Empirical relationship between pulsating and fully reversed fatigue strength amplitudes

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Abstract

The fatigue behavior of metallic materials is significantly influenced by superimposed static mean stresses, which must be taken into account in service life calculations or in structure design. This requires a suitable engineering rule to estimate the allowable stress amplitude for any non-zero mean stress. Traditionally, the drop of endurable amplitude with increasing mean stress is approximately calculated from the so-called mean stress sensitivity, which is believed to be a linear function of the tensile strength. In this paper it is shown that a power law between fully reversed and pulsating stress amplitudes is much better suited to predict unknown pulsating tension or bending and torsion amplitudes in the high cycle fatigue range, including endurance limits, from existing fully reversed amplitudes. Based on an extensive compilation of fatigue data, the empirical parameters of the new function are provided for various ductile and brittle iron base and non-ferrous materials and for cast iron and steel weldments. Beyond the approximate description of tension and torsion mean stress effects, the proposed relationship permits also a better fatigue prediction for multiaxial stress combinations by suitable strength hypotheses.

1 Introduction

Mean stress effects in fatigue are often presented as plots of a stress amplitude $\sigma_a$ versus the corresponding mean stress $\sigma_m$, with the life to failure $N$ as a parameter. For a particular number of cycles to failure it is usually observed that the amplitude decreases with growing mean or static stress, irrespective of the loading mode. In axial testing, for instance, an absolutely limiting engineering stress
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would be the tensile strength of the material. For this case, Fig. 1 would give a schematic overview of the amplitude reduction with increasing mean stress and extending number of cycles to failure in the range of positive mean stresses. In general, however, the fatigue performance of materials is not sufficiently known to describe the behaviour in the form of Fig. 1.

![Figure 1: Mean stress effect in the S-N diagram and in the Haigh diagram (schematic).](image)

**Figure 2:** Linear description of the mean stress effect between $R = -1$ and $R = 0$ for a given cyclic life (here endurance limits) in nondimensional form of the amplitude versus mean stress plot according to Haigh.

For the sake of better comparison and - if possible - generalization, Haigh diagrams can be presented in a dimensionless version by normalizing the stress amplitude to the fully reversed amplitude at $\sigma_m = 0$ and formulating a stress ratio also for the mean stress on the abscissa, e.g. in axial tests by dividing the mean stress by the tensile strength. A schematic example is given in Fig. 2 for endurance limits $\sigma_A$, where the stress ratio $R = \sigma_{\text{min}}/\sigma_{\text{max}}$ is the ratio of lowest...
and highest stress value during a loading cycle. For generalization purposes several procedures have been proposed. Goodman suggested to estimate the endurable amplitude from a straight line between $\sigma_n/\sigma_A (R = -1) = 1$ and $\sigma_n/R_m = 1$ which gives reasonable results for brittle materials, yet, underestimates the behavior of ductile materials.

Gerber introduced a parabola through $\sigma_A/\sigma_A (R = -1) = 1$ and $\sigma_n/R_m = 1$ which was in fair agreement with experimental observations of ductile materials with positive mean stresses. With at least one additional experimental result more realistic parabolic descriptions became possible. To this end usually the pulsating fatigue strength $\sigma_A (R = 0)$ is determined, Fig. 2 [1, 2, 3]. In other approaches the endurable amplitude is discontinuously described by straight lines between certain defined stress ratios $R$ [4, 5] or by higher order polynomials with respect to static stress in compression [6]. Often, however, for a given material, there are no reliable pulsating fatigue strength results at hand. It is, therefore, the aim of this paper to provide a statistically backed estimate of the pulsating fatigue strength if only the alternating strength amplitude is known.

