image)

Figure 2: Microstructural models with $p = 0.15$, (a) Model 1 with $r = 1.0$, (b) Model 2 with $r = 2.0$, (c) Model 3 with $r = 3.0$. 
The matrix A356 aluminum alloy was modeled as an isotropic elastic-plastic material. The elastic modulus was 72,400.0 MPa (10.5 x 10^6 psi) and Poisson ratio was 0.33. The plastic behavior of the alloy followed a von Mises yield behavior. Uniaxial yield stress was 147.5 MPa (21,400.0 psi) and uniaxial yield strain was 0.0021. Strain hardening behavior is given by Fig. 3. All models were run in plane stress.

Figure 3: Al 356 true stress versus true strain curve

Elastic analyses were performed for two basis deformation cases. The two basis deformation cases, representing orthogonal applied deformations, were constructed by independently applying \((u_1, u_2)\) deformations to the \((x_1 \text{ and } x_2)\) boundaries, respectively (Fig. 4). The local stress component (i.e. \(\sigma_{11}, \sigma_{12}, \sigma_{22}, \text{ and } \sigma_{33}\)) databases were saved for each loading case for the representative section of the model as seen in Fig 2. This information was used to generate the estimated yield surface. The post processing was done with ABAQUS/Post 5.6 (HKS[6]) to generate the elastic local stress component databases, the load vs displacement data and the yield point data.

The basis load databases can be linearly superimposed to create combined loading states using the scalar multiplier \(\lambda\) to scale the basis stresses \((\sigma_i)_1\) and \((\sigma_i)_2\), using eqn. (1) to create arbitrary biaxial loading paths.
where \( \sigma_{ij} = \lambda_1 (\hat{\sigma}_{ij})_1 + \lambda_2 (\hat{\sigma}_{ij})_2 \) (1) and \( \lambda^2 = \lambda_1^2 + \lambda_2^2 \) (2) defines the radial direction in strain space.

A limited number of fully nonlinear analyses were conducted to establish yield response benchmarks. The applied deformations are referred to by an \((u_1, u_2)\) ordered pair. A "+" sign refers to applied tensile deformation and "-" sign refers to applied compressive deformation. Applied deformations of \((+,0)\), \((0,+), (+,+), \) and \((-,+)\) were applied to each of the three models. The normalized load versus normalized displacement results for the basis loads from these cases is shown in Fig. 5. The curves are labeled sequentially in order from the highest to the lowest value. The

\[
\theta = \tan^{-1}(\lambda_2 / \lambda_1)
\]

defines the radial direction in strain space.

Figure 5: Finite element analysis. Normalized load versus normalized displacement \((\hat{\sigma}_{ij} - \hat{\varepsilon}_{ij})\) results for models featuring (a) \(r = 1.0\), (b) \(r = 2.0\), (c) \(r = 3.0\).
post processing was done with ABAQUS/Post 5.6 (HKS[6]) to generate the elastic plastic load vs displacement data and the yield point data.

Connectivity testing

The connectivity of local yielding can be established by an iterative application of a local yield near neighbor test to the von Mises stress database. This is shown schematically in Fig. 6. The scaled database is flagged with all (zero stiffness) pore elements and elements with von Mises stresses greater than yield. A template with a seed column (or row) of elements is created and the near neighbor test is applied. If the seed column (or row) can propagate across the model, local yield percolation has occurred.

![Diagram of connectivity testing](image)

Figure 6: Schematic diagram of x₁-percolation test showing (a) yield and initial seed elements, (b) element near neighbors.

The radial resolution of the yield surface, at an angle θ in the biaxial strain plane, is defined to a tolerance given by eqn. (4).

\[
\Delta \lambda = \alpha (\lambda_{last} - \lambda_{first})
\]  

(4)

where \( \lambda_{last} \) is the last point in the model to yield, \( \lambda_{first} \) is the first point in the model to yield and \( \alpha \) is a constant factor, taken to be 0.01 for this study. Therefore, the estimated yield surface will be defined to within one percent of the distance between first and last local yield of the model for that particular applied deformation state. These bounds are very conservative, in the sense that they only reflect the absence and totality of yield, and do not reflect local plastic connectivity which is key to yield response.

An angular increment \( \Delta \theta \) of 10 degrees is used to create each radial direction on which yield percolation is determined. It follows that 36 points are used to define the yield surface. All database calculations were performed using MATLAB (Math Works[7]).

