Fatigue damage analysis of welded structures based on continuum damage mechanics

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Abstract

In this study, the fatigue damage analysis and life prediction of welded structures are carried out by using continuum damage mechanics. For high cycle fatigue, it is considered that plastic deformation and damage occurs at a microscopic scale. Therefore, a two-scale model presented by Lemaitre is introduced to evaluate the high cycle fatigue damage evolution. As it is difficult to identify the parameters directly at the micro scale, the identification method is proposed to obtain the reliable material parameters for weldment. In order to consider the effect of residual stress on fatigue behavior of welded joints, the inherent strain method is applied to determine the residual stress. The analytical results by the proposed method are compared to experimental results and it is confirmed that the proposed method could give the reliable fatigue lifetime of welded structures.

1 Introduction

Decision concerning the maintenance and replacement of the existing bridges gives serious impact on the traffic patterns and economy of the surrounding community. Thus, extending the lifetime of bridge structures plays an important role in reducing the total life cycle costs. Therefore, it is necessary to evaluate the durability and lifetime of bridge structures accurately. In case of steel bridge, structures are commonly fabricated by welding. These welded structures are often subjected to high cyclic loading and subsequently failures may occur due to fatigue damage at the welded joints [1, 2]. The lifetime of welded structures subjected to cyclic loading, consists of two phases: crack initiation and crack propagation to final failure. The problem of crack propagation up to failure of structures has received much attention through development of fracture mechanics [3]. In this framework, the assumption of a pre-existing crack is required to estimate the growth of a crack through the final failure. However, it is difficult to assume the pre-existing crack size in advance. Thus, an effective
procedure for life prediction of welded structures based on the continuum damage mechanics (CDM) [4] is proposed. CDM could describe both crack initiation and propagation in multi-axial loading condition with appropriate failure criteria. For high cycle fatigue (HCF), materials do not exhibit such a macroscopic plasticity and damageable behavior. However, it is considered that plastic deformation and damage occur at microscopic scale. Therefore, a two-scale model presented by Lemaitre [4,5] is introduced to evaluate the damage evolution at the microscopic scale by using the law of localization of self-consistent scheme. As it is difficult to identify the parameters directly at the microscale from the material experiment, the identification method is proposed to obtain the reliable material parameters.

The fatigue damage behavior of welded joints is complicated by numerous factors with respect to welding condition. Residual stresses that exist in the welded joints could be regarded as the result of incompatible thermal strains. These residual stresses reduce the fatigue life of welded structures, particularly when a tensile residual stress exceeds yield stress at the weld toe regions [6]. Therefore, the effect of residual stress on fatigue behavior of welded structures is considered in this analysis. In this study, the simplified prediction of residual stresses based on the inherent strain method [7,8] is carried out. The fatigue damage analysis of T-joint is demonstrated to clarify the accuracy of the proposed method. The analytical results are compared with experimental results to confirm the validity.

2 Fatigue damage analysis procedure

The HCF damage analysis and life prediction of welded structures are performed by the following two steps: 1) The welding residual stresses are estimated by using the inherent strain method. 2) The fatigue damage analysis is carried out based on the continuum damage mechanics.

2.1 Determination of residual stresses

The residual stresses can be obtained by several experimental measuring methods. These methods can be classified into destructive and non-destructive methods. Furthermore, the thermal finite element analysis can also be applied to the simulation of welding process. Although all the methods above based on measurements or thermal elastic-plastic analysis can give accurate results, it is time-consuming. Therefore, a simplified procedure for evaluating the residual stresses in thin walled structures is applied in this study. This method is based on the inherent strain method, which was proposed by Ueda and Yuan [7, 8].

