Seismic analysis of isolated structures by the response spectrum approach

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**Abstract**

A method able to determine some modal combination rules for the application of the response spectrum approach for linearly base-isolated structures is introduced. Three different modal combination rules have been introduced, each of them characterised by good applicability and accuracy level.

**1 Introduction**

The use of base isolated structures in the design of civil buildings has become very important in the last years. This is due to the necessity of finding always new technical approaches able to protect new and existing structures against earthquake effects. Base isolation lies on avoiding that damaging components of earthquake motion reach the structure by introducing flexibility and energy absorption capacity through some particular mechanisms placed between the structures and their foundations.

The use of isolators implies some important problems from an analytical point of view. In fact, even in the simplest case, here treated, of linear behaviour of the isolator, its presence makes the structural system non-classically damped, even if the structure is classical when it is not isolated. For this reason some recent codes, such as the Eurocode 8 [1], state that the response spectrum approach cannot be applied for the isolated structural systems. The same codes indicate that it is necessary to apply some other alternative methods, such as the stochastic analysis or the dynamic analysis with artificial accelerograms, that can be very heavy from a computational point of view.
In some recent papers (Falsone & Muscolino [2,3]) the authors have presented a method able to apply the response spectrum approach for non-classically damped structures by the use of a modal combination rule in which the cross-correlation coefficients are evaluated without the introduction of complex quantities, as it happens when other literature methods are used (Villaverde [4], Sinha & Igusa [5]). This method lies on the common assumption that both the earthquake input and the structural responses are stationary gaussian stochastic processes, using the properties of these processes in the estimation of the maximum value of a response quantity. The fundamental idea consists of stochastically relating the original non-classically damped system with an opportune classically damped one for which the application of the response spectrum approach by a modal combination rule is possible. For clarity’s sake, the input is here considered to be a gaussian white noise, but the extension to the filtered excitations is not difficult, as it was shown in Falsone & Muscolino [3] for common non-classically damped systems.

In the present paper the above cited method is particularised to the case of linear base-isolated structures, by using three different classically damped systems related to the original one, and consequently obtaining three different modal combination rules. In the first rule here presented, the classically damped system stochastically related to the isolated structure is the original structure considered as non-isolated. In the second one, it is the system characterised by the same modal frequencies of the non-classically damped system and by a diagonal damping matrix obtained by the original one setting zero the non-diagonal elements. Lastly in the third rule the stochastically related system has the same modal frequencies as the non-classically damped one and fixed modal damping ratios which, for convenience, can be chosen equal to that one of the response spectrum. This last rule is very comfortable from a practical point of view because it requires the values of the response spectrum only for a fixed damping ratio that can be that one for which the code spectrum is given.

The application of these approaches to a simple but significant example evidences important characteristics about the level of accuracy of each of the combination rule here introduced.

2 Preliminary concepts

The motion equations of an $n$-DOF linear structural system subjected to earthquake excitations can be written as follows:

$$ M \ddot{u}(t) + C \dot{u}(t) + K u(t) = -M \tau \ddot{u}_g(t) $$

where $u$ is the $n$-vector collecting the structural DOF, the dot on a variable indicates its time derivative, $M$, $C$ and $K$ are the system mass, damping and stiffness $n \times n$ matrices respectively and $\tau$ is the load $n$-vector characterising the influence of the ground acceleration $\ddot{u}_g(t)$ on the structural DOF.
If the structure is isolated by a SDOF linear base-isolator, the equation of motion of the whole system, composed by the structure and the isolator, can be written in the following form:

\[ \ddot{\mathbf{u}}(t) + \mathbf{C} \dot{\mathbf{u}}(t) + \mathbf{K} \mathbf{u}(t) = -\mathbf{M} \mathbf{\tau} \mathbf{u}_g(t) \]  

where:

\[
\mathbf{M} = \begin{pmatrix} \mathbf{m} & \mathbf{m} \\ \mathbf{m}^T & m_i \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} \mathbf{C} & \mathbf{0}_{n,1} \\ \mathbf{0}_{1,n} & c_i \end{pmatrix}, \quad \mathbf{K} = \begin{pmatrix} \mathbf{K} & \mathbf{0}_{n,1} \\ \mathbf{0}_{1,n} & k_i \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} \mathbf{x} \\ x_i \end{pmatrix}, \quad \mathbf{\tau} = \begin{pmatrix} \mathbf{\tau} \\ 1 \end{pmatrix}
\]  

\[ m \] being the \( n \)-vector whose each element is equal to the mass of the isolator; \( m_i \) is the mass of the whole system; \( c_i \) and \( k_i \) are the damping and stiffness coefficients of the isolator; lastly \( x_i \) is the isolator displacement.

