A study of wing-body aerodynamic interference by the method of singularities

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Abstract

The development of any new design requires an extensive use of resources. During the preliminary design phase the changes of shape and subsequent analysis must be performed as fast as possible. The method of singularities, or panel method is still the fastest three-dimensional method of solution for potential flows and therefore is still widely used in the aerospace industry. This paper describes a robust doublet based method that accurately describes the flow around the junction region. It allows the imbrication (intersection) of the separate components while computing the exact flow solution correspondent to the external surface. The intensity of all internal panels are automatically zero by the use of the doublets. To demonstrate its application the method is applied to compute wing-body interference coefficients for typical missile configurations in supersonic flow. The results are compared with previous analytical methods and present an excellent match. The use of this technique allows the computation of the interference coefficients for any arbitrary combination of body shape and fins.

1 Introduction

One of the basic objectives of Aerodynamics, as applied science, is to accurately predict the pressure distribution on the surface of any arbitrary body, in motion through a fluid media. To accomplish such complicated task, the models used have to undergo several simplifications or the geometries analyzed must be very simple. Such balance has been altered due to increasingly improvements on computational hardware and algorithms,
which allow more realistic solutions. Despite the evolution of Computational Fluid Dynamics, Boundary Element Methods, dealing with Potential Flow, are computationally unbeatable and still very useful in full three-dimensional analysis of aerospace vehicles, observing, of course, the limitations of the potential flow hypothesis.

One of the most time consuming tasks in modeling the geometry of any aerodynamic shape resides the junction between wing and body. A suitable boundary-element method, like the method of singularities requires a careful discretization of the junctions. The proximity of the panels can promote the ill-conditioning of the solution matrix and therefore spurious solutions.

The lack of detail of the preliminary configuration can inaccurately predict the aerodynamic interaction between wing and body. Such effect can be of the same order of magnitude of the isolated aerodynamic coefficients, particularly in the case of low aspect ratio wings such as missile fins.

The work of Hess and his co-workers [1, 2, 3, 4, 5] is the basis for the use of boundary-element methods in fluid dynamics. In several of his work the problem of junction, where the distance between the control points of the panels is smaller than the smallest dimension of the panel, which give rise to spurious solutions due to the ill-conditioning of the matrix problem.

The proposed alternative, to increase the efficiency of the geometrical definition of the wing-body junction, is to compute directly the interference effects by merely imbricating or intersecting the wing into the body. Such approach will only be valid if the intensity of the internal panels is zero, therefore only the remaining external surface would be active.

Such alternative was proposed initially by Ryan and Morchoisne [6]. The use of doublet, or dipole, singularities which automatically produce zero flow inside the body, allows the direct imbrication of independent surfaces. This problem was also explored by the author [7] and the major results will be presented in this paper. The doublet method used is similar to the one presented by Hunt [8].

2 Theoretical Formulation

Considering the incompressible and irrotational flow of a perfect fluid, continuity is given by:

$$\nabla \cdot V = 0 \quad (1)$$

Also since the flow irrotational:

$$\nabla \times V = 0 \quad (2)$$

There exists a scalar potential which gradient $\nabla \Phi = V$, therefore resulting in Laplace’s Equation:

$$\nabla^2 \Phi = 0 \quad (3)$$
To determine the solution of eq. (3) boundary conditions must be enforced to make the solution unique. There are three possibilities: potential specified (Dirichlet), velocity specified (Neumann) and potential and velocity specified at non-coincident regions of the same boundary (Robin or Poincaré).

The Method of singularities consists on superimposing elementary solutions of Laplace’s Equation, such that the integral of the influence of all singularities satisfies the boundary conditions [9].

Green’s Second Theorem states that for two scalar functions $U$ and $W$ the following relation is valid:

$$
\iint_{\Omega} (U \cdot \nabla^2 W - W \cdot \nabla^2 U) d\Omega = - \iint_{\Gamma} (W n \cdot \nabla U - U n \cdot \nabla W) dS
$$

(4)

Replacing $U$ by the velocity potential $\Phi$ and $W$ by an elementary tridimensional solution for Laplace’s Equation, $W = \frac{1}{r}$, where $r$ is the distance between the singularity and any point inside the domain $\Omega$, bounded by surface $S$, results:

$$
k\Phi P = \iint_{S} \left( \frac{1}{r} n \cdot \nabla \Phi - \Phi \cdot \nabla \frac{1}{r} \right) dS
$$

(5)

where, $k = 4\pi$ for $P$ inside the domain, $k = 2\pi$ for $P$ on the surface and $k = 0$ for $P$ outside the domain.