In this idea, considering the internal stress $\sigma_{ij}^0$ as residual stresses in the weldment. Then total strains $\varepsilon_{ij}$ are evaluated by the sum of elastic strains $\varepsilon_{ij}^e$ and inherent strains $\varepsilon_{ij}^*$ as:

$$\varepsilon_{ij} = \varepsilon_{ij}^e + \varepsilon_{ij}^*$$

and the Hook’s law is written as

$$\sigma_{ij} = E_{ijkl}^e \varepsilon_{kl}^e = E_{ijkl}^e (\varepsilon_{ij} - \varepsilon_{ij}^e)$$

where $E_{ijkl}^e$: elastic modulus tensors.
The residual stresses $\sigma^0_{ij}$ are the self-equilibrium stresses with no existence of external forces, thus the equilibrium condition yields

$$\sigma^0_{ij}n_j = (E_{ij}e_{ij})n_j - (E_{ij}e_{ij})n_j = 0$$

and

$$\sigma^0_{ij} = (E_{ij}e_{ij}) - (E_{ij}e_{ij}) = 0$$

(3)

The residual stresses can thus be evaluated by the elastic finite element method using the above equations, if the inherent strains are given. The simplified method to determine the inherent strains is given as the following procedure.

### 2.1.1 Inherent strain in butt welds

The longitudinal inherent strain distribution in the transverse section of a butt weld can be approximated by a trapezoid as shown in Figure 1 and Figure 2a.

![Figure 1: Butt welded plate.](image)

![Figure 2: Inherent strain and residual stress distributions.](image)

From the numerical studies, Ueda and Yuan [7] established some formulas for description of the approximated distribution and magnitude: the peak value $\varepsilon^*$, the width of inherent strains zone $b$ and the width of heat-affected zone (HAZ) $y_n$. The formulas were derived for a butt joint in a plate with a width of $2B$ as follows, assuming that the heat source is applied simultaneously to the entire weld line ($y=0$), so that the material deforms uniformly in the direction perpendicular to the weld line [7]:

$$y_n = 0.242Q/[c_p(T_m - T_0)]$$

$$b = \xi b_0$$

$$\varepsilon^* = \xi \varepsilon^*_{in}$$

and

$$b_0 = 0.242\alpha E Q/(c_p h \sigma_y)$$

$$\varepsilon^*_{in} = \sigma_{yw}/E$$

$$\xi = 1 - 0.27\alpha T_m/\sigma_y$$

$$\zeta = 1 - 0.27\alpha T_m/\sigma_y$$

$$T_m = Q/c_p A$$

where $Q$: line heat input (J/mm); $T_m$: mechanical melting point over which yield stresses disappear (°C); $T_0$: room temperature(°C); $c$: specific heat (J/g°C); $p$: density(g/mm³); $h$: thickness of plate (mm); $\alpha$: linear thermal expansion coefficient(1°C); $E$: Young’s modulus (GPa); $\sigma_{yw}$: yield stress of weld metal.
and HAZ (MPa): $\sigma_{yb}$: yield stress of base metal (MPa); $A$: area of transverse cross section ($\text{mm}^2$).

A simple example of a residual stress distribution is shown in Figure 2b. The magnitude of $\tilde{\varepsilon}_s$ is found from the equilibrium condition.

2.1.2 Inherent strain in T-joints and I-joints

The formulas in the previous section can be applied to T-joints and I-joints with appropriate modifications [8]. The magnitude of the inherent strain is practically the same for a butt weld and a T-joint. Thus, only the width of the inherent strain zone $b$ has to be modified. The heat input in a T-joint is different from a butt joint. This effect can be taken into account by replacing the heat input $Q$ by an equivalent heat input $Q'=2Qh/(2h_f+h_w)$ (see Figure 3) and the value of $b_0$ by the following equation:

$$b_0 = 0.484\alpha E \bar{Q} / \left[ c \rho (2h_f + h_w) \right] \sigma_{yb}$$

(6)

where $h_f$ and $h_w$ are the plate thickness of the web and the flange, respectively.

![Diagram of T-joint and I-joint](image)

Figure 3: Determination of equivalent heat input.

