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IME: a general method to analyse linear systems and electric circuits

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Abstract

This paper is intended to show a general method that relies on the superposition principle to analyse linear systems and electric circuits, based on hierarchical sequences of more simple topologies with some inhibited elements and on a short recollecting logic to calculate the final solution from the previous partial and more elementary solutions.
Sections 1, 2 and 3 will mention the origin of the method, why it is general and some its global features, for which there will probably be not enough space to explain in detail, like previous partial calculations reuse.
Section 4 will expose the non-disciplinary theory of the method. This originally anonymous theoretical formulation can be particularised to other field of interest by specific formulations, and due to its intrinsic symbolic constitution this method can be directly used to get symbolic solutions from the system which it is being applied to, or even be re-formulated in different algorithmic ways to optimise some aspects, like memory or time consumption. Examples on this matter will be given in a separate companion paper.

1 Introduction

The Inhibition Method (IME) is an iterative exact non-inverting method to solve any linear system, derived from the Cross method, which is, on the contrary, a non-exact method and from which IME inherits the fundamental characteristic of decomposing the problem into easier to solve sub-problems. It produces a hierarchical sequence of sub-systems [1], at the end of which only

elementary systems can be found. Therefore, they can be solved rapidly with the minimum of knowledge, that is, few program code lines are needed.
The problem splitting process is founded, in fact, on the possibility (see Fig. 1) to deduce the properties of the system (a) from two more simple sub-systems (b) and (c), obtained from (a) by suppressing or inhibiting a component.

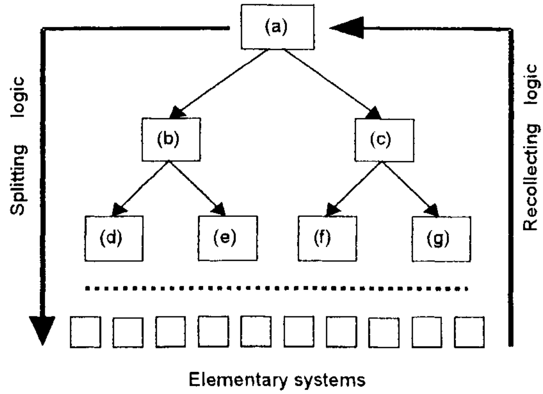


Fig. 1: IME logic.

The same idea can work on the sub-systems (b) and (c), each of which can be deduced from other couples, respectively (d-e) and (f-g), by suppressing other components, and so on. At the end of the splitting process we come to sub-systems so simple that they can be solved immediately. For example, a linear electric network can be repeatedly split into more simple networks with an inhibited branch until they become one-loop circuits. A similar ground basis is used in [2].
The recollecting logic, which will be demonstrated later in detail, is very simple as well. As a matter of fact, if a variable (a) consists of the sum of other three variables (b), (c) and (x),

$$a = b + c + x,$$

where (x) is proportional to (a) through a constant (k),

$$x = k \cdot a,$$

it follows that

$$a = \frac{b + c}{1 - k},$$

which is the symbolic formula that synthesises the link between a generic system (a) and the two sub-systems (b) and (c) derived from it. The demonstration is not difficult either, because it is organised following the "Short Didactics" (SD) criteria [3]: the unifying element of several steps of the proof is

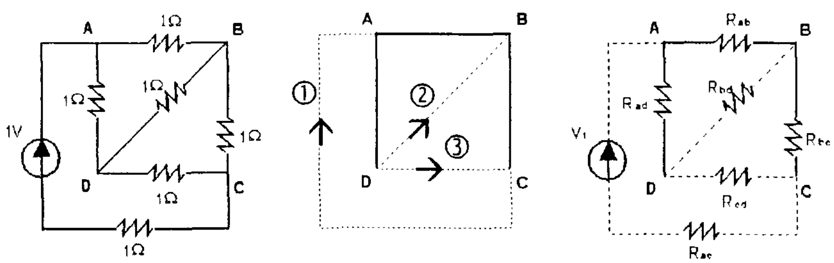
put in evidence and collected to shorten and clarify the proof itself, realising a kind of "internal contraction" [4]
The first IME version [5] was only for numeric calculations and it was based on a laborious algorithm which had been working fine because it allowed one to get around the heavy memory constraints for huge systems to solve. Here, a new version of the method is presented that really becomes a general method with the use of modern software tools.

