# Estimation of the low cycle fatigue life for a submarine pressure hull

C.-C. Liang<sup>1</sup>, C.-Y. Hsu<sup>2</sup>, T.-L. Teng<sup>1</sup> & C.-Y. Jen<sup>3</sup>

<sup>1</sup>Department of Mechanical and Automation Engineering,
Da-Yeh University, Taiwan, ROC

<sup>2</sup>Department of Marine Mechanical Engineering,
Chinese Naval Academy, Tsoying, Taiwan, ROC

<sup>3</sup>Institute of System Engineering, University of National Defense Chung
Cheng Institute of Technology, Ta-Shi, Tao-Yuan 335, Taiwan, ROC

#### **Abstract**

An accurate estimate of the fatigue life of a submersible pressure hull is a valuable reference for the maintenance or construction. This work presents a novel means of estimating the low-cycle fatigue life of a submarine pressure hull. A finite element procedure is initially developed to determine the area of high stress and the strain spectrum of this area after the cyclic hydrostatic pressure loading is obtained. The Coffin-Manson non-linear experience equation associated with the strain-life method is then applied and combines the Miner damage accumulation rule to calculate the fatigue life. In so doing, the Rainflow counting method and the Newton-Raphson method are applied to simplify the diving load history and derive the Coffin-Manson non-linear experience equation. Finally, a submarine pressure hull is considered as an illustrative example to predict its fatigue life. The estimated schedule of fatigue life developed herein can provide the structural designer and the operator of the submersible pressure hull with an effective tool for both design and periodical maintenance. Estimates schedule can also provide a valuable reference for renovating the submarine.



#### 1 Introduction

Studies of fatigue problems of the submarine pressure are limited. In 1954, Coffin [1] and Manson [2] established the theorem of material damage due to plastic strain. In 1965, the US navy performed a large-scale fatigue test used a full-size model of the pressure hull of the submarine to elucidate fatigue problems associated with a large submarine structure [3]. In 1979, Riggs [4] considered a fatigue analysis of semi-submersible structures. Kilpartrick [5], in 1986, presented fatigue test results on two series of fatigue test models – the S series and the L series, used by the Admiralty Research Establishment of the UK. A crack was found to occur initially at the point of welding between the plate and stiffeners.

In this work, the fatigue life of a Guppy like class submarine is estimated. The estimated fatigue life schedule developed herein can provide designer of the submarine pressure hull structure, and its operator, with an effective tool for design and periodical maintenance.

### 2 Theoretical background

The procedure for estimating the crack initiation fatigue life of the submarine pressure hull is described as follows. First, the finite element method is initially used to determine the position of the high-stress area of the pressure hull and the strain spectrum over this under cyclic hydrostatic pressure loading [6]. Second, the Coffin-Manson nonlinear empirical equation associated with the strain-life method is applied; it combines the Miner damage accumulation rule with the front strain spectrum [1-2, 7-8]. The fatigue problem of the submarine pressure hull is a multiaxial fatigue problem, so the total strain is calculated by an ASME code procedure [9]. A fatigue life estimation schedule is thus determined. In this schedule, the Rainflow counting method and the Newton-Raphson method are used to simplify the diving load history and derive the Coffin-Manson nonlinear experience equation [10]. Figure 1 presents the procedure in detail.

## 3 Fatigue analysis of the pressure hull of a Guppy like class submarine

### 3.1 Static strength analysis of the pressure hull of the Guppy like class submarine

The first task is to determine the position of high-stress area of the pressure hull of the Guppy like class submarine. A finite element model of the pressure hull of the Guppy like class submarine is established.

#### 3.1.1 Geometry of the submarine pressure hull

Figure 2 depicts the configuration of the Guppy like class submarine. The pressure hull has eight parts- the forehead and the after torpedo compartment, the forehead and the after main engine room, the forehead and the after storage



battery compartment, the motor compartment and the control room. Table 1 presents the geometry of the pressure hull. The dimensions of the circumferential stiffener of the pressure hull are 6"×6"×22.5". It is made of H-section steel, as shown in Fig. 3.