In the case that the average temperature $T_{av}$ rises ($T_{av}>30^\circ\text{C}$), significant bending deformations may occur as a consequence of the uniform heat input in the web of the T-joint or I-joint. Therefore, a subsequent modification has to be made by dividing the longitudinal strain in the flange into two parts: one is due to the uniform deformation and another the bending deformation. The effect of the uniform deformation can be evaluated by the formulas derived for a butt weld. The bending part can be taken into account the increase of the average temperature as:

$$T_{av}' = T_{av} + \Delta T_{av} = (1+\phi)T_{av}$$

$$\phi = \bar{z}^2 A / I$$

(7)

hence

$$\xi = 1 - 0.27\alpha E T_{av} (1+\phi) / \sigma_{yb}$$

(8)

Then the magnitude of longitudinal inherent strain in a T-joint can be evaluated by using Eqns (4) and (5), while the distribution can be obtained by Eqns. (6) and (8). The inherent strain distribution in I-joint can also be evaluated by means of the same procedure.
2.2 Continuum damage mechanics

Material deterioration is an irreversible phenomenon during the damage evolution process, which may be interpreted as the growth of micro-defects and micro-cavities. The effect of these micro-defects is evaluated in the mesoscale plane that can be defined in a representative volume element (RVE) [4]. The damage variable is considered as the degree of degradation of material in the homogeneous field. Therefore, the basic image of damage variable $D$ is defined as the loss of effective area in the meso-scale (RVE):

$$D = \frac{A_D}{A_0} \tag{9}$$

where $A_0$: the total area of considered plane, $A_D$: the area of all micro-defects.

Figure 4: Damage evolution.

The damage evolution is derived from the associated flow rule with the strain energy density release rate $\dot{Y}$ and the existence of potential of dissipation $F_D$ based on the thermodynamics framework. Then, it is proportional to the increment in accumulated plastic strain and obtained directly by

$$dD = \frac{\partial F_D}{\partial \dot{Y}} d\lambda = \left(\frac{\dot{Y}}{S} \right) dp \tag{10}$$

where $S$ and $n$ are the damage energy strength and exponent of damage evolution, respectively.

The accumulated plastic strain caused by micro defects is defined as:

$$dp = \sqrt{\frac{1}{2} d\varepsilon_p^p d\varepsilon_p^p} \tag{11}$$

where $dp$ and $d\varepsilon_p^p$ are the accumulated plastic strain increment and plastic strain increments, respectively.

It is assumed that damage occurs when the accumulated plastic strain $p$ reaches a certain value $p_D$. The subsequent damage growth up to rupture occurs at the meso-scale (macrocrack initiation), when damage variable reaches the critical value $Dc$ (see Figure 4). If the stored energy in an arbitrary stress state is the same as in the uniaxial tension, the damage threshold is determined as:

$$p_D = \varepsilon_{np}^{p} \left(\sigma_u - \sigma_f \right) / \left(\sigma_{eq} - \sigma_f \right) \tag{12}$$

where $\sigma_u$, $\sigma_f$, $\sigma_{eq}$ are the ultimate, fatigue limit and von Mises equivalent stresses, respectively; $\varepsilon_{np}$ is the damage threshold in pure tension.
2.2.1 Damage model for HCF (Two-scale model)

When the amplitude of the loading is lower than yield stress, the plastic strain cannot be observed at the mesoscale. Consequently, for high cycle fatigue, material is regarded as a damageable micro-inclusion embedded in an elastic macro-element. Therefore, the two-scale model is applied to evaluate the damage evolution in a weak micro-inclusion embedded in a RVE [5].

![Diagram showing the two-scale model](image)

Let us consider that a meso-volume element is elastic everywhere except in a micro-volume inclusion $\mu$ under the elastoplastic state as shown in Figure 5. The inclusion has the same properties as the meso-domain except for the plasticity threshold $\sigma_{\mu}^{y}$ ($=\sigma_{\mu}^{f}$, fatigue limit of the material).