In order to reduce the number of variables for both the systems (1) and (2), it is possible to operate in the modal subspace by means of the following coordinates transformations:

\[ \mathbf{u}(t) = \Phi \mathbf{q}(t); \quad \ddot{\mathbf{u}}(t) = \ddot{\Phi} \ddot{\mathbf{q}}(t) \]  

where \( \mathbf{q} \) and \( \ddot{\mathbf{q}} \) are the modal coordinates vectors, of order \( m \times 1 \) and \( \overline{m} \times 1 \) respectively (with \( m \leq n \) and \( \overline{m} \leq n + 1 \)), while \( \Phi \) and \( \ddot{\Phi} \) are the modal matrices, normalised with respect to the mass matrices and given by the solution of the following eigenproblems respectively:

\[ \mathbf{K} \Phi = \mathbf{M} \Phi \Omega^2; \quad \ddot{\mathbf{K}} \ddot{\Phi} = \mathbf{M} \ddot{\Phi} \ddot{\Omega}^2 \]  

where \( \Omega \) and \( \ddot{\Omega} \) are the diagonal matrices listing the first \( m \) and \( \overline{m} \) radian frequencies \( \omega_i \) and \( \ddot{\omega}_i \) of the systems (1) and (2) respectively. By using these transformations, we can rewrite the differential equations of motions reduced as follows:

\[ \ddot{\mathbf{\bar{q}}}(t) + \Xi \dot{\mathbf{\bar{q}}}(t) + \Omega^2 \mathbf{\bar{q}}(t) = \mathbf{p} \ddot{\mathbf{u}}_g(t); \quad \ddot{\mathbf{\bar{q}}}(t) + \ddot{\Xi} \dot{\mathbf{\bar{q}}}(t) + \ddot{\Omega}^2 \mathbf{\bar{q}}(t) = \ddot{\mathbf{p}} \ddot{\mathbf{u}}_g(t) \]  

where \( \mathbf{p} \) and \( \ddot{\mathbf{p}} \) are the participation coefficients vectors given by:

\[ \mathbf{p} = -\Phi^T \mathbf{M} \mathbf{\tau}; \quad \ddot{\mathbf{p}} = -\ddot{\Phi}^T \mathbf{M} \ddot{\mathbf{\tau}} \]  

and \( \Xi \) and \( \ddot{\Xi} \) are the generalised damping matrices given by:
Earthquake Resistant Engineering Structures

\[ \Xi = \Phi^T C \Phi; \quad \Xi = \Phi^T \Xi \Phi \]  

(8a,b)

It is important to note that the matrix \( \Xi \) is not diagonal, even if in the very common case (here considered) of classically damped structure the matrix \( \Xi \) is diagonal. Hence, while the system (1) is classically damped, characterised by the damping ratios \( \zeta \), the system (2) is a non-classically damped one; consequently, while eqn(6a) defines a system of \( m \) uncoupled differential equations, eqn(6b) represents a system of \( \overline{m} \) coupled differential equations. Nevertheless for both the system it is possible to apply the modal state variable approach, that allows us to rewrite eqns(6) as follows:

\[ \dot{z}_0(t) = D_0 z_0(t) + v_0 \ddot{u}_g(t); \quad \dot{\Xi}(t) = \overline{D} \Xi(t) + \overline{v} \ddot{u}_g(t) \]  

(9a,b)

where:

\[ z_0(t) = \begin{pmatrix} q_0(t) \\ \dot{q}_0(t) \end{pmatrix}, \quad D_0 = \begin{bmatrix} 0_{m \times m} & I_{m \times m} \\ -\Omega^2 & -\Xi \end{bmatrix}, \quad v_0 = \begin{bmatrix} 0_{m \times 1} \\ p \end{bmatrix}; \]

\[ \Xi(t) = \begin{pmatrix} \Xi(t) \\ \dot{\Xi}(t) \end{pmatrix}, \quad \overline{D} = \begin{bmatrix} 0_{m \times m} & I_{m \times m} \\ -\Omega^2 & -\Xi \end{bmatrix}, \quad \overline{v} = \begin{bmatrix} 0_{m \times 1} \\ p \end{bmatrix} \]  

