A new software tool for analysis and design of induction machines

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1 Introduction

The guidelines of the research in the field of software tools for electrical machines oriented to the design and to the analysis, in last years, have followed very often divergent aims. In fact for the design of standard or new machines numerical codes, on f.e.m. analysis based, are adopted, especially for magnetostatic or eddy currents analysis of preliminary or final arrangement of magnetic path of actuators. In this case, starting from possible solutions proposed by specialists of electrical machines design, these arrangements usually are processed by using different commercial codes in order to verify the magnetic and electrical stress or the localisation of Joule losses in massive iron core. These indications suggest possible modification of magnetic structure or windings shape and, by a step by step procedure, the optimisation of the design can be reached. These optimisation regards, mainly, only static operation conditions or, in the case of frequency analysis, particular steady state operations. The complexity of commercial pre-processors, oriented to the definition of suitable mesh, constrains the designer to limit the possible test or to invoice other specialists. The trend in last years is the availability of user friendly pre-processor codes that can be adopted directly by designers. However the information that can be deduced by these tools does not involve the achievement of electrical and mechanical characteristics of actuators but only indication about a good exploitation of materials. The achievement of the electrical and mechanical behaviour of the machine can be get after an identification phase of the actuators by means of
mathematical model of machine and experimental test. The evaluation of the parameters used in the model could be very complex, especially for new geometry motors, and, in this case, it would be necessary a full validation test of the model. In this field many model have been proposed by different Authors: the technical literature is full of different proposal for all the machines that have interest in applications.

The phase of analysis is quite separated from the phase of design because need a different approach in the use of software tools. For this reason integrated tools that permit the quick analysis of the characteristic of a machine, in particular operated conditions too, starting from the geometrical structure of the actuator, from the properties of materials and from the knowledge of the load and the electrical supply, are rather unusual.

This is the reason that has induced the Authors, from many years involved in the field of the research of mathematical models of electrical machines, into the development of suitable tools that give the right solution to the problems generally met in the design of a new geometry machine or in the modification of the structure of build machines. Furthermore the availability of a flexible mathematical model that can be easily used on standard processors by no expert people is a suitable tools in different analysis of electrical machines. For example the knowledge of the electrical and mechanical behaviour of an actuator in particular no standard conditions (as faults, asymmetrical arrangement of active conductors, missing of phases and so on) allows to collect suitable data for the arrangement of an expert system able to identify the running operations during diagnostic activities.

This tool would be really user friendly if full interactive procedures are involved.

The first realisation of this tool, that in the paper is presented, is oriented to multiphase induction machines. The mathematical background is summarised in the first paragraph. In the second one the different drives of the procedure are presented and some optimisation criteria are sketched. Examples are at least shown in order to demonstrate the possible use of the tool and the fields of application.

2 Mathematical background

The machine model adopted in this note uses the Window formulation of the electric and magnetic quantities in the air-gap.

As in previous papers shown \(^1,2,3\), in the case of cylindrical geometry of machine surfaces, this method bases itself on a formulation of air-gap field distribution by means of the superimposition of the most elementary components of m.m.f. The elementary component, that represents the quantum of all the air-gap magnetic quantities that are present in the electrical machines, is the unitary Window function.

For the application of the proposed mathematical model the following simplificative assumptions are considered:
the air-gap g is constant and the fields have only a radial component;
the value of air-gap is so small with respect to the machine diameter that the radial components of the field are practically constant on the air-gap line;
the behaviour of iron is linear with $\mu_r=\infty$;
the dimension of the slots is supposed to be infinitesimal, so that the primary winding can be represented, in the case of single layer winding, by means of an impulsive distribution of currents density:

$$\Theta_s = \sum_{i=1}^{m} \sum_{h=1}^{q_h} \pm z_h \delta (\gamma - \gamma_{k,h}) i_h$$  \hspace{1cm} (1)

where $z_h$ is the number of conductors in each slot, $q_h$ the number of the slots for each phase, $m$ the number of the phases, $\gamma_{k,h}$ the angular abscissa, in the stator frame, of a single slot, (see fig. 1); $i_h$ is the current in the $h$-th phase and $\delta(.)$ is the Dirac's function*.

the width of the bar is supposed to be infinitesimal, so that the rotor winding can be represented by means of an impulsive distribution of bar currents density (see fig. 1):

$$\Theta_r = \sum_{i=1}^{s} \pm \delta (\rho - \rho_{k}) i_{r,k}$$  \hspace{1cm} (2)

Figure 1: The considered structure of stator and rotor conductors.

According to these simplificative assumptions the Maxwell equation of the air-gap flux density can be written as:

$$\frac{\partial}{\partial \gamma} [gB(\gamma,t)] = \mu_0 [\Theta_s(\gamma,t) + \Theta_r(\gamma,t)]$$  \hspace{1cm} (3)

To this equation it is necessary to associate the boundary condition:

\[ \theta(t) = \theta_0 + \int_0^t \omega(\tau) d\tau \]
By considering an asymmetrical rotor cage as represented in Fig. 2 and the external rings as equipotential sides, it is possible to associate to each mash (from 0 to s-2) a current J.