The damage evolution law is now written in terms of microscopic fields as

$$D = \left(\frac{Y^\mu}{S}\right) \dot{\rho}^\mu$$

(13)

The strain energy release rate $Y^\mu$, taking into account the different behaviors in tension and compression, is written as

$$Y^\mu = \frac{1}{E} \left[ \frac{\langle \sigma_{\mu}^T \rangle^2}{(1-D)^2} + \frac{\langle \sigma_{\mu}^C \rangle^2}{(1-D)^2} \right] + \frac{\langle \sigma_{\mu}^C \rangle^2}{(1-D)^2} + h \frac{\langle \sigma_{\mu}^T \rangle^2}{(1-D)^2}$$

(14)

where $h$ is the crack closure parameter (for most metal $h=0.2$), and $\langle \rangle$ means the positive part, i.e., $\langle x \rangle = x$ if $x > 0$, $\langle x \rangle = 0$ if $x \leq 0$.

In order to perform the calculation based on the macroscopic response, the stresses at microscopic $\sigma_{\mu}^{y}$ are evaluated from the stresses at macroscopic $\sigma_{\mu}^{y}$ by mean of the self-consistent scheme $[5, 9]$:

$$\sigma_{\mu}^{y} = \sigma_{\mu}^{y} - a \epsilon_{\mu}^{yp}$$

(15)

where $a$ is given by Eshelby’s analysis of a spherical inclusion $[10]$,

$$a = (1-\beta)/(1+\nu), \quad \beta = 2(4-5\nu)/(4\nu)$$

(16)

The constitutive equations at the microscopic scale that apply the kinematic hardening $X^{\mu}$ are written as:

$$f^\mu = (\sigma^{yp} / (1-D) - X^{\mu})_{\eta \mu} - \sigma_{\mu}$$

$$\epsilon_{\mu}^{yp} = \frac{\partial f^\mu}{\partial \sigma_{\mu}^{yp}}$$

$$\dot{X}^{\mu} = 2 \dot{\epsilon}_{\mu}^{yp} / (1-D)/3$$

(17)

Finally, it is possible to compute damage evolution up to failure (crack initiation, $D=D_C$) as a function of the macroscopic stresses for any loading at micro-scale.
2.2.2 Simplified model

In order to obtain the material properties, the simplified identification is proposed under the following hypotheses: 1) There is no coupling between damage and elasticity. 2) The quasi-unilateral effect is not taken into account, \( h = 1 \). 3) The damage occurs with plasticity simultaneously. 4) The influence of damage on the effective stress at microscale in the yield surface is neglected. 5) The critical value of damage is assumed to be \( D_t = 1 \). The simplified damage evolution rate can be expressed as a function of the macroscopic stresses [5]:

\[
\frac{dD}{dN} = \frac{(\sigma_{\sigma} + k\sigma_f)^{R_u^\nu}}{2ES(1+k)^2(1-D)^2} \frac{d\sigma_{\sigma}}{C(1+k)} \quad \text{if} \quad \sigma_{\sigma} \geq \sigma_f
\]  

(18)

where the triaxiality function is

\[
R_u^\nu = \frac{2}{3}(1+\nu)+3(1-2\nu) \left( \frac{\sigma_{\mu}(1+k)}{(\sigma_{\sigma} + k\sigma_f)} \right)^{2n+1}
\]  

(19)

and

\[
k = \frac{3\alpha E}{2C}
\]  

(20)

where \( C \) is a kinematic hardening parameter.

2.2.3 Identification of material parameters

The main difficulty is the identification of the material properties at the microscale. Two main material parameters, damage energy strength \( \bar{S} \) and exponent \( n \) for damage evolution of Eqn (12) need to be defined. The parameters \( (\bar{S}, n) \) can be identified by using Eqn (18) with a set of fatigue test data [5].