(10a-f)

As said in the Introduction and as it will be clearer in the next section, it is useful to introduce, together with the systems governed by eqns(9), the other two classically damped systems governed by the following differential equations:

\[ \dot{z}_1(t) = D_1 z_1(t) + v_1 \ddot{u}_g(t); \quad \dot{z}_2(t) = D_2 z_2(t) + v_2 \ddot{u}_g(t) \]  

(11a,b)

where:

\[ D_1 = \begin{bmatrix} 0_{m \times m} & I_{m \times m} \\ -\Omega^2 & -\Xi \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0_{m \times m} & I_{m \times m} \\ -\Omega^2 & -2\xi_{ref} \Xi \end{bmatrix}, \quad z_i(t) = \begin{pmatrix} q_i(t) \\ \dot{q}_i(t) \end{pmatrix}, \quad i = 1,2 \]  

(12a-c)

and \( v_1 = v_2 = v \). In eqn(12a) \( \Xi_d \) is a diagonal matrix in which the non-zero diagonal components are the corresponding components of the matrix \( \Xi \). In eqn(12b) \( \xi_{ref} \) is a reference damping ratio which, for convenience, can be chosen equal to that one characterising the code response spectrum considered to represent the earthquake excitation. Hence it is not difficult to verify that both the systems governed by eqns(11) are classically damped, like the original non-isolated one, governed by eqn(9a), while, as said before, the isolated one,
governed by eqn(9b), is non-classically damped. In particular, both the systems
given in eqns(11) are characterised by the same radian frequencies of the isolated
structure, while they differ for the modal damping ratios, those of the system
(11b) being all equal to the reference one.

If the stochastic analysis of the systems given into eqns(9) and (11) is
performed, the response statistics have to be evaluated once that the stochastic
characters of the input are defined. If the input $\tilde{u}_k(t)$ is assumed to be a gaussian
stationary white noise process, as the structural system is linear, the response is
gaussian, too and, after the transient phase, it is also stationary. Under these
assumptions the response is completely characterised from a probabilistic point
of view by the stationary second order moments. For the various systems before
considered the modal response second order moments are given by (Falsone &
Muscolino [2,3]):

$$E[\bar{z}^{(2)}] = - (\bar{D} \oplus \bar{D})^{-1} \bar{v}^{(2)}q;$$
$$E[z_i^{(2)}] = -(D_i \oplus D_i)^{-1} v_i^{(2)}q; \quad i = 0,1,2 \quad (13a,b)$$

where the exponent into square brackets indicates the Kronecker power, that is
$E[z^{(2)}] = E[z \otimes z]$ ($\otimes$ indicating the Kronecker product), while $\oplus$ is the
Kronecker sum. Hence it is not difficult to verify that the vectors $E[\bar{z}^{(2)}]$ and
$E[z_i^{(2)}]$ collect all the second order moments of the components of the state
variables vectors $\bar{z}$ and $z_i$, respectively. At last in eqns(13) $q$ is the white noise
intensity.

As it will be seen in the next section, with the aim of determining a modal
combination rule for the isolated structures, it is useful to find the relationships
between the second order moments of the isolated structure modal responses and
the standard deviations of the corresponding quantities of either the non-isolated
structure or one of the other two classically damped systems considered before.
If, for convenience, we assume $m = \bar{m}$, it is always possible to find the
coefficients $\alpha_{i,j,k}$ relating these quantities in the following way:

$$E[\bar{z}_j \bar{z}_k] = \alpha_{i,j,k} \sigma_{z_i,j} \sigma_{z_i,k}; \quad \sigma_{z_i,j} = \sqrt{E[z_{i,j}^2]} \quad (14a,b)$$

$\bar{z}_j$ and $z_{i,j}$ being the $j$-th components of $\bar{z}$ and $z_i$, respectively (with $i = 0,1,2$).