For multiple domains equation (5) can be written as the summation for each individual domain. Considering also the influence of the non-disturbed flow it can be written as:

$$
k\Phi P = 4\pi \Phi_{\infty} + \sum_{i=1}^{N} \iint_{S_{ij}} [K_{1} n_{i} \cdot (\nabla \Phi_{i} - \nabla \Phi_{j}) - K_{2} (\Phi_{i} - \Phi_{j})] dS
$$

(6)

where $K_{1} = -\frac{1}{r}$ and $K_{2} = n_{i} \cdot \nabla \left( \frac{1}{r} \right)$ which are scalar kernel functions, which depend only on geometry and therefore can be computed independently from any flow or boundary condition.

Analysis of equation reveals that potential at an arbitrary $P$ point depends on jumps of potential and and normal velocities across each of the frontiers defining each sub-domain.

Considering now Imbricated domains, as shows fig.1, the equations can be rewritten, expanding the summation.

$$
k\Phi P = 4\pi \Phi_{\infty} + \iint_{S_{01}} [K_{1} n_{0} \cdot (\nabla \Phi_{0} \nabla \Phi_{1}) - K_{2} (\Phi_{0} - \Phi_{1})] dS + \iint_{S_{02}} [K_{1} n_{0} \cdot (\nabla \Phi_{0} - \nabla \Phi_{2}) - K_{2} (\Phi_{0} - \Phi_{2})] dS + \iint_{S_{13}} [K_{1} n_{1} \cdot (\nabla \Phi_{1} - \nabla \Phi_{3}) - K_{2} (\Phi_{1} - \Phi_{3})] dS + \iint_{S_{23}} [K_{1} n_{2} \cdot (\nabla \Phi_{2} - \nabla \Phi_{3}) - K_{2} (\Phi_{2} - \Phi_{3})] dS
$$

(7)
If the potential inside the sub-domains $\Omega_1$, $\Omega_2$ and $\Omega_3$ is continuously equal to $\Omega_I$, equation (7) becomes:

$$k\Phi_P = 4\pi \Phi_\infty + \int_{S_{01}} [K_1 n_0 \cdot (\nabla \Phi_0 \nabla \Phi_I) - K_2 (\Phi_0 - \Phi_I)] dS + \int_{S_{02}} [K_1 n_0 \cdot (\nabla \Phi_0 - \nabla \Phi_I) - K_2 (\Phi_0 - \Phi_I)] dS$$

Figure 1: Imbricated domains

which corresponds to the same equation for a single domain, bounded by surface $S_{0I} = S_{01} + S_{02}$, as illustrates fig.2.

Figure 2: Resulting domain
The use of a continuous internal potential, as shows eq.(8), allows the aerodynamic analysis of the external surface, by imbricating several bodies. This allows the use of a single method and no special care of the junction between each of those bodies. Such feature represents an enormous advantage in speeding up the geometric definition of aeronautical vehicles.

3 Wing-Body Aerodynamic Interference

An aerodynamic configuration, like an aircraft or missile, can be considered as a composition of the aerodynamic effects of several elements like wing, body, tail plus an interference effect, that is the aerodynamic interaction of each of these parts.