2 Mean stress sensitivity

Assuming as a first approximation a linear amplitude drop $\Delta \sigma_A$ in the mean stress interval $\Delta \sigma_m$, the slope $\tan \alpha$ in Fig. 2 is $\Delta \sigma_A/\Delta \sigma_m$. In the special case that pure alternating ($R = -1$) and pulsating ($R = 0$) loading conditions are considered, the slope can be expressed as:

$$\tan \alpha = \frac{\sigma_n (R = -1) - \sigma_n (R = 0)}{\sigma_m (R = 0)}$$  (1)

Under these special conditions $\sigma_m (R = 0)$ is equal to $\sigma_A (R = 0)$ and eq. (1) becomes

$$\tan \alpha = \frac{\sigma_n (R = -1)}{\sigma_n (R = 0)} - 1$$  (2)

The expression eq. (1) was called mean stress sensitivity $M$ by Schuetz [7] and was applied for amplitudes $\sigma_n$ in the high cycle fatigue range above the endurance limit. Schuetz found that high tensile strength steels are more sensitive to mean stresses than low tensile strength steels and postulated from the test results available at that time a linear relationship between mean stress sensitivity $M$ and ultimate tensile strength $M = 0.00035 \ R_n$ [MPa] – 0.1 [7]. This conclusion was predominantly based on test results between $N = 10^4$ and $N = 10^6$ with notched specimens which had stress concentrations up to $K_t = 5$. Later, similar approximations were proposed for cast iron [8] and aluminum materials. Since then the linear dependence of $M$ on $R_n$ was generally accepted and the coefficients were adopted as material dependent parameters in engineering guidelines mainly for endurance limits [5].

In the meantime fatigue test results have become much more numerous, and a recent evaluation by the authors [9] indicates that the assumption of a linear
increase of $M$ with $R_m$ is misleading, Fig. 3. At best a weak dependency can be recognized with an extremely large scatter between the experimental mean stress sensitivity $M_{exp}$ and values which are estimated from the recommendations in [5, 7, 8]. In general, $M$ is underestimated for lower strength steels suggesting stress amplitudes on the unsafe side for positive mean stresses.

![Figure 3: Literature evaluation on the effect of tensile strength on mean stress sensitivity.](image)

According to eq. (2) the mean stress sensitivity is governed by the ratio of alternating and pulsating stress amplitudes. Therefore, in [10] a linear relationship

$$2 \sigma_A (R = 0) = 1.55 \sigma_A (R = -1) + \sigma_0$$

was proposed from a regression analysis of a larger volume of results with $\sigma_0 = 15$ MPa for steels and light metals and $\sigma_0 = -10$ MPa for cast iron materials. The data evaluated were based on a compilation of multiaxial long life fatigue test results which are listed in [10]. Eq. (3) gives good estimates if one of the two endurance limits is known, but is not fully satisfactory for low strength materials, because the straight line suggested by (3) does not intersect the origin.
3 Correlation between pulsating and fully reversed fatigue strength

In double logarithmic coordinates S-N curves can be assumed as straight lines as depicted in Fig. 4 in the high cycle fatigue regime up to the knee number of cycles $N_K$:

$$N = C \sigma_a^k = N_K \sigma_a^k/\sigma_A^k$$  \hspace{1cm} (4)

From the ratio of the stress amplitudes $\sigma_a (R = -1)$ and $\sigma_a (R = 0)$ at equal number of cycles $N = N (R = -1) = N (R = 0)$ the following power law eq. (5) can be derived:

$$\sigma_a (R = 0) = K_a [\sigma_a (R = -1)]^q$$  \hspace{1cm} (5)

where

$$q = k (R = -1) / k (R = 0)$$  \hspace{1cm} (6)

is the ratio of the slopes of the two S-N curves and

$$K_a = [C (R = -1) / C (R = 0)]^{1/k (R = 0)}$$  \hspace{1cm} (7)

is a power of the ratio of the abscissa intercepts extrapolated from the two S-N curves beyond $N_K$. The parameters $K_a$ and $q$ can be gained by regression analysis if a sufficient number of data pairs is available, Fig. 5. To this end, a considerable number of own results on steels, aluminum alloys, cast iron and powder metallurgy steels was combined with earlier compilations of S-N curves and latest test results.
from the literature. A complete list of the references is available on request from the authors. All data were re-evaluated, based on the log-normal distribution, with the aid of the commercial software SAiFD (Statistical Analysis of Fatigue Data) developed by the authors [11]. Only data pertaining to 50% survival probability were further processed. From complete S-N curves the amplitudes at cycle numbers of full decades beginning at $10^4$ were used up to $N_K$. Beyond $N_K$ only the endurance limits at 50% probability were inserted. Between the full decades additional values at 2 and 5 times the decade were selected. In some cases also single values could be included depending on reliability and way of evaluation by the authors.