2 Why "general"

It is a general method for several reasons. It can be applied in every field if the system to solve is linear (or made linear). The following anonymous formulation emphasizes this fact, because there is no reference to the physical nature of the system, thus, "anonymous" stands for inter-disciplinary and indicates a general method all over the linear world.
From this formulation, discipline-specific methods can be derived simply by carrying out the related rules, intrinsic to that particular discipline, as could be seen in one of the next sections, where the electric formulation is given. Furthermore, adding to this some specific knowledge, like the modified nodal analysis, a complete software tool can be realized for electric circuits.
Due to the presence of symbolic processors, like Mathematica™, which can work with much more abstract quantities, the anonymous formulation can be directly transferred to a computer program to make a general tool for linear system solving and, last but not least, to solve it in complete symbolic form, giving a more general character to the solution found. Symbolic programming is a quite recent phase in the evolution of programming but has now reached relevance and importance in engineering [6,7,8,9,10,11].

3 An introductory example

The anonymous formulation is not too complicated, but it could be a little hard



(a): resistive circuit (b): tree, co-tree, loops (c): symbolic names

Fig. 2: Sample circuit.

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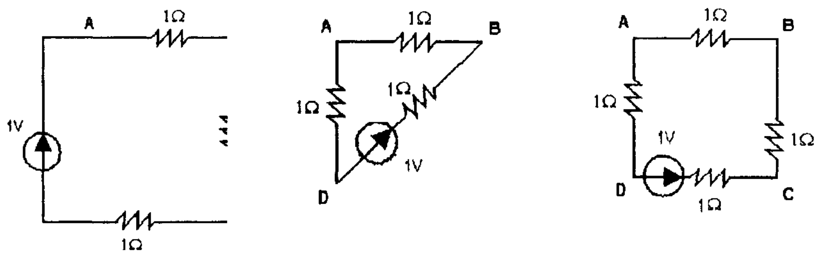


Fig. 3: Elementary one-loop circuits from the sample circuit.

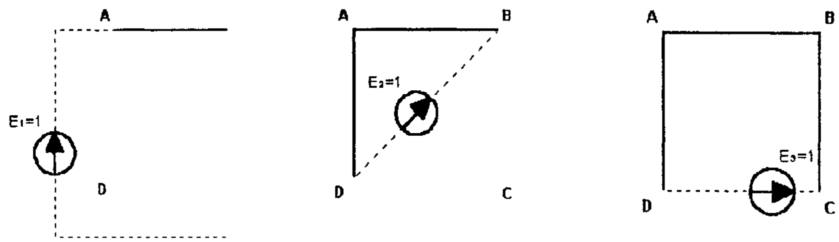
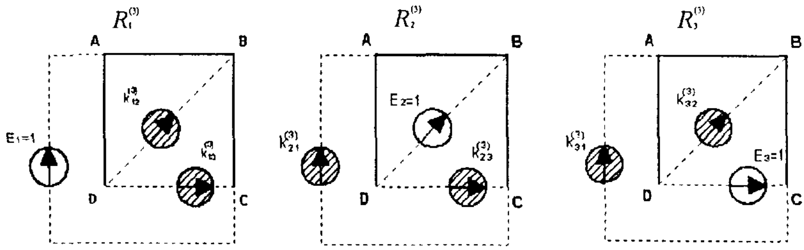


Fig. 4: Simplified schemes for the elementary one-loop circuits.

to understand because it is really abstract. An introductory example should show the method basics in a practical way. Let's consider the very simple resistive circuit of Fig. 2.

This is a 3-loop circuit: a possible tree could be that one depicted in Fig. 2b, with numbered main loops. IME performs a decomposition of the starting circuit into three elementary (one-loop) circuits corresponding to the main loops, shown in Fig. 3, where each loop is supplied with an unitary voltage source in the co-tree branch, regardless of the original supply value. These three elementary circuits are first solved independently as reported (symbolically) in Fig. 4, and then the procedure continues rebuilding the complete solution of the starting circuit, following a recollecting logic. This logic uses "special" quantities to link and deduce the general solution from the different elementary ones: *inhibition quantities*. In this case, as depicted in Fig. 5 with slashed greyed symbols, inhibition quantities are voltage sources. Their values have to be calculated to get just zero current in the branch (seat) where they are placed, in order to render the starting circuit a virtual one-loop circuit, that is, a three-loop circuit with two co-tree branches inhibited (Fig. 3 and Fig. 5 compared). This is called *inhibition level 3*. The next generation of circuits (*inhibition level 2*) basically consists of the same three-loop circuit with only one co-tree branch inhibited, as shown in Fig. 6. The solution of these circuits is deduced from level 3 via *Inhibition Theorem* (see Introduction or following Section 4.3). In the same way, level 1, which is the final level, with no inhibited branches, can be deduced from level 2, giving the final solution for the starting circuit of Fig. 2.