#### 3.1.2 Material properties

The pressure hull of the submarine and its stiffener are made of high tensile steel (HTS). The material properties of HTS are as follows; Young's modulus,  $E=2.96\times10^7$  psi; yield stress,  $S_y=4.70\times10^4$  psi; ultimate strength,  $S_u=9.00\times10^4$  psi; density,  $\rho=0.283$  lb/in<sup>3</sup>; Poisson's ratio,  $\nu=0.30$ .

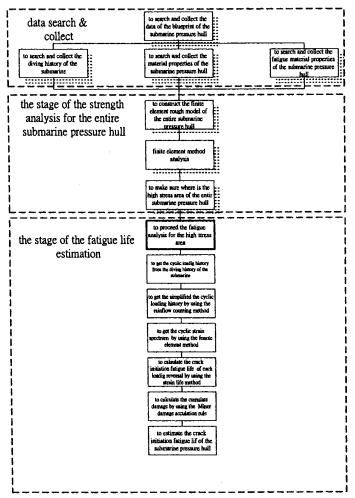


Figure 1: Procedure for estimating the fatigue life estimation of the submarine pressure hull.



Figure 2: Guppy like class submarine.

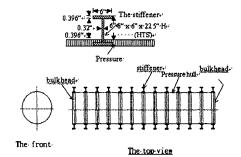


Figure 3: Guppy like class submarine pressure hull.

Table 1: Size of the main compartment in the Guppy like class submarine.

	Section 1	Section 2	Section 3	Section 4	Section 5	Section 6
Compartment	Forehead	Control room	After head	Forehead main	After head main	Motor compartment
	storage battery		storage battery	engine room	engine room	
	compartment		compartment			
Frame location	9" AFT. FR.47 TO FR.35	3" AFT. FR.58 TO 9" AFT. FR.47	7.25" AFT. FR.77 TO 3" AFT. FR.58	7.25" AFT. FR.77 TO FR.88	FR.88 TO 13.5" FWD. FR.99	13.5" PWD. FR.99 TO FR. 107
Radius, R(in.)	96.00	96.00	96.00	96.00	96.00	96.00 →91.00
Length, L(in.)	369.00	324.00	532.25	322.75	316.50	241.50
shell thickness, t(in.)	0.875	0.875	0.875	0.875	0.875	0.875
Frame spacing	Fr. 0-Fr. 35: 24in.	Fr. 35-Fr. 62: 30in.	Fr. 62-Fr. 69: 24in.	Fr. 69-Fr. 103: 30in.	Pr.103-Fr. 104: 24in.	Fr. 104-Fr. 106: 30in.
	Pr. 106-Pr. 137: 24if1.					



Figure 4: Finite element model diagram of the Guppy like class submarine pressure hull.

#### 3.1.3 Finite element model

Figure 4 presents the finite element model of the pressure hull of the Guppy like class submarine. Part of the pressure hull, in the finite element model, is meshed by four nodes doubly curved thin shell elements with 1848 elements. The other part of the stiffener in the finite element model is meshed by 1872 two node linear I-ship beam elements. The finite element model of the submarine pressure hull includes 2028 nodes.



#### 3.1.4 Loading of the submarine pressure hull

This work considered a series of diving depths of the submarine to probe the depth caused the yielding (using the HTS yielding stress as a criterion) and failure (using the HTS ultimate stress as criterion).

#### 3.1.5 Boundary condition

The bulkheads of the submarine are very strong structures, so the radial displacement of the divided part of the compartment is constrained.

#### 3.1.6 Results and discussion

Table 2 shows the maximum von Mises stress of each compartment in the submarine pressure hull at diving depths of 200 ft., 400ft., 1121ft and 2180ft. Table 2 shows that the stress was almost higher in the after storage battery compartment at diving depth. The maximum von Mises stress in the after storage battery compartment of the pressure hull with stiffeners are 8437psi, 16874psi and 44668psi at diving depths of 200ft, 400ft and 1121ft, respectively.

Table 2: Maximum von Mises stress (psi) on the compartment in the submarine pressure hull at various diving depths.