As it will be seen in the next section, the relationship given in eqn(14a) is
fundamental in each of the modal combination rules regarding the isolated
structures introduced in the present paper. It stochastically relates the original
system with a classically damped one.
3 Peak response of isolated structures

It is well known that any modal combination rule for the evaluation of the maximum value of a response quantity of interest is based on the assumption that the earthquake is a stationary gaussian process and on the property that an estimate of the maximum value assumed by a gaussian stationary process is proportional to its standard deviation by the so-called peak factor (Der Kiureghian [6], Vanmarcke [7]).

A generic response quantity of interest of the isolated structure, such as the stress at a point or the internal force in a member, can be generally expressed as a linear combination of the nodal displacements and velocities, that is:

\[ s(t) = \mathbf{I}_1^T \mathbf{u}(t) + \mathbf{I}_2^T \mathbf{\dot{u}}(t) = \mathbf{r}^T \mathbf{\bar{Z}}(t) = \sum_{j=1}^{2\bar{m}} r_j \mathbf{\bar{z}}_j(t) \]  

(15)

where \( \mathbf{I}_1 \) and \( \mathbf{I}_2 \) are \((n+1)\times 1\) vectors of coefficients and \( \mathbf{r}^T = \left( \mathbf{I}_1^T \mathbf{\Phi} \quad \mathbf{I}_2^T \mathbf{\Phi} \right) \). The standard deviation of \( s(t) \) can be obtained as the square root of its variance, which can be expressed in terms of the second order moments of the modal responses as follows:

\[ \sigma_s^2 = \sum_{j=1}^{2\bar{m}} \sum_{k=1}^{2\bar{m}} r_j r_k E[\mathbf{\bar{z}}_j \mathbf{\bar{z}}_k] \]  

(16)

This relationship is not useful for the determination of a modal combination rule as the equations governing the modal response \( \mathbf{\bar{Z}} \) are coupled, so that the generic modal displacement \( \mathbf{\bar{q}}_j \) can not be considered as the response of a SDOF oscillator. But, if eqn(14a), which implies \( m = \bar{m} \), is taken into account, eqn(16) can be rewritten in the following form:

\[ \sigma_s^2 = \sum_{j=1}^{2\bar{m}} \sum_{k=1}^{2\bar{m}} r_j r_k \alpha_{i,jk} \sigma_{z_{i,j}} \sigma_{z_{i,k}} \]  

(17)

The terms of the second summation in eqn(17) can be rearranged in order to express them by the variances of \( \mathbf{q}_i \) and \( \mathbf{\dot{q}}_i \), that is:

\[ \sigma_s^2 = \sum_{j=1}^{\bar{m}} \sum_{k=1}^{\bar{m}} \alpha_{j,k}^{(0)} \sigma_{q_{i,j}} \sigma_{q_{i,k}} + \alpha_{j,k}^{(1)} \sigma_{q_{i,j}} \sigma_{\dot{q}_{i,j}} + \alpha_{j,k}^{(2)} \sigma_{\dot{q}_{i,j}} \sigma_{q_{i,k}} + \alpha_{j,k}^{(3)} \sigma_{\dot{q}_{i,j}} \sigma_{\dot{q}_{i,k}} \]  

(18)

where the coefficients \( \alpha_{j,k}^{(i)} \) in this equation are the coefficients \( r_j r_k \alpha_{i,jk} \) of eqn(17) suitably rearranged. By taking into account that the equations governing all the modal variables \( \mathbf{q}_i \) (with \( i = 0,1,2 \)) are uncoupled, the three eqns(18) can be rewritten as follows:
\[ \sigma_s^2 = \sum_{j=1}^{m} \sum_{k=1}^{m} \left( a_{0,j,k}^{(0)} + a_{0,j,k}^{(1)} \omega_j + a_{0,j,k}^{(2)} \omega_j \omega_k \right) \left| p_j \right| \left| p_k \right| \sigma_{d_{ij}, \sigma_{d_{jk}}} \]

\[ \sigma_s^2 = \sum_{j=1}^{m} \sum_{k=1}^{m} \left( a_{1,j,k}^{(0)} + a_{1,j,k}^{(1)} \omega_j + a_{1,j,k}^{(2)} \omega_j \omega_k \right) \left| \overline{p}_j \right| \left| \overline{p}_k \right| \sigma_{d_{ij}, \sigma_{d_{jk}}} \]

\[ \sigma_s^2 = \sum_{j=1}^{m} \sum_{k=1}^{m} \left( a_{2,j,k}^{(0)} + a_{2,j,k}^{(1)} \omega_j + a_{2,j,k}^{(2)} \omega_j \omega_k \right) \left| \overline{p}_j \right| \left| \overline{p}_k \right| \sigma_{d_{ij}, \sigma_{d_{jk}}} \]  