Table 1 and Table 2 report the glossary needed to understand the formulation. In many cases, the word or the definition used in the anonymous formulation is followed on the same row by the corresponding one used in the electrical formulation, both in terms of branch current and node voltage. In this manner it is possible to figure out the abstract formulation with the help of suitable entities taken from the more usual electrical topics.

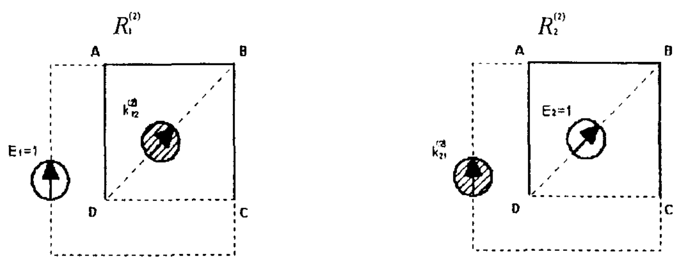


$[I_1]_1^{(3)} = \frac{1}{3}$ $\quad$ $[I_2]_2^{(3)} = \frac{1}{3}$ $\quad$ $[I_3]_3^{(3)} = \frac{1}{4}$

$k_{12}^{(3)} = -R_{ab}\cdot[I_1]_1^{(3)} = -\frac{1}{3}$ $\quad$ $k_{21}^{(3)} = -R_{ab}\cdot[I_2]_2^{(3)} = -\frac{1}{3}$ $\quad$ $k_{31}^{(3)} = -(R_{ab}+R_{bc})\cdot[I_3]_3^{(3)} = -\frac{1}{2}$

$k_{13}^{(3)} = -(R_{ab}+R_{bc})\cdot[I_1]_1^{(3)} = -\frac{2}{3}$ $\quad$ $k_{23}^{(3)} = (R_{ab}+R_{ad})\cdot[I_2]_2^{(3)} = \frac{2}{3}$ $\quad$ $k_{32}^{(3)} = -(R_{ab}+R_{ad})\cdot[I_3]_3^{(3)} = \frac{1}{2}$

Fig. 5: IME elementary regimes of the sample circuit (inhibition level 3).



$k_{12}^{(2)} = [k_{12}^{(3)} - k_{13}^{(3)}\cdot k_{32}^{(3)}]\cdot\alpha^{(3)}$ $\quad$ $k_{21}^{(2)} = [k_{21}^{(3)} - k_{13}^{(3)}\cdot k_{31}^{(3)}]\cdot\alpha^{(3)}$

Fig. 6: Hierarchical deduction of circuits "less inhibited" (inhibition level 2) and sample symbolic calculus via Inhibition Theorem.

4 "Anonymous" formulation

IME can be used in solving any physical linear system. Here we'll talk about linear systems in general, because the intention is to give a more abstract formulation of the method, with no reference to real systems, from which we get the notion of "cause", "effect" and "seat". The word "seat" will indicate the place where some causes act to produce some effects, in general.

To get a more compact dissertation, some other symbolic representations are given in Table 3 to understand all the figures in the following sections.

	Seat	Cause	Effect	Supplied seat	Free seat	Inhibited seat
Anonymous	j	Y_j	X_j	$Y_j \neq 0$	$Y_j = 0$	$Y_j = k \to X_j = 0$
Electrical (branch current)	j branch	E_j voltage source	I_j branch current	$E_j \neq 0$	$E_j = 0$	$E_j = k \to I_j = 0$
Electrical (node voltage)	j node	I_j current source	V_j node voltage	$I_j \neq 0$	$I_j = 0$	$I_j = k \to V_j = 0$
Comment	$j = 1, 2, .. n$, where n is the total number of seats	The cause in the seat j	The effect in the seat j	A seat is *supplied* when both cause and effect are present on it.	A seat is *free* when the cause is not present but the corresp. effect is.	A seat is *inhibited* when a quantity, dimensionally homogeneous with the causes, is present and set to a value so that it cancels the effect in the seat .

Table 1: Fundamental glossary.