Diving depth(ft.)	200	400	1121	2180
Forehead storage battery compartment	8432	16863	44640	75609
Control room	8379	16757	44361	75514
After head storage battery compartment	8437	16874	44668	75346
Forehead main engine room		16740	44316	75463
After head main engine room		16725	44275	75636
Motor compartment	7908	15815	41867	73937

The results concerning high stress area of the Guppy like class submarine, diving to various depths, show that the stress is often in the after storage battery compartment. Therefore, a follow-up should be conducted to estimate the initial crack fatigue life, focusing on the opening location of the after battery compartment.

#### 3.2 Fatigue analysis of the pressure hull of the Guppy like class submarine

#### 3.2.1 Problem description

The high stress area is found to be in the afterward storage battery compartment by the static strength analysis of the Guppy like class submarine pressure hull. This work considers the afterward storage battery compartment as an object in estimating the initial crack fatigue life of the area around its opening.

#### 3.2.2 Geometry of the afterward storage battery compartment

The afterward storage battery compartment is located in the intermediate zone of the submarine pressure hull. It is a cylinder with a length is 532.25 in. Twentyone stiffeners (frame) encircled the cylindrical pressure hull. The numbers of the frames are from FR.59 to FR.77. The circumferential stiffeners are made of



6.00"×6.00"×22.50" H section steel. The radius (R) of the compartment is 96.00 in. Its thickness (t) is 0.875 in. The opening area on top of the cylinder is located between FR.64 and FR.65. It is a manhole through which people can access the submarine. Figure 5 presents the detail configuration of the afterward storage battery compartment.

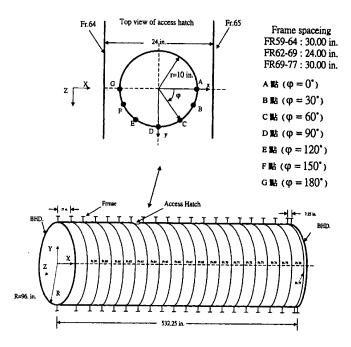


Figure 5: Afterward storage compartment in the Guppy like class submarine.

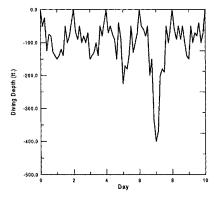


Figure 6: Simulated diving history of the Guppy like class submarine.



### 3.2.3 Fatigue material properties of the afterward storage battery compartment

The fatigue material properties of the steel are described as follows; The fatigue ductility coefficient,  $\varepsilon_f = 0.26$ ; fatigue ductility exponent, c = -0.75; fatigue strength coefficient,  $\sigma_f = 1.33 \times 10^5$  psi; fatigue strength exponent, b=-0.075.

#### 3.2.4 Fatigue loading histories

In this work, the analysis of fatigue life is based on the real operating conditions of the submarine, in the simulation of diving history, as plotted in Fig. 6. In this diving history, the load associated with a ten-day dive is taken as one loading block. The diving history includes five voyages.

#### 3.2.5 Meshing

The opening through which people access is on the centreline of the top of the afterward storage battery compartment. This symmetry was used in the preceding fatigue analysis: only half of the compartment needs to be considered for estimating the fatigue life of the pressure hull. Figure 7 shows the finite element model. The finite element model is meshed by four- node doubly curved thin shell elements and includes 1380 elements. The finite element model includes 630 elements in the part of stiffener in the finite element model meshed by two-node linear I-ship beam elements. The finite element model includes 1457 nodes for the afterward storage battery compartment.

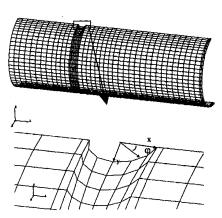


Figure 7: Finite element model of he afterward storage compartment in the Guppy like class submarine.

#### 3.2.6 Boundary condition

The geometry and loading condition of the opening in the pressure hull are symmetrical about the center of the cylindrical hull; the symmetry constitutes the boundary condition.

#### 3.2.7 Strain spectrum

In the procedure followed herein to estimate the initial crack fatigue life the rain flow method is applied to simplify the loading history. The simplified strain spectrum of the pressure hull under the loading history, is calculated using the finite element method.