Local plastic percolation in the \( x_1 \) direction will produce strong indications of plastic flow in the \( \sigma_{22} - \varepsilon_{22} \) response. Conversely, local
plastic percolation in the $x_2$ direction will produce strong indications of plastic flow in the $\sigma_{11} - \varepsilon_{11}$. The orthogonal coupling between the stress and strain component responses, when strong as in a low porosity solid, produces simultaneous indications of yield. As the porosity increases, in conjunction with pore anisotropy or applied deformation anisotropy, the orthogonal yield indications will diverge.

**Yield surface estimates**

The results for the yield surfaces are shown in the biaxial strain representations of Fig. 7. The symbols indicate three different definitions of yield obtained from the normalized load versus normalized displacement responses of the nonlinear finite element simulations. These definitions of yield are: the initial loss of proportionality, the onset of plastic flow and 0.002 plastic strain offset. These points lie on the eight radial paths corresponding to the basis load combinations.

On those radial paths where two sets of points for a given definition are plotted, the yield strains observed in the $\sigma_{11} - \varepsilon_{11}$ components of stress and strain precedes that for the yield strains observed in the $\sigma_{22} - \varepsilon_{22}$ components of stress and strain. This disparity in observed yield strain in different directions is a function of the pore shape and pore pattern morphology. The longer aspect ratio pores generate higher stress concentrations and accelerate yield observed in the $x_1$ direction.

The lines in Fig. 7 represent estimates of the yield surface behavior derived from the elastic basis deformation cases, scaling and local yield percolation. The specific definitions of yield used in these plots are discussed next. Three definitions of yield are described in each plot: the first point for which the von Mises yield stress reaches the matrix yield stress, plastic percolation in the $x_1$ direction generating yield behavior in the $x_2$ direction, and plastic percolation in the $x_2$ direction generating yield behavior in the $x_1$ direction in the model. Observations on these results can be made in terms of yield surface shape, effect of pore aspect ratio, differences in $x_1$ and $x_2$ percolation and the comparison of the elastic estimates to the nonlinear analyses.

General observations on the shape of strain yield surfaces suggest that the shape becomes distorted due to the presence of the pores. Equal biaxial loading in tension and compression produces lower values of yield strain than would be expected in a pore free material. This is due to the pore induced stress concentrations and plastic zone linking under equal biaxial tension. Pore collapse under compression is not accounted for at the strains produced in these analyses, so both equal biaxial tension and
Figure 7(a): Strain yield surfaces from elastic and non-linear results for Model 1 with $p = 0.15, r = 1.0$.

Figure 7(b): Strain yield surfaces from elastic and non-linear results for Model 2 with $p = 0.15, r = 2.0$. 

Nonlinear analysis
- Loss of proportionality
+ Plastic flow
* 0.002 strain offset
Elastic analysis
- First microyield
- Perc $x_2$/ yield $x_1$
compression responses are identical.

The $x_1$ and $x_2$ percolation yield surfaces are relatively smooth, with some irregularity that can be attributed to the irregular pore patterns and the size of the model databases. These yield surfaces are quite comparable in the equal biaxial strain states but tend to be different for other strain states as the pore aspect ratio increases. Also, as the pore aspect ratio increases, the yield surface expands in the $\bar{\varepsilon}_{11}$ and contracts in the $\bar{\varepsilon}_{22}$ directions.

The nonlinear proportional yield and elastic first microyield generally show good agreement. For pore aspect ratios of 2.0 and 3.0, the nonlinear proportional yield in the $x_1$ direction occurs at smaller $\bar{\varepsilon}_{11}$ strains. Some of this tendency to smaller yield strains may be due to increasing elastic compliance in the $x_1$ direction.

The elastic local yield percolation estimates generally exceed the nonlinear bulk yield values. Agreement in the equal biaxial direction is generally good. The nonlinear 0.002 offset yield is much greater than the elastic yield percolation, as might be expected since the former represents

Figure 7(c): Strain yield surfaces from elastic and non-linear results for Model 3 with $p = 0.15$, $r = 3.0$. 

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well developed plastic flow and the latter is more closely associated with the onset of plastic flow.

Summary

In this paper, two dimensional computational simulations of initial yield and plastic flow in the microstructure were used evaluate the effect of different multiaxial deformations on the onset, connectivity and saturation of plasticity and yield surface characteristics. The results were used to demonstrate the sensitivity of the yield surface to pore geometry and to compare different definitions of yield. The results suggest that percolation based estimates of yield surfaces, derived from elastic analyses, can be useful for investigating microstructural effects in porous metals.

References