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Name and symbol	Anonymous	Electrical (branch current)	Electrical (node voltage)
Regime $R(\cdots)$	State of the system individuated by the indication of the (given) causes, supplied seats, free seats and inhibited seats.	State of the circuit individuated by the set of currents and voltages	
Multi-supplied regime	Regime with more than one supplied seat.	(State of the) circuit with more than one branch having a voltage source	(State of the) circuit with more than one node having a current source
Single-supplied regime	Regime with only one supplied seat.	(State of the) circuit with only one branch having a voltage source	(State of the) circuit with only one node having a current source
Unitary regime	Single-supplied regime where the supplying cause has unitary value.	(State of the) circuit with only one branch having a voltage source set to "one"	(State of the) circuit with only one node having a current source set to "one"
Principal regime of order v $R_j^{(v)}$	Unitary regime supplied in the *j-th* seat, whose free seats are all and only the seats with index greater than v. This means that the first v seats are all inhibited but one, which is unitary supplied.		
Inhibition quantity $k_{jl}^{(v)}$	Considering a principal regime of order v supplied in j, it is the quantity, in the seat l, dimensionally homogeneous with the causes set to a value so that it cancels the effect in the same seat (the effect is not present) (*).	.. it is the voltage source in the branch l, set to the value so that the current is zero in the branch l.	.. it is the current source in the node l, set to the value so that the node voltage is zero.
Inhibition level of order v	The set of all principal regimes of order v.		
Inhibition sequence	The sorted set of all inhibition levels, sorted by decreasing values of the order v.		

(*) This definition is tautological with respect to the inhibited seat, but it is given anyway to underline that the inhibition quantity itself is an effect as well, even if it causes the principal effect cancellation.

Table 2: Fundamental definitions.

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<table>
<tr><td>Generic quantity</td><td>$Q_j^{(v)}$</td><td>It is the value assumed by the quantity Q in the regime of order v, supplied in the j-th seat. It follows the same notation of the regime, as shown in Table 2.</td></tr>
<tr><td>Unitary regime</td><td><table><tr><td>1</td><td>1</td></tr><tr><td>2</td><td>$k_{12}^{(2)}$</td></tr><tr><td>3</td><td></td></tr><tr><td></td><td>$R_1^{(2)}$</td></tr></table></td><td>A regime is depicted by a table: the last row shows the regime name; the other (numbered) rows indicate the seats; inhibited seats have greyed background; free seats are empty and supplied seats have supply value reported.</td></tr>
<tr><td>Non-unitary regime</td><td><table><tr><td>1</td><td>5</td></tr><tr><td>2</td><td>$5\,k_{12}^{(2)}$</td></tr><tr><td>3</td><td></td></tr><tr><td></td><td>$5\,R_1^{(2)}$</td></tr></table></td><td>If, for the previous regime, the supplying value is not set to 1, let's say 5, and then it will be depicted by the table on the left, due to the linearity between causes and effects (it is a multiple of an unitary regime).</td></tr>
</table>

Table 3: Useful representations utilized in the anonymous formulation.

Level 3	Q	Seat	$k_{\_1}$	$k_{\_2}$	$k_{\_3}$
$R_1^{(3)}$	$Q_1^{(3)}$	1		$k_{12}^{(3)}$	$k_{13}^{(3)}$
$R_2^{(3)}$	$Q_2^{(3)}$	2	$k_{21}^{(3)}$		$k_{23}^{(3)}$
$R_3^{(3)}$	$Q_3^{(3)}$	3	$k_{31}^{(3)}$	$k_{32}^{(3)}$	
Level 2					
$R_1^{(2)}$	$Q_1^{(2)}$	1		$k_{12}^{(2)}$	
$R_2^{(2)}$	$Q_2^{(2)}$	2	$k_{21}^{(2)}$		
Level 1					
$R_1^{(1)}$	$Q_1^{(1)}$	1			

Fig. 7: Sample inhibition sequence tableau with symbols.

4.1 Inhibition sequence tableau

This tableau gives a complete view of all principal regimes of the same inhibition sequence and of all effect values in the respective regimes:

- in the first column are indicated all principal regime names, grouped by decreasing inhibition order;
- the column headed with "Seat" reports the supplied seat index for more readability;
- the columns between regime names and seat index contain effect values;
- the columns on the right of the seat index column contain inhibition quantities.

In Fig. 7 it's reported a tableau for a 3-seat system with only one effect quantity.

4.2 ICS operator

ICS stands for Inhibition-Compensation-Separation and indicates a particular kind of decomposition from a given unitary regime into other two, due to the superposition principle.