The equation (3), can be treated using the Window method, by referring all the quantities to the stator reference frame. If a generic distribution for the stator winding and for the rotor bars is considered, the expression of the air-gap resultant field will be:

\[ B(\gamma, t) = B_s(\gamma, t) + B_r(\gamma, t) \]  

where:

\[ B_s(\gamma, t) = \frac{\mu_0}{g} \left\{ \sum_{k=1}^{m} N_{h_k} i_k(t) \sum_{s=1}^{q/2} W(x_s, x_s, \gamma) \left[ \frac{H}{2} \right] \right\} \]  

\[ B_r(\gamma, t) = \frac{\mu_0}{g} \sum_{r=0}^{s-2} J_r(t) \left\{ W(x + \theta(r) + 2\pi(v - 1)r, x_r, \gamma) + \frac{x_r}{2\pi} W(\pi + 2\pi(v - 1), 2\pi, \gamma) \right\} \]

The expressions of the voltage balance and of the thrust will be:

stator voltage balance:

\[ v_h(t) = \left( R_h + L_h \frac{d}{dt} \right) i_h(t) - e_h \]
rotor voltage balance for the $\zeta$-th rotor mesh:

$$e_s = -R_{\zeta} J_{\zeta-1} + \left( R_s + R_{\zeta+1} \right) J_{\zeta} - R_{\zeta+1} J_{\zeta+1} +$$

$$- L_{\zeta} \frac{d}{dt} J_{\zeta-1} + \left( L_{\zeta} + L_{\zeta+1} \right) \frac{d}{dt} J_{\zeta} - L_{\zeta+1} \frac{d}{dt} J_{\zeta+1}$$

(7)

torque expression:

$$T(t) = i \frac{D}{2} \sum_{\nu} I_{\nu}(t) B_s(\gamma_{\nu}, t)$$

(8)

where $R_h$ and $L_h$ are, respectively, the resistance and the leakage inductance of stator phase, $R_{\zeta}$ and $L_{\zeta}$ are the resistance and the leakage inductance of the $\zeta$-th rotor bar, $\gamma_{\nu}$ represents the angular abscissa of the $\nu$-th rotor bar at generic instant $t$ and:

$$e_h = -N_h l \frac{D}{2} \frac{d}{dt} \sum_{\nu} I_{\nu}(t) \int_{\gamma}^{\gamma+\phi(t)} B(\gamma, t) d\gamma$$

(9)

$$e_s = -l \frac{D}{2} \frac{d}{dt} \int_{\gamma}^{\gamma+\phi(t)} B(\gamma, t) d\gamma$$

(10)

represent the e.m.f. in the $h$-th phase of stator and in the $\zeta$-th rotor mesh.

The resulting differential equations systems has, as unknowns, the instantaneous values of phases current (stator) and of mesh currents (rotor). It is possible to note as this approach is particularly suitable in the case of loss of geometrical or electrical symmetry in the stator windings or in the rotor cage and it is really convenient because the proposed technique approaches and solves any problem directly in the time domain if a proper software tool is setup.

3 The MADES tool for the analysis and the design of induction machines

The mathematical model summarised in previous section has found its easy implementation into a proper tool that allows a friendly utilisation both by no expert people and by specialist. This software code has been called MADES (Machine Analysis and DESign). MADES is an interactive tool that allows to cover a wide variety of applications: in design phase activity, in the computation of performances in steady state or in transient operations, in test
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conditions. The proposed software is equipped by many procedures that makes easier its uses, as:

- a modelling and data analysis support;
- a visualisation system for functions and data;
- a procedure to create interactive documents;
- an embedded system that allows to call MADES from other programs;

The source code of the MADES has been written in Matlab® language, therefore MADES can use all the performances and the facilities of this mathematical program.

The system software is divided into two parts: the kernel and the front end. The kernel actually performs computations; the front end handles interaction with the user and it is optimised for particular computers and graphical interfaces. The MADES front end incorporates a graphical interface for the input of geometrical data of the structure and for the results presentation. By means of this interface it is possible:

- to give symbolic representations of geometrical object by using particular primitives computing aided design;
- to input all the data of the electrical machines (for example the positions of the stator windings and of the rotor bars, the length of the air-gap, the properties of the materials and so on);

The main steps of the MADES kernel and its functions are resumed as following:

- Input machine description:
  Converter the graphical description of stator areas and electrical connections given by the front end;
  identification of rotor areas and materials;
- Precode matrix generation:
  set up of initial conditions;
  generation of constant matrices depending on geometrical or electrical parameters;
- Matrix code generator:
  automatic generation of the Dynamic Connection Matrix (DCM);
  By DCM it is possible to obtain any information about the reconstruction of air-gap function and the configurations of the rotor surfaces in front of stator belts with respect to different angular positions during the rotation.
- Unknowns Evaluation Routine:
  Calculates, for each step, all unknowns as phase currents, torque and so on.