The crack initiation fatigue life at the points at the various angle  $\varphi$  from the opening is estimated. Angle  $\varphi$  is measured in degrees from 0° to 180°. The following seven location angles are considered.  $\varphi=0^\circ$ , 30°, 60°, 90°, 120°, 150°, 180°.

The points on the boundary of the opening are A, B, C, D, E, F, G. (shown in Fig. 5).

Table 3 lists the upper and lower bounds on the equivalent strain ( $\epsilon_{eq}$ ) spectrum and the equivalent strains ( $\Delta\epsilon_{eq}$ ) at points A ( $\phi$ =0°), B( $\phi$ =30°), C( $\phi$ =60°), D( $\phi$ =90°), E( $\phi$ =120°), F( $\phi$ =150°), G( $\phi$ =180°). Table 3 shows the maximum upper bound and the minimum lower bound on the equivalent strain spectrum ( $\epsilon_{eq}$ ) at point F—4.58×10<sup>-4</sup> and -4.58×10<sup>-7</sup>, respectively. The maximum upper bound on the equivalent strain range ( $\Delta\epsilon_{eq}$ ) at the point F is determined as 9.16×10<sup>-4</sup>. The minimum lower bound on the equivalent strain range ( $\Delta\epsilon_{eq}$ ) at point C is 2.54×10<sup>-4</sup>.

#### 3.2.8 Results and discussion

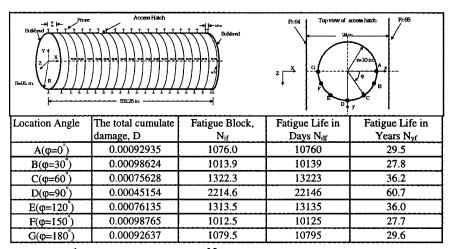
Table 4 and Fig. 8 show results of the analysis, the total cumulative damage and fatigue life at the points around the opening on the pressure hull.

Table 3: Equivalent strain ( $\varepsilon_{eq}$ ) spectrum and equivalent strain amplitude ( $\Delta \varepsilon_{eq}$ ) of each axis strain spectrum at various location angles.

Prince  Prince	Access links	2 \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \	of access tatch  Pt.65  24,m  Pt.65
Location angle	The upper and lower bound of the strain	$\epsilon_{eq}$	Δε <sub>eq</sub>
A(φ=0°)	The upper bound	4.29×10 <sup>-4</sup>	8.58×10 <sup>-4</sup>
	The lower bound	-4.29×10 <sup>-4</sup>	3.24×10 <sup>-4</sup>
B(φ=30°)	The upper bound	4.57×10 <sup>-4</sup>	9.14×10 <sup>-4</sup>
	The lower bound	-4.57×10 <sup>-4</sup>	3.42×10 <sup>-4</sup>
C(φ=60°)	The upper bound	3.38×10 <sup>-4</sup>	6.76×10 <sup>-4</sup>
	The lower bound	-3.38×10 <sup>-4</sup>	2.54×10 <sup>-4</sup>
D(φ=90°)	The upper bound	1.32×10 <sup>-4</sup>	2.64×10 <sup>-4</sup>
	The lower bound	-1.32×10 <sup>-4</sup>	9.88×10 <sup>-5</sup>
E(φ=120°)	The upper bound	3.41×10 <sup>-4</sup>	6.82×10 <sup>-4</sup>
-(7 /	The lower bound	-3.41×10 <sup>-4</sup>	2.56×10 <sup>-4</sup>
F(φ=150°)	The upper bound	4.58×10 <sup>-4</sup>	9.16×10 <sup>-4</sup>
1(4-150)	The lower bound	-4.58×10 <sup>-4</sup>	3.44×10 <sup>-4</sup>
G(φ=180°)	The upper bound	4.28×10 <sup>-4</sup>	8.56×10 <sup>-4</sup>
Δ(Ψ-100)	The lower bound	-4.28×10 <sup>-4</sup>	3.20×10 <sup>-4</sup>



Table 4: Total cumulate damage, D and the fatigue life of the each axis strain spectrum at various location angles.