(19a-c)

where \( d_{ij} \) are the \( j \)-th components of the vectors \( q_i \) particularised for unitary participation coefficients. In these equations it has been taken into account that \( \sigma_{d_{ij}} \) is proportional to \( \sigma_{d_{ij}} \) by the corresponding participation factor and that the ratio between the velocity and displacement standard deviations of a SDOF oscillator subjected to a white noise input is equal to its radian frequency.

Another common assumption made for the determination of a modal combination rule is that all the peak coefficients, both for the quantity of interest and for all the modal responses, are equal. This assumption implies that eqns(19) generate the following three different combination rules:

\[ \max_0 |s(t)| = \sqrt{\sum_{j=1}^{m} \sum_{k=1}^{m} \rho_{0,j,k} D(\omega_j, \xi_j) D(\omega_k, \xi_k)} \]

\[ \max_1 |s(t)| = \sqrt{\sum_{j=1}^{m} \sum_{k=1}^{m} \rho_{1,j,k} D(\overline{\omega}_j, \overline{\xi}_j) D(\overline{\omega}_k, \overline{\xi}_k)} \]

\[ \max_2 |s(t)| = \sqrt{\sum_{j=1}^{m} \sum_{k=1}^{m} \rho_{2,j,k} D(\overline{\omega}_j, \overline{\xi}_{\text{ref}}) D(\overline{\omega}_k, \overline{\xi}_{\text{ref}})} \]  

(20a-c)

where:

\[ \rho_{0,j,k} = \left( a_{0,j,k}^{(0)} + a_{0,j,k}^{(1)} \omega_j + a_{0,j,k}^{(2)} \omega_j \omega_k \right) \left| p_j \right| \left| p_k \right| \]

\[ \rho_{1,j,k} = \left( a_{1,j,k}^{(0)} + a_{1,j,k}^{(1)} \omega_j + a_{1,j,k}^{(2)} \omega_j \omega_k \right) \left| \overline{p}_j \right| \left| \overline{p}_k \right| \]

\[ \rho_{2,j,k} = \left( a_{2,j,k}^{(0)} + a_{2,j,k}^{(1)} \omega_j + a_{2,j,k}^{(2)} \omega_j \omega_k \right) \left| \overline{p}_j \right| \left| \overline{p}_k \right| \]  

(21a-c)

In eqns(20) \( D(\omega, \xi) \) is the response spectrum corresponding to the considered input for the radian frequency \( \omega \) and the damping ratio \( \xi \).

In this way three different modal combination rules able to give an approximated maximum value of the response quantity of interest by using the response spectrum approach have been introduced for isolated structures. In
particular, the first combination rule, indicated with the index (0), requires the values of the response spectrum for a given input corresponding to the radian frequencies and the damping ratios of the structure considered as non-isolated. Obviously the influence of the isolator is included in the correlation coefficients $\rho_{0,jk}$, and, in particular, in the coefficients $a^{(l)}_{0,jk}$. The second modal combination rule here introduced, indicated with the index (1), requires the values of the response spectrum for the radian frequencies of the isolated structure and for the damping ratios deriving from the diagonalization of the damping matrix. This implies that the code response spectrum, which is given for a fixed damping ratio, has to be modified, introducing further approximations. This drawback is overcome by using the third modal combination rule here introduced and indicated by the index (2). Obviously in both these last two approaches the effects connected to the fact that the structure is non-classically damped are included in the coefficients $a^{(l)}_{i,jk}$ ($i=1,2$).

The author feeling is that the third rule is the more comfortable from a practical point of view for two reasons: a) as the second one, it requires only one eigenproblem solution, that is that one of the isolated structure, while the first one requires the solution of the non-isolated structure eigenproblem solution, too; on the contrary of the second one, it requires only one response spectrum at a fixed damping ratio, that can be the one of the code. Obviously each of the rules introduced, giving approximated results, is characterised by a level of accuracy which has to be investigated. The numerical results reported in the next section evidence this aspect.