By assuming \( R_u^\nu \) is constant and integrating Eqn (18) over one cycle symmetric loading, \( \pm \sigma_{\mu} \) (Figure 6), the damage increment per cycle written as:

\[
\frac{\delta D}{\delta N} = \frac{2(R_u^\nu)^n}{(2n+1)(2ES)^n(1-D)^{2n}C} \left[ \left( \frac{\sigma_{\mu} + k\sigma_f}{(1+h)} \right)^{2n+1} - \sigma_f^{2n+1} \right]
\]  

(21)

Integrating Eqn (21) over the whole periodical loading up to \( D = D_C = 1 \), gives Number of failure loading cycle as,

\[
N_f = \frac{(2ES)^nC}{2(R_u^\nu)^n \left[ \left( \frac{\sigma_{\mu} + k\sigma_f}{(1+h)} \right)^{2n+1} - \sigma_f^{2n+1} \right]}
\]  

(22)

where

\[
R_u^\nu = \frac{2}{3}(1+\nu)+3(1-2\nu) \left( \frac{1+k}{3(1+k\sigma_f / \sigma_{\mu})} \right)^2
\]  

(23)

\[\text{Figure 6: S-N curve of Eqn (22).} \quad \text{Figure 7: Damage evolution of Eqn (25).}\]
Fatigue Damage of Materials: Experiment and Analysis

Two main parameters $S$ and $n$ can be obtained by inverse calculation of Eqn (22) under the condition that analytical results agree with fatigue test data. However, if the damage exponent $n$ is given in advance, the damage energy strength can be identified by the explicit formula.

In order to determine the damage exponent $n$, an additional equation is derived by substituting Eqn (22) into Eqn (21) as

$$\frac{\delta D}{\delta N} = \frac{1}{(2n+1)(1-D)^n N_f}$$

(24)

then integrating Eqn (24) over the periodical loading to a certain number of cycle $N$, for damage $D < D_c$. Finally the damage is obtained as a function of normalized cycle loading:

$$D = 1 - \left[1 - \frac{N}{N_f}\right]^{\frac{1}{2n+1}}$$

(25)

The damage exponent $n$ can be identified by Eqn (25) with the experimental data of HCF (see Figure 7). After that, the damage energy strength $S$ can be simply identified by Eqn (22).

2.3 Analysis procedure

The HCF analysis procedure is shown in Figure 8. At first, the welding residual stresses are evaluated by the inherent strain method coupling with elastic FE analysis as outlined in section 2.1. Then the residual stresses are introduced as the initial stresses into the FE-structure analysis. By using the two-scale model, the fatigue damage analysis is carried out for the welded structure with modification of initial Young’s modulus $E$. The initiation of a crack at macro-scale is assumed when damage reaches its critical value (numerically taken $D_c = 0.9$) at any Gauss points. Consequently, the crack area is assumed to coincide with the critical damaged zone ($D = D_c$). The overall failure condition is assumed that the length of crack size reaches a certain critical value.

![Figure 8: HCF analysis procedure.](image-url)
3 Numerical analysis and results

The HCF damage analysis of a T-joint is demonstrated by the proposed method. The analytical geometry of T-joint is shown in Figure 9. The length and thickness of the plate for flange and web are 600mm and 12mm, respectively. The fillet weld legs are 6mm. The three-dimensional solid FE-mesh is selected to model a quarter of T-joint. The material data in this analysis are taken from experiment of Ueda and Yuan [7, 8].

<table>
<thead>
<tr>
<th>Heat input (J/mm)</th>
<th>Q</th>
<th>2400</th>
</tr>
</thead>
<tbody>
<tr>
<td>Melting temperature (°C)</td>
<td>Tm</td>
<td>700</td>
</tr>
<tr>
<td>Room temperature (°C)</td>
<td>T0</td>
<td>15</td>
</tr>
<tr>
<td>Average temperature (°C)</td>
<td>Tw</td>
<td>90</td>
</tr>
<tr>
<td>Specific heat (J/g°C)</td>
<td>C</td>
<td>0.63</td>
</tr>
<tr>
<td>Density (g/mm³)</td>
<td>ρ</td>
<td>7.82x10³</td>
</tr>
<tr>
<td>Thermal expansion coefficient (1/°C)</td>
<td>α</td>
<td>1.2x10⁻³</td>
</tr>
<tr>
<td>Yield stress of weld metal &amp; HAZ (MPa)</td>
<td>σyw</td>
<td>520</td>
</tr>
<tr>
<td>Yield stress of base metal (MPa)</td>
<td>σyb</td>
<td>330</td>
</tr>
</tbody>
</table>

