Cone model for a pile foundation embedded in a soil layer

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Abstract

In this work the cone model is used to analyze the dynamic behaviour of piles embedded in a soil layer, after introducing several mirror-image disks placed at the other side of the free surface and of the interface with bedrock. Principles of symmetry, anti-symmetry and superposition permit satisfaction of the boundary conditions. By comparison with rigorous solutions, the model exhibits adequate accuracy. The unavoidable loss of precision is fully compensated by simplicity in application. The calculated impedance function of the pile is used to analyze the pile group behaviour.

1 Introduction

The cone model is a simple physical representation of the unbounded soil in a dynamic soil-structure interaction analysis. The cone model for translational mode of vibration was introduced half a century ago by Ehlers; the cone model for rotational mode considerably later by Meek & Veletsos. From the experience gained through extensive development in the field of foundation vibration during the last 25 years, Wolf has rearranged and extended the concept of the cone model to cover a complete range of dynamic excitations and physical situations.

For each degree of freedom of the foundation, an equivalent rigid massless disk on the surface of a homogeneous halfspace is considered. The halfspace below the disk is modeled as a truncated semi-infinite cone with the same material properties: mass density, \( \rho \), shear modulus, \( G \), and Poisson’s ratio,
v. The opening angle of the cone (with $z_0$ the apex height of the cone and $r_0$ the radius of the disk) follows from equating the static-stiffness coefficient $K$ of the cone to the well-known closed-form solution of the disk on a halfspace. Considering one-dimensional wave propagation with only body waves, the displacement pattern over the cross-section of the cone is determined by the corresponding value on the axis of the cone, using the theory of strength of materials (plane sections remain plane). Depending on the nature of deformation, it is necessary to distinguish between the translational cone for vertical and horizontal modes and the rotational cone for rocking and torsion, with the appropriate wave-propagation velocities $V$. From a prescribed mode of vibration of the disk, the interaction force-displacement relationship follows. As a typical result of one-dimensional wave propagation models, the specific radiation damping coefficient (equivalent viscous dashpot) $C$ equals to density times appropriate wave velocity $\rho V$ per unit area. In a vibration problem involving dilatational waves, velocity $V_p$ tends to infinity as $\nu$ approaches 0.5. To avoid an unrealistic infinite damping, Meek & Wolf\textsuperscript{4} limit to $2V_s$ the dilatational-wave velocity in the range $0.33 \leq \nu \leq 0.5$.

Wolf et al.\textsuperscript{5} expanded the concepts of the cone model to the analysis of pile foundations. A double cone model is introduced to represent a disk in the interior of a homogeneous fullspace. The only change consists in doubling the static-stiffness coefficient $K$ of the disk. The double cone’s displacement field defines approximate Green’s function for use in a matrix formulation of structural mechanics.

Wolf & Meek\textsuperscript{6} used the double cone model to determine the complex dynamic-stiffness matrix of a single layer. For a disk on the surface of a horizontally layered soil halfspace, assembling the dynamic-stiffness matrices of the layers and the dynamic-stiffness coefficient of the underlying halfspace, results in the dynamic-stiffness matrix of the site. Solving the dynamic equilibrium equation leads to the impedance function of the disk. As a consequence, the impedance function of a disk embedded in a layered halfspace can be calculated, but the procedure, called backbone cone, reveals certain shortcomings which deserve further research.

In this study the double cone model for a pile embedded in a soil layer resting on rigid rock is addressed.

![Figure 1. Cone and double cone models. (From Wolf, 1994).](image)
2 Double cone model

The representation of halfspace with the cone is illustrated in Figure 1a. Figure 1b shows the modified double cone for the fullspace.

For a disk embedded in the halfspace at a distance \( e \) from the surface, a mirror-image disk to model the free surface of the soil is introduced. Both disks are considered embedded in the fullspace and also represented by double cones, loaded by the same harmonic force (or moment) with unit amplitude and the same direction (Figure 2a). The same procedure can be used to model a fixed boundary, but the force on the mirror-image disk acts in the opposite direction (Figure 2b).

![Figure 2. Image disks and double cones. (From Wolf, 1994).](image)

For a disk embedded in a soil layer the concept of anti-symmetrical and symmetrical loaded mirror-image disks is applied repeatedly (Figure 3).