![Graph showing fatigue data correlations](image)

Figure 5: Correlation between pulsating and fully reversed fatigue strength in uniaxial loading and bending of metals.

Fig. 5 proves a well developed material dependent correlation between the pulsating and the fully reversed stress amplitude, where especially $K_a$ in eq. (5) and (7) is affected by the material ductility. With more brittle materials, like high strength steels, surface hardened materials or cast irons, a more pronounced mean stress sensitivity is to be expected than from predominantly ductile materials, which endure accordingly relatively higher pulsating stress amplitudes to failure. Size effects are of minor significance because the few reliable studies on size indicate more or less proportionality between the amplitudes at $R = -1$ and $R = 0$ for all geometries. Stress concentrations can be taken into account via an extension of eq. (5):

$$K_i \sigma_a (R = 0) = K_a [K_i \sigma_a (R = -1)]^q$$

(8)
The coefficient of correlation for this extended version will even further improve, yet, eq. (8) has little practical impact as long as the exponent q is close to 1, which is the case here. Thus, the parameters $K_a$ and q from Fig. 5 are valid for smooth geometries and notched geometries up to notch factors $K_a < 5$ irrespective of size. This permits the application to structural components in design. In the case of sharp notches $K_a > 5$ the material ductility seems to lose its importance: All data fit into the scatter band of brittle steels and titanium alloys.

![Graph: Predicted S-N Curve](image)

**Figure 6:** Example for a predicted S-N curve for pulsating load ($R = 0$) by the proposed relation in comparison with fatigue tests.

Knowing $K_a$ and q, from a given S-N curve at $R = -1$ the missing S-N curve at $R = 0$ can now be estimated with $\sigma_A (R = 0)$ from eq. (5)

$$\lg C (R = 0) = \lg C (R = -1) - \lg K_a \cdot k (R = -1) / q$$

(9)

and provided both curves have the same knee cycle number $N_K$

$$\lg N = \lg C (R = 0) + \lg \sigma_a (R = 0) \cdot k (R = -1) / q$$

(10)

An example is given in Fig. 6. Here the prediction from eq. (8) and (9), based on a statistically evaluated S-N curve of an alloy steel at $R = -1$, is compared with measured results under pulsating conditions $R = 0$. The agreement is fair and fully consistent with Haibach’s concept of so-called uniform scatter bands for analyzing S-N curves [4, 12]. In this concept, the stress amplitudes at different mean stress, e.g. $R = -1$ and $R = 0$, are normalized to the fully reversed amplitude at 50%
survival probability. The resulting scatter bands usually coincide quite satisfactorily and can be evaluated together.

Further to the materials in Fig. 5, eq. (5) is suitable to describe the behavior of weldments from constructional steels as well, Fig. 7. The parameters resulting from the data in [13] are $K_a = 1$ and $q = 0.96$. Separate evaluations for the different forms of weldments mentioned in Fig. 7 proved not to be justified nor necessary.

![Graph showing correlation between pulsating and fully reversed fatigue strength of constructional steel weldments in tension and bending.](image)

**Figure 7**: Correlation between pulsating and fully reversed fatigue strength of constructional steel weldments in tension and bending.

### 4 Relationship for torsional loading

Only few data pairs are available in the literature for the quantification of the mean stress effect under torsional loading conditions with $\tau_a$ (R = −1) and $\tau_a$ (R = 0). In total 110 sets could be extracted for an analogous regression analysis according to eq. (5) the result of which is summarized in Fig. 8. The parameters $K_a$ and $q$ are stated in Fig. 8, thus, also under shear stresses reasonable predictions of endurable pulsating conditions are possible from fully reversed S-N curves for several material groups.