3.1 Residual stresses evaluation

The welding condition of T-joint is shown in Table 1. Then the inherent strains for T-joint are evaluated by the simplified formulas in section 2.1. Figures 10(a) and 11(b) show the distribution of residual stresses in the flange and web, respectively. It is confirmed that the analytical results agree well with the experimental results [8].

![Figure 10: Residual stress distribution in T-joint.](image)

3.2 Fatigue life prediction

The HCF damage analysis and life prediction of a T-joint is carried out. The T-joint is subjected to cyclic loading with constant amplitude (see Figure 9). The material parameters of base metal (SM400B), HAZ and weld metal are defined as in Tables 2 and 3. Two main parameters: damage energy strength S and exponent n were identified for base metal, HAZ and weld metal by the simplified model, respectively. Due to the lack of the test data that can be applied to Eqn
(25), the damage exponent is assumed \( n=2 \) for all material. Under this assumption, the damage energy strength \( S \) was defined by Eqn (22) directly. The damage energy strength for base metal and weld metal are obtained \( S = 2.0 \) MPa and 6.0 MPa, respectively.

The fatigue damage analysis is carried out until the critical damage zone \( (D=D_c) \) spreads through the whole longitudinal cross section of T-joint (Figure 11). Figure 12 shows the results of fatigue life prediction compared with the experimental results from JSSC [11]. It is observed that the proposed method can predict the accurate lifetime. Therefore, the proposed method can simulate the HCF damage behavior of the typical welded structure.

\[
\begin{array}{|c|c|c|}
\hline
\text{Elastic modulus (MPa)} & E & 2 \times 10^5 \\
\hline
\text{Poisson's ratio} & \nu & 0.3 \\
\hline
\text{Fatigue limit stress (MPa)} & \sigma_f & 160 \\
\text{Yield stress (MPa)} & \sigma_y & 330 \\
\text{Ultimate stress (MPa)} & \sigma_u & 465 \\
\text{Damage threshold} & \varepsilon_{\text{th}} & 0.0 \\
\text{Critical damage} & D_c & 0.9 \\
\text{Hardening parameter (MPa)} & C & 470 \\
\text{Crack closure} & h & 0.2 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|}
\hline
\text{Elastic modulus (MPa)} & E & 2 \times 10^5 \\
\hline
\text{Poisson's ratio} & \nu & 0.3 \\
\hline
\text{Fatigue limit stress (MPa)} & \sigma_f & 220 \\
\text{Yield stress (MPa)} & \sigma_y & 520 \\
\text{Ultimate stress (MPa)} & \sigma_u & 650 \\
\text{Damage threshold} & \varepsilon_{\text{th}} & 0.0 \\
\text{Critical damage} & D_c & 0.9 \\
\text{Hardening parameter (MPa)} & C & 500 \\
\text{Crack closure} & h & 0.2 \\
\hline
\end{array}
\]

Figure 11: Damage distribution.

Figure 12: Fatigue life prediction.

4 Conclusions

The high cycle fatigue damage analysis based on the continuum damage mechanics were carried out. The influence of the residual stresses on the fatigue damage behavior of T-joint was considered. The residual stresses were evaluated by the inherent strain method. For high cycle fatigue, a two-scale damage model is applied to evaluate the damage evolution. As it is difficult to obtain the damage parameters at the micro scale, the identification of material parameters for weldment is proposed by using the simplified model.

From the analytical results, good agreement between the fatigue life prediction of T-joint and experimental data was confirmed. Therefore, The high cycle fatigue damage analysis and life prediction of typical welded structure could be carried out by the proposed method. In order to obtain the reliable fatigue life, the identification method of material properties needs to be improved.
References