Let the given unitary regime be $R_j^{\cdot\cdot}$, where "$\cdot\cdot$" has to be understood as non specified inhibited seats.

The first two steps of the ICS operator, applied to a generic free seat *m* are equivalent to add and subtract the correspondent inhibition quantity $k_{jm}^{\cdot\cdot m}$ in the same seat (Fig. 8). The third step consists in separating the two quantities in two different regimes (Fig. 9). The inhibition quantity $k_{jm}^{\cdot\cdot m}$ is left in the original regime (which now becomes $R_j^{\cdot\cdot m}$) and its opposite $-k_{jm}^{\cdot\cdot m}$ now acts as supplying quantity in the seat *m* of a second regime that, superimposed to the first, gives back the original one.

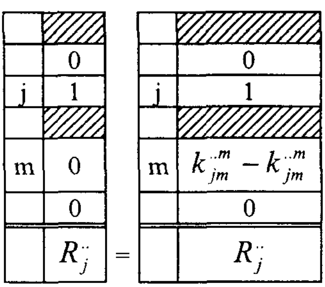


Fig. 8: Adding and subtracting the same inhibition quantity in the seat *m*.

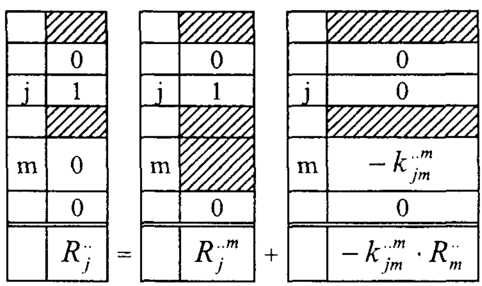


Fig. 9: Separating the two compensating regimes.

4.3 Inhibition theorem

4.3.1 Proof

It is derived from the relationship of Fig. 9 applying the ICS operator again on the last regime. If we inhibit its seat j by adding the inhibition quantity $-k_{jm}^{..m} \cdot k_{mj}^{..j}$ (remembering that this regime is a multiple of $R_m^{..}$ through the factor $-k_{jm}^{..m}$) and if we compensate it with $k_{jm}^{..m} \cdot k_{mj}^{..j}$, which will be then separated and made acting as supply cause in the next regime $k_{jm}^{..m} \cdot k_{mj}^{..j} \cdot R_j^{..}$, we will get the symbolic equation:

$$R_j^{..} = R_j^{..m} - k_{jm}^{..m} \cdot R_m^{..j} + k_{jm}^{..m} \cdot k_{mj}^{..j} \cdot R_j^{..}.$$

The superposition principle allow to arrange this symbolic equation as any other equation, and thus:

$$R_j^{..} - k_{jm}^{..m} \cdot k_{mj}^{..j} \cdot R_j^{..} = R_j^{..m} - k_{jm}^{..m} \cdot R_m^{..j},$$

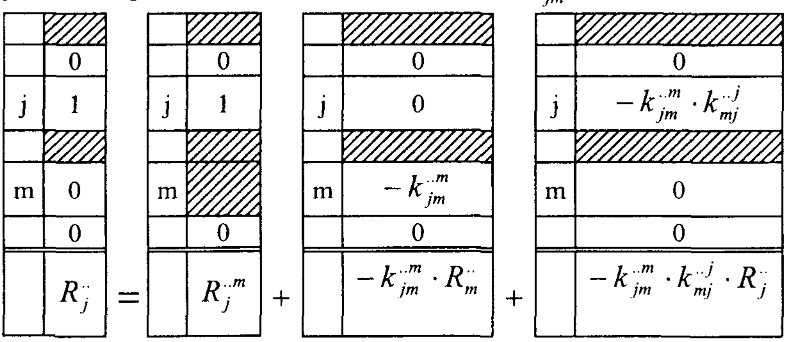
collecting the common factor:

$$(1 - k_{jm}^{..m} \cdot k_{mj}^{..j}) \cdot R_j^{..} = R_j^{..m} - k_{jm}^{..m} \cdot R_m^{..j},$$

and at the end:

$$R_j^{..} = (R_j^{..m} - k_{jm}^{..m} \cdot R_m^{..j}) \frac{1}{1 - k_{jm}^{..m} \cdot k_{mj}^{..j}} \qquad (1)$$

as reported in Fig. 10, with the obvious definition of $\alpha_{jm}^{..}$.



$$R_j^{..} = \left[R_j^{..m} - k_{jm}^{..m} \cdot R_m^{..j}\right] \cdot \alpha_{jm}^{..} \quad , \quad \alpha_{jm}^{..} = \frac{1}{1 - k_{jm}^{..m} \cdot k_{mj}^{..j}}$$

Fig. 10: Proof of the Inhibition Theorem.