4 Numerical example

In order to investigate the accuracy of the approaches here proposed, the same five-stories shear-frame considered in the example 3 of a paper by Chopra [8] is here taken into account. In tab(1) (case a) the radian frequencies of the structure, considered with and without base-isolation, are reported, together with the damping ratios of the non-isolated structure and those used for the combination rule (2).

The analysis is made considering the input as a white noise. As response quantity of interest $s(t)$ the nodal displacement of the fifth story $u_5(t)$ and the relative displacements between the first and second story $u_2(t) - u_1(t)$ have been considered. The exact estimation of the reference nodal response peak has been obtained by means of the following steps: a) evaluation of the exact nodal response power spectral density; b) evaluation of all the quantities, such as the spectral moments, necessary for determining the peak coefficient in conformity with Vanmarcke [7] expression; c) estimation of the maximum response by means of the Vanmarcke [7] formula. Each maximum modal displacement $D(\omega, \xi)$ to insert into the modal combination rules given in eqns(22) has been
obtained by means of the same steps a), b) and c) previously considered, obviously making reference to the power spectral density of the response of a SDOF oscillator, characterised by a radian frequency $\omega$ and a damping ratio $\xi$, and excited by the fixed earthquake excitation. In tab(2) (case a) the results of this analysis are reported for the three different rules here introduced. It is important to note that the rules (1) and (2) are characterised by an excellent level of accuracy, while the rule (0) gives result that differ from the exact ones of about 10%. This is due to the fact that the hypothesis of equal peak factors is too restrictive when the radian frequencies are changed from those of the isolated structure to those of the non-isolated one. This is confirmed by the fact that, as it can be evidenced in tab(2) (case b), the accuracy level of the rule (0) increases when an isolator ten times stiffer is introduced; in fact, the presence of this stiffer isolator reduces the differences between the radian frequencies of the isolated and non-isolated structure, as it is evidenced in tab(1) (case b).

<table>
<thead>
<tr>
<th>Non-isolated structure</th>
<th>Isolated structure (case a)</th>
<th>Isolated structure (case b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>mode</td>
<td>$\omega$</td>
<td>$\xi$</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>15.69</td>
<td>0.020</td>
</tr>
<tr>
<td>2</td>
<td>45.81</td>
<td>0.058</td>
</tr>
<tr>
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<td>72.21</td>
<td>0.092</td>
</tr>
<tr>
<td>4</td>
<td>92.76</td>
<td>0.118</td>
</tr>
<tr>
<td>5</td>
<td>105.81</td>
<td>0.135</td>
</tr>
</tbody>
</table>

Table(1): frequencies and damping characteristics of non-isolated and isolated structure: case(a): Chopra example; case(b): ten times stiffer isolator.

<table>
<thead>
<tr>
<th>case(a)</th>
<th>Case(b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>exact</td>
<td>rule(0)</td>
</tr>
<tr>
<td>$u_s$</td>
<td>0.17385</td>
</tr>
<tr>
<td>$u_z - u_1$</td>
<td>0.04632</td>
</tr>
</tbody>
</table>

Table(2): Exact and approximated reference responses.

5 Conclusions

A method that allows us to apply the response spectrum approach to linear base-isolated structures has been introduced. This application, that is discouraged by the Eurocode 8, is made possible by the use of particular modal combination rules obtained by relating the original non-classically system, formed by the structure and the linear isolator, with a classically damped system. In particular
three different classically damped systems have been taken into account. The first one is the original structure considered as non-isolated, granted that it is classically damped. The second one has the same modal frequencies of the total system and a diagonal damping matrix whose non-zero elements are the diagonal elements of the original full damping matrix. Lastly the third one has the same modal frequencies too, but the damping ratios are all equal to a fixed one that, for convenience, can be that one for which the response spectrum is given.

The application to a simple example has evidenced a good level of accuracy of the first combination rule, which becomes optimum when the other two rules are considered. Between these two approaches, the third has the further advantage of considering only one fixed modal damping ratio that makes the application of the response spectrum approach very comfortable.

References