A flow-chart representation of these steps is represented in fig. 3.
The Matrix code generation is the main routine of the kernel procedure. In order to explain the construction of the DCM is necessary the consideration of the eqs (9),(10). These eqs, by considering separately the interaction stator-stator and stator-rotor, can be written as follow:

\[ e_{h}^{(S,S)}(t) = e_{h}^{(S,S)}(t) + e_{h}^{(R,S)}(t) \] (11)

\[ e_{c}^{(R,R)}(t) = e_{c}^{(R,R)}(t) + e_{c}^{(S,R)}(t) \] (12)

with:

\[ e_{c}^{(R,R)} = -X_{c}J_{c} \frac{D}{2} \mu_{0} \frac{d}{dt} \left( J_{c}(t) \left( 1 - \frac{X_{c}}{2\pi} \right) - \sum_{r=0}^{r=2} J_{c}(t) \frac{X_{c}}{2\pi} \right) \] (13)

\[ e_{h}^{(S,S)} = -\frac{D}{2} g l \sum_{k=1}^{n} N_{h,k} N_{h,k} \lambda_{k,h} \frac{d}{dt} i_{k}(t) \] (14)

\[ e_{e}^{(R,S)} = W_{h,c}(t) \cdot [A_{e}(t)]_{(1,4)} \cdot [I]_{(x,1)} \]

\[ e_{c}^{(S,R)} = W_{c,h}(t) \cdot [A_{c}(t)]_{(1,m)} \cdot [I]_{(m,1)} \] (15)
where:

\[ l = \begin{bmatrix} \frac{d_i(t)}{dt} & \ldots & \frac{d_i(t)}{dt} \end{bmatrix} \]

\[ J = \begin{bmatrix} \frac{d_j(t)}{dt} & \ldots & \frac{d_j(t)}{dt} \end{bmatrix} \]

\( \lambda_{kh} \) is a constant, \([A_s(t)], [A_r(t)]\) are time function matrices depending on the position of the elementary coil with respect to the flux density distribution; \([W_z, h], [W_{h, \zeta}]\) are the Window matrices that have all the elements equal zero or equal one according to the presence of a coil in front of the considered flux density window. The DCM matrix has for elements the matrices \([W(.)]\).

### 4 Simulation results

The MADES tool is applied to analyse the performances of an induction motor whose parameters are reported in tab. 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>V</td>
<td>380 [V]</td>
</tr>
<tr>
<td>f</td>
<td>50 [Hz]</td>
</tr>
<tr>
<td>(X_m)</td>
<td>74 [Ω]</td>
</tr>
<tr>
<td>(X_s)</td>
<td>2.91 [Ω]</td>
</tr>
<tr>
<td>(R_s)</td>
<td>1.078 [Ω]</td>
</tr>
<tr>
<td>(R_r)</td>
<td>0.898 [Ω]</td>
</tr>
<tr>
<td>(J_r)</td>
<td>0.07 [kgm²]</td>
</tr>
</tbody>
</table>

In particular the behaviour of a phase current, of the torque and of the slip relevant to the inertial starting transient are reported. This case is an interesting test in order to prove the performances of the proposed software for the complex electromagnetic interaction that involves the machine completely.

![Figure 4: Torque vs slip](image)

![Figure 5: Slip vs time](image)
The trends of the considered quantities is just coincident with the trends get by using traditional model referred to Fourier's series expansion. The test has been developed by considering other characteristic trends of the induction machine too (as slip torque load during different steady state operations) in order to verify the quality of the MADES in all the cases where the results are known. For the complexity of the whole procedure the runtime spent by this application, for this easy analysis, is surely higher than the runtime spent by the traditional approach. However this runtime is not depending by the symmetry of the machine as it occurs for the models that consider the Fourier's series. By Window model, in fact, the number of equations and unknown are depending on the number of slots and bars and are not depending by the particular symmetry conditions. For this reason MADES is quite suitable in all that cases where a new particular geometry of machine, a particular no symmetry condition or fault have to be analysed.

5 List of symbols

\[
\begin{align*}
\mu & \quad \text{permeability} \\
\chi & \quad \text{central abscissa of a generic window function} \\
c & \quad \text{generalised first integration limit} \\
d & \quad \text{generalised second integration limit} \\
X & \quad \text{generalised range of window function} \\
p & \quad \text{pole pairs} \\
D & \quad \text{Diameter} \\
f & \quad \text{supply frequency} \\
X_m & \quad \text{magnetizing reactance per phase} \\
X_s & \quad \text{stator leakage reactance per phase} \\
X_r & \quad \text{rotor leakage reactance per phase} \\
J_r & \quad \text{moment of inertia of the rotor} \\
V & \quad \text{r.m.s. value of the line voltage} \\
R_r & \quad \text{rotor phase resistance}
\end{align*}
\]
References


4. Schartz, *Theories des Distributions*, Hermann - Cie, Paris vol. 1,2; 1957