Note:  $N_{if} = \frac{1}{D}$  ,  $N_{df} = 10N_{if}$  ,  $N_{yf} = \frac{N_{df}}{365}$ 

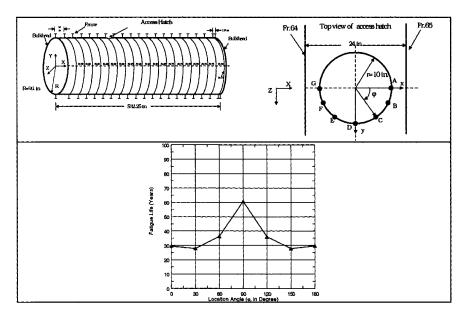


Figure 8: Initial crack fatigue life at location angle, φ in the area around the opening in the afterward storage compartment.

The results in Table 4 and Fig. 8 show the following.

- 1. The total cumulative damage at points around the opening on the pressure hull is from  $4.51\times10^{-4}$  to  $9.87\times10^{-4}$ . The maximum cumulative damage at point  $F(\phi=150^{\circ})$  is  $9.87\times10^{-4}$ . The minimum cumulative damage at point  $D(\phi=90^{\circ})$  is  $4.51\times10^{-4}$ .
- 2. The fatigue life around the opening in the pressure hull is symmetrical about the angle  $\phi$ =90°. For examples, the fatigue life at point A equals that at point G, and so on.
- 3. In the diving history, the load experienced during ten days of diving is a single loading block. The history includes five voyages. The maximum damage is at point  $F(\phi=150^{\circ})$  where the fatigue life is 1012.5 blocks or 10125 days, about 27.7 years.
- 4. The fatigue life of 27.7 years assumes that the submarine operates continuously during the diving history. If the submarine employed is operated for six months and in port of six months in a year, then its fatigue life becomes 55.4 years.

#### 4 Conclusions

Results of this study support the following conclusions.

- 1. The high stress area of the Guppy like class submarine is in the after storage battery compartment.
- 2. The fatigue life around the opening on the pressure hull is symmetrical about the location angle  $\omega=90^{\circ}$ .
- 3. Ten days of diving is specified as one loading block. The history included five voyages. The fatigue life of the opening in the after storage battery compartment in the Guppy like class submarine pressure hull is 1012.5 blocks or about 27.7 years.
- 4. The fatigue life of 27.7 years assume that the submarine is operated continuously. If the submarine is operated for six months and is six months in ports throughout the years, then the fatigue life of the Guppy like class submarine becomes to 55.4 years.

#### References

- [1] Coffin, L. F., A Study of Effects of Cyclic Thermal Stresses on a Ductile Metal, Transactions of the American Society of Mechanical Engineers, 76, pp. 931-950, 1954.
- [2] Manson, S. S., Behavior of Materials under Conditions of Thermal Stress, National Advisory Commission on Aeronautics, Report 1170, Cleveland: Lewis Flight Propulsion Laboratory, 1954.
- [3] Dunham, F. W., Fatigue Testing of Large-Scale Models of Submarine Structural Details, Marine Technology, pp. 299-307, 1965.
- [4] Riggs, R. P. Fatigue Considerations for Semisubmersible Structures, Marine Technology, 16(1), pp. 49-62, 1979.

- [5] Kilpatrick, The Fatigue Characteristic of Submarine Structures Subjected to External Pressure Cycling, In: Advance in Marine Structures, pp. 305-324,1986.
- [6] Pawtucket, N. J., ABAQUS Theory Manual, Hibbitt, Karlsson & Sorensen, (1997).
- [7] Palmgren, A., Die Lebanstaver von Kugellagern, ZVDI, 68, 339, 1924.
- [8] Miner, M. A., Cumulative Damage in Fatigue, Journal of Applied Mechanics, 12, A-159, 1945.
- [9] American Society of Mechanical Engineers, Cases of the ASME Boiler and Pressure Vessel Code, Case N-47-12, ASME, New York, 1977.
- [10] Rice, R. C., Leis, B. N., Drew V. N., Berns, H. D., Lingenfelser, D., Mitchell, M. R., Fatigue Design Handbook, 2<sup>nd</sup> Edition, SAE Fatigue Design and Evaluation Technical Committee, 1988.