Consider the geometry presented on fig.3, a slender missile in supersonic flow.

\[ \left( M_{\infty}^2 - 1 \right) \frac{\partial^2 \Phi}{\partial x^2} - \frac{\partial^2 \Phi}{\partial y^2} - \frac{\partial^2 \Phi}{\partial z^2} = 0 \]  
\[ \left( M_{\infty}^2 - 1 \right) \frac{L^2}{l^2} \frac{\partial^2 \Phi}{\partial \bar{x}^2} - \frac{\partial^2 \Phi}{\partial \bar{y}^2} - \frac{\partial^2 \Phi}{\partial \bar{z}^2} = 0 \]

where \( \bar{x} = \frac{x}{L} \), \( \bar{y} = \frac{y}{l} \) and \( \bar{z} = \frac{z}{l} \). \( L \) and \( l \) are characteristic lengths along the longitudinal and transversal directions. Therefore for an slender body, \( \frac{L}{l^2} \ll 1 \), eq.(10) becomes the Laplace's equation in the transversal plane:
The classical solution for this problem and geometry is found in several references, [10, 11, 12, 13]. The interference coefficient, $K_w$, which is the ratio between the lift of the wing in the presence of the body and the lift of the isolated wing, is given by:

$$
K_w = \frac{1}{\pi(\lambda - 1)^2} \left[ \frac{\pi}{2} \left( \frac{\lambda^2 - 1}{\lambda} \right)^2 + \left( \frac{\lambda^2 + 1}{\lambda} \right) \arcsin \left( \frac{(\lambda^2 - 1)}{(\lambda^2 + 1)} \right) - \frac{2(\lambda^2 - 1)}{\lambda} \right]
$$

(12)

where $\lambda = \frac{s}{a}$ is the wing span ($s$) over body radius ($a$) ratio.

Consequently, $K_b$, the ratio between the lift of the body in the presence of the wing and the lift of the isolated wing, is given by:

$$
K_b = (1 + \lambda)^2 - K_w
$$

(13)

Table 1. Interference Coefficients - Theoretical Values

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>0.0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_w$</td>
<td>1.000</td>
<td>1.162</td>
<td>1.349</td>
<td>1.555</td>
<td>1.774</td>
<td>2.000</td>
</tr>
<tr>
<td>$K_b$</td>
<td>0.000</td>
<td>0.278</td>
<td>0.611</td>
<td>1.005</td>
<td>1.467</td>
<td>2.000</td>
</tr>
</tbody>
</table>

4 Numerical Solution using Prandtl’s Lifting Line Theory

To evaluate the imbrication technique in the calculation of interference coefficients, the method of singularities will be numerically implemented (panel method) using the transversal geometry presented in fig.4.

The use of surface distribution of doublets, like in fig.4, corresponds to pointwise distribution of vortexes, similar to Prandtl’s lifting line model, therefore the lift of the wing can be written using, the Kutta-Joukowski expression:

$$
L_w = \rho(\alpha V_\infty) \sum_{i=1}^{N} \Gamma_i \Delta y_i
$$

(14)

where the circulation, $\Gamma_i = \mu_i - \mu_{i+1}$, is given by the difference between neighbouring doublet intensities. Therefore the interference coefficient $K_w$ is given by:

$$
K_w = \frac{L_w(b)}{L_w} = \frac{\left[ \sum_{i=1}^{N} \Gamma_i \Delta y_i \right]_{w(b)}}{\left[ \sum_{i=1}^{N} \Gamma_i \Delta y_i \right]_{w}}
$$

(15)
A total of 104 panel were used on the wing, which thickness was limited to 0.5%. On the body 128 panels were used. The results for the interference coefficients are found in Table 2.

![Figure 4 - Geometrical model for the numerical analysis](image)

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>0.0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_w$</td>
<td>1.000</td>
<td>1.161</td>
<td>1.347</td>
<td>1.552</td>
<td>1.781</td>
<td>2.000</td>
</tr>
<tr>
<td>$K_b$</td>
<td>0.000</td>
<td>0.279</td>
<td>0.613</td>
<td>1.008</td>
<td>1.459</td>
<td>2.000</td>
</tr>
</tbody>
</table>

The maximum relative error between analytical and numerical values is 0.7%. Such level is considered very satisfactory, verifying the accuracy of the imbrication procedure.

5 Conclusions

The claims of the imbrication technique are verified, and the numerical procedure allows the evaluation of interference coefficients for any cross section of an slender configuration in supersonic flow. Research proceeds in incorporating this technique to existing panel methods, allowing a faster geometrical definition of aeronautical vehicles, during the preliminary design phase.

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Bibliography