4.3.2 Goal

If we consider that:

- it has been proved that any unitary regime can be deduced from two regimes which have one more inhibited seat each;

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- regimes with higher order of inhibition are easier to study, because inhibition practically means suppression;
- the theorem can be applied on the new two regimes, getting other more simple (inhibited) regimes, and so on, until they are no more simplifyable; then the goal is to move the study of the (real) given system to regimes of maximum inhibition and thus of maximum solution simplicity.

4.4 Fundamental formula

The proof is valid for any unitary regime, and thus for any principal regime of order v. The fundamental formula can be deduced directly substituting m by v in eqn (1). Starting from:

j	1
m	0
	0
	$R_j^{\cdot}$

with m=v we get:

$$R_j^{(v-1)} = \left[R_j^{(v)} - k_{jv}^{(v)} \cdot R_v^{(v)}\right] \cdot \alpha_{jv}^{(v)}$$

where:

$$\alpha_{jv}^{(v)} = \frac{1}{1 - k_{jv}^{(v)} k_{vj}^{(v)}}$$

which can be particularized for any quantity:

$$Q_j^{(v-1)} = \left[Q_j^{(v)} - k_{jv}^{(v)} \cdot Q_v^{(v)}\right] \cdot \alpha_{jv}^{(v)} \text{ and } k_{jr}^{(v-1)} = \left[k_{jr}^{(v)} - k_{jv}^{(v)} \cdot k_{vr}^{(v)}\right] \cdot \alpha_{jv}^{(v)}.$$

Level 3	Q	Seat	$k_{\_1}$	$k_{\_2}$	$k_{\_3}$	α
$R_1^{(3)}$	**A'**	1		**A**	**B**	**D**
		2				
$R_3^{(3)}$	**C'**	3		**C**		***
Level 2						
$R_1^{(2)}$	**a'**	1		**a**	$\alpha_{12}^{(2)}$	
		2			**	
Level 1						
		1		*		

Fig. 11: Tableau with auxiliary column and symbol positions (topological formula).

4.5 Topological rule

The previous expression are not properly user friendly, but it is possible to give a topological formula of them, avoiding the annoying use of indexes. We need to add a new column to the tableau, called the *auxiliary column*, where to put the values of $\alpha_{jv}^{(v)}$, in correspondence with $R_j^{(v)}$, as depicted in Fig. 11.

These values have to be calculated through the product between the inhibition quantity $k_{jv}^{(v)}$ (adjacent cell to $\alpha_{jv}^{(v)}$, on the left) and the inhibition quantity $k_{vj}^{(v)}$ (symmetrically located to previous cell, with respect to the greyed diagonal of the tableau).

After a look to Fig. 12, where two examples (*a*, for inhibition quantities and *a'*, for generic quantities) are reported, it is easy to understand the topological rule:

$$a = [A - B \cdot C] \cdot D$$

Every cell "a" can be calculated from the homologous cell "A" (same row and column in the immediately upper level) subtracting the product between the right end of row cell "B" and the lower end of column cell "C", and multiplying this difference by the value of cell "D" adjacent to "B" in the auxiliary column.

$k_{12}^{(2)}$	**a**
$k_{12}^{(3)}$	**A**
$k_{13}^{(3)}$	**B**
$k_{32}^{(3)}$	**C**
$\alpha_{13}^{(3)}$	**D**

$Q_1^{(2)}$	**a'**
$Q_1^{(3)}$	**A'**
$k_{13}^{(3)}$	**B**
$Q_3^{(3)}$	**C'**
$\alpha_{13}^{(3)}$	**D**

$$a = [A - B \cdot C] \cdot D \qquad a' = [A' - B \cdot C'] \cdot D$$

Fig. 12: Correspondence between symbols for the topological formula.

4.6 Solving mono-supply regimes

The topological rule allows to deduce all effects in a given inhibition level when all principal regimes of the immediately upper level are known. The level of maximum inhibition has to be studied directly, of course, but its regimes are the most simple ones (elementary regimes).

When all elementary regimes are known, then the iterative application of the topological formula leads to the final evaluation of the Level 1 quantities. This Level 1 corresponds to the regime, mono-supplied in seat 1.