For about three quarters of the data in Fig. 8 also the axial fatigue behavior is reported in the original works. This permits to calculate the ratios $\tau_a / \sigma_a$ (R = −1) and $\tau_a / \sigma_a$ (R = 0) which are plotted in Fig. 9. Obviously, the ratio of the pulsating amplitudes $\tau_a / \sigma_a$ (R = 0) is systematically higher than the ratio of the fully reversed amplitudes $\tau_a / \sigma_a$ (R = −1) and, surprisingly, under pulsating conditions R = 0 the torsional amplitude can become larger than the axial amplitude. This observation implies that the mean stress sensitivity under torsion is lower than under axial stresses. Nevertheless, the data scatter is too large in Fig. 9. In most investigations the pulsating S-N curves had not been measured with the same care...
as the fully reversed data. Usually a much lower number of specimens was used and, therefore, the reliability of the pulsating amplitudes is generally lower than that of the alternating amplitudes. For this reason, from the experimental fully reversed amplitudes the pulsating amplitudes were calculated from eq. (5) with the coefficients in Fig. 5 and Fig. 8. The calculated ratio $\tau_a / \sigma_a (R = 0)$ scatters much less versus the experimental ratio $\tau_a / \sigma_a (R = -1)$ than the experimental ratio $\tau_a / \sigma_a (R = 0)$ does in Fig. 9. As a rule of thumb, the presently available information can be described by

$$\tau_a(R=0) = K_{a,a} \left[ \sigma_a(R=-1) \right]^{0.8} \cdot 2\sqrt{3} \left[ \tau_a/\sigma_a(R=-1) \right]$$

(11)

Figure 8: Correlation between pulsating $\tau_a (R = 0)$ and fully reversed fatigue strength in torsion $\tau_a (R = -1)$ at the same number of cycles.

As a consequence of the limited reliability due to the large scatter in Fig. 9 we suggest to reinvestigate this relationship further with a variety of ductile and more brittle materials in a study complying with high quality standards.

5 Accuracy and prediction quality

The accuracy of the proposed eq. (5) with the parameters from Fig. 5, 7 and 8 can be tested by comparing the calculated with the predicted pulsating stress amplitude as shown in Fig. 10 for about 1700 pairs of data. The percentage deviation from the predicted amplitude obeys approximately a normal distribution with a standard deviation of 8.8%. Thus 95.5% of the calculated amplitudes are within ±17.6% around the experimental value. Bearing in mind that many experimental
investigations leave room for improvements with regard to reliability and reproducibility, the precision of prediction can be considered quite satisfactory.

Figure 9: Relationship for the ratio of torsion to normal stress between pulsating and fully reversed loads at the same number of cycles.

Figure 10: Predicted pulsating fatigue strength versus experimental data in pulsating tension, bending and torsion.
6 Summary

Estimating mean stress sensitivities from ultimate tensile strength can be misleading in many cases. An equation combining pulsating and fully reversed stress amplitudes via a power law can be derived from the mathematical description of S-N curves in double logarithmic coordinates. The parameters in this relation are material specific, particularly responsive to brittleness, and can be determined by regression analysis of a larger quantity of pairs of S-N curves.

The type of equation developed, together with the coefficients found for different classes of materials, can be used to predict pulsating S-N curves from fully reversed S-N curves or vice versa, which are valid for the high cycle fatigue region \( N > 10^4 \) including the endurance limit.

Presently, the difference in mean stress sensitivity between torsional and axial loading cannot yet satisfactorily be taken into account because of the extreme scatter of experimental results. Therefore we recommend to start an experimental investigation of greatest care and reliability comprising pulsating and fully reversed loading under torsional and axial stresses with ductile and brittle materials to shed more light onto this interdependence.

References