So, the basic algorithm is:

1. Choice, enumeration and orientation of seats.

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2. Direct solution of elementary regimes.
3. Calculation of inhibition quantities.
4. Calculation of effects.

4.6.1 Remarks

- choice of seats is normally arbitrary;
- enumeration of seats is arbitrary, except for the supplied seat, which has to be indexed with "1";
- orientation of seats (if necessary) is normally arbitrary;
- inhibition quantities have to be calculated first, because they are needed for the effect calculation;
- effects can be calculated separately, because the calculus is row independent.

4.7 Solving multi-supply regimes

It is possible to solve regimes with more than one supplied seat because they can be reduced to mono-supply regimes, due to the superposition principle, following a similar procedure (not reported here) based on the ICS operator as well. The final formula (given for the same three-loop circuit) is a very simple expression:

$$R(Y_1, Y_2, Y_3) = F_1 \cdot R_1^{(1)} + F_2 \cdot R_2^{(2)} + F_3 \cdot R_3^{(3)} \tag{2}$$

where the following quantities

$$F_3 = Y_3,$$
$$F_2 = Y_2 - k_{32}^{(3)} F_3,$$
$$F_1 = Y_1 - k_{31}^{(3)} F_3 - k_{21}^{(2)} F_2$$

are called "auxiliary causes". We notice that all needed quantities in this formula are present or can be directly derived from the tableau calculated for the mono-supply regime, with a minimal calculation overhead.

4.8 Flexibility

There some general feature of the method that make it very flexible.

4.8.1 Change of supply

If the auxiliary causes are left symbolic, then we get expressions which are function of the supply values only. Once the full tableau is completed, then this fact allows to solve the originary system with the supply values configuration changed (not the seat displacement, of course), without redoing the entire tableau calculations: we simply need to recalculate the auxiliary causes by introducing the new supply values and then to use eqn (1) to get the requested effects corresponding to the new configuration.

4.8.2 Inverse matrix calculation

Once the full tableau is completed, inverse problem solving becomes immediate. An inverse problem is of the kind: "find the causes so that the given effects are produced" and it is depicted for a three-seat system, with the usual symbols, below:

$$X_1 = b_{11}Y_1 + b_{12}Y_2 + b_{13}Y_3$$
$$X_2 = b_{21}Y_1 + b_{22}Y_2 + b_{23}Y_3$$
$$X_3 = b_{31}Y_1 + b_{32}Y_2 + b_{33}Y_3$$

The coefficients b_{ij} constitute the inverse matrix of the originary system. The element displacement in this matrix can be interpreted as follows (see Fig. 13)

- row (1): effects in regime $R(1,0,0)$;
- row (2): effects in regime $R(0,1,0)$;
- row (3): effects in regime $R(0,0,1)$.

Thus, the inverse problem is reduced to the calculation of the last two regimes by a supply change (see previous section) each, because the first one corresponds to $R_1^{(1)}$, already calculated at the end of the full tableau.

coefficients interpreted as:	Calculation needed:
$b_{11} = [X_1]_{Y_1=1,Y_2=0,Y_3=0}$ $b_{21} = [X_2]_{Y_1=1,Y_2=0,Y_3=0}$ $b_{31} = [X_3]_{Y_1=1,Y_2=0,Y_3=0}$	$R(1,0,0)$ (no further calc.)
$b_{12} = [X_1]_{Y_1=0,Y_2=1,Y_3=0}$ $b_{22} = [X_2]_{Y_1=0,Y_2=1,Y_3=0}$ $b_{32} = [X_3]_{Y_1=0,Y_2=1,Y_3=0}$	$R(0,1,0)$ (one supply change)
$b_{13} = [X_1]_{Y_1=0,Y_2=0,Y_3=1}$ $b_{23} = [X_2]_{Y_1=0,Y_2=0,Y_3=1}$ $b_{33} = [X_3]_{Y_1=0,Y_2=0,Y_3=1}$	$R(0,0,1)$ (one supply change)

Fig. 13: Regime correspondence for inverse problem.

5 Example

Let us consider a 3-loop circuit with one dependent voltage generator of Fig. 14. The mutual induction between L1 and L2 is not considered (even if it is possible), and the parameter values are not given because we work in a completely symbolic way.

If the tree is composed by the branches 4 and 5, then the elementary regimes are those summarized in Fig. 15.

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If we apply the procedure explained in the previous section we get the full inhibition sequence until the final solution row, which is partially reported only for the quantity i_1:

$$i_1 = \frac{E\left(1+\alpha Cs + CL_2 s^2\right)}{R_1 + \left(L_1 + L_2 + \alpha CR_1\right)s + \left(CL_2 R_1 + \alpha CL_1\right)s^2 + CL_1 L_2 s^3}$$

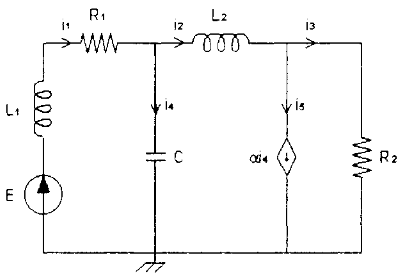

Fig. 14: Sample circuit.

	i_1	i_2	i_3	$k_{\_1}$	$k_{\_2}$	$k_{\_3}$
$R_1^{(3)}$	$\frac{1}{Z_{L1}+Z_C+R_1}$	0	0		$-(Z_C+\alpha)\cdot i_1$	$\alpha\cdot i_1$
$R_2^{(3)}$	0	$\frac{1}{Z_{L2}+Z_C+\alpha}$	0	$-Z_C\cdot i_2$		$-\alpha\cdot i_2$
$R_3^{(3)}$	0	0	$\frac{1}{R_2}$	0	0	

Fig. 15: Elementary regimes of the sample circuit

6 Conclusions

Symbolic programming can be used either for new methodologies or to renew old ones. In this paper an obsolete method from the point of view of numerical computing (called IME) has been reformulated symbolically in a complete abstract way, because its intrinsic nature of hierarchical problem decomposer helps to avoid some typical limitations of the symbolic analysis of electric circuits. As a matter of fact, among the Mathematica evironment, it allows to solve quite wide networks, to deduce (semi-)symbolic relationship between circuit parameters (very useful in circuit diagnosis, for example), or to utilize inhibition tableau data to easily deduce other quantities (related to the circuit, of course) like sensitivities and inverse matrix. Furthermore, it is a flexible method

because it is interdisciplinary and general in finding solutions. Also it allows to solve "inverse problems", even if it is a non-inverting method.
The paper showed the theoretical basis of the method and some features, while the software implementation is shown in a companion paper.

References

[1] Tan, X. & Shi, C. J. R. Hierarchical symbolic analysis of large analog circuits with determinant decision diagrams. *IEEE 1998.*

[2] Van Bokhoven, W. M. G. & Leenaerts, D. M. W. Explicit formulas for the solutions of piecewise linear networks. *IEEE Trans. on Circuits and System,* vol.46, n.9, 1999.

[3] Artioli, M. & Ciampolini, F. & Filippetti, F. Short Didactics (SD): a feasible proposal to shorten Learning times. Accepted at *EPE* $E = T_e M^2$ *Tomorrow's Education in Electrical Technologies:revisited Methods & Tools for renewed Motivation,* Liège 2001.

[4] Artioli, M. & Ciampolini, F. & Filippetti, F. Inhibition Method: a "Short Didactics" example for an engineering approach to didactics. Accepted at *EPE* $E = T_e M^2$ *Tomorrow's Education in Electrical Technologies:revisited Methods & Tools for renewed Motivation,* Liège, 2001.

[5] Ciampolini, F. A method for linear electric circuit analysis (in Italian), *L'Elettrotecnica*, n.10, 1968.

[6] Germundsson, R. Viewpoint: The importance of Symbolic Programming. *IEEE Spectrum 1999.*

[7] Filippetti, F. & Martelli, M. Symbolic techniques addressed to the fault diagnosis of large linear electric circuits. *IEEE Mediterranean Electrotechnical Conf. MELECON'96*, Bari 1996.

[8] Filippetti, F. & Martelli, M. & Artioli, M. Improvements to the fault diagnosis of large linear electric circuits via IME method. *Int. Symp. on Theoretical Electrical Engineering ISTET'97*, Palermo 1997.

[9] Filippetti, F. & Martelli, M. Network sensitivity analysis of large circuits in symbolic form. *European Conf. on Circuit Theory and Design ECCTD'95,* Istanbul 1995.

[10] Liberatore, A. & Manetti, S. SAPEC a personal computer program for the symbolic analysis of electrical circuits", *IEEE ISCAS'88*, Helsinki 1988.

[11] Fernàndez, F. V. & Rodrìguez-Vàzquez, A. Symbolic analysis tools - The state-of-the-art. *IEEE 1996.*