![Figure 3. Cone models for a disk embedded in soil layer. (From Wolf, 1994).](image)
Green’s function at a distance $a$ beneath the disk embedded in the layer of depth $H$ results
\[
g(a, \omega) = \frac{0.5}{S(\omega)} \left[ \frac{\exp(-i \frac{\omega}{V} a)}{1 + \frac{a}{z_0}} + \frac{\exp(-i \frac{\omega}{V} (2e + a))}{1 + \frac{2e + a}{z_0}} + \sum_{k=1}^{\infty} (-1)^{k+1} \frac{\exp(-i \frac{\omega}{V} (2kh \pm a))}{1 + \frac{2kh \pm a}{z_0}} + \frac{\exp(-i \frac{\omega}{V} (2e + a))}{1 + \frac{2kh \pm a}{z_0}} \right]
\]
for translational mode, with
\[
S(\omega) = \frac{\rho V^2 \pi r^2}{z_0} \left( 1 + i \frac{\omega}{V} z_0 \right)
\]  
the impedance function for translational cone, and
\[
g_s(a, \omega) = \frac{0.5}{S_s(\omega)} \left[ \frac{\exp(-i \frac{\omega}{V} a)}{\left( 1 + \frac{a}{z_0} \right)^2} + \frac{\exp(-i \frac{\omega}{V} (2e + a))}{\left( 1 + \frac{2e + a}{z_0} \right)^2} + \sum_{k=1}^{\infty} (-1)^k \frac{\exp(-i \frac{\omega}{V} (2kh \pm a))}{\left( 1 + \frac{2kh \pm a}{z_0} \right)^2} + \frac{\exp(-i \frac{\omega}{V} (2e + a))}{\left( 1 + \frac{2e + a}{z_0} \right)^2} \right]
\]
for rotational motion, with
\[
S_s(\omega) = \frac{3 \rho V^2 \pi r^2}{z_0} \left[ 1 - \frac{1}{3} \left( \frac{\omega}{V} \right) z_0 \right] + i \frac{\omega}{V} z_0
\]
the impedance function for rotational cone. The sums over $k$ are limited to 2000 disks.

Modeling the soil region, which will later be replaced by the pile, by a stack of rigid disks equally spaced, the complex dynamic-flexibility matrix $[G(\omega)]$ of the free field, discretized in the nodes of the pile corresponding to the disks, is calculated. The generic coefficient $g_{ij}(\omega)$ of this matrix specifies the displacement of the $i$-th disk (receiver) with amplitude $u_i(\omega)$ caused by harmonic force of unit amplitude $P_j(\omega)$ applied to the $j$-th disk of the stack and its mirror-image disks (sources). The complex dynamic-stiffness matrix $[S(\omega)]$ of the free field is determined by inverting $[G(\omega)]$. Replacing the cylindrical soil region by the pile results in a dynamic-stiffness matrix $[\Delta S(\omega)]$ with the same discretization and the difference of the properties of the pile and the soil
\[
[\Delta S(\omega)] = [\Delta K] - \omega^2 [\Delta M]
\]
where $[\Delta K]$ and $[\Delta M]$ are the standard pile static-stiffness and mass matrices respectively, calculated based on beam theory after subtracting the stiffness and mass of the (excavated) soil cylinder. Assembling $[S(\omega)]$ and $[\Delta S(\omega)]$ leads to the complex dynamic-stiffness matrix $[S(\omega)]$ of the pile. For a harmonic load with unit amplitude applied at the head of the pile, the dynamic equilibrium equation $[S(\omega)] \{u(\omega)\} = \{P(\omega)\}$ is solved. The reciprocal of the first element of the displacement vector $\{u(\omega)\}$ is equal to the impedance function $S(\omega)$ of the pile. The horizontal and rocking modes of a single pile are coupled. For the free field, the complex dynamic-stiffness matrix of horizontal and rocking modes,
both calculated with double cone, are independent. The expected coupling is introduced through beam theory applied to the pile elements.

3 Applications

The impedance function of a floating or end-bearing pile with diameter, \( d \), length, \( L \), slenderness ratio, \( L/d \), modulus of elasticity ratio, \( E_p/E_s \), mass density ratio, \( \rho_p/\rho_s \), in a homogeneous soil layer of depth, \( H \), with modulus of elasticity, \( E_s \), density, \( \rho_s \), Poisson’s ratio, \( \nu \), and hysteretic damping ratio, \( \zeta \), is determined. The latter affects Green’s functions through the correspondence principle, modifying the elastic modulus \( E_s(1+2i\zeta) \) and the wave velocity \( V(1+i\zeta) \). The pile is discretized with 25 rigid disks separated by 24 soil layers with thickness \( \Delta e = L/24 \).

For a given vibration mode, the normalized impedance function can be written in the form

\[
S(a_0) = k(a_0) + ia_0 c(a_0)
\]

in which \( a_0 = \omega d/V_s \) is a dimensionless frequency defined on the basis of the shear velocity, \( k(a_0) \) the stiffness and \( c(a_0) \) the damping coefficients, normalized by the static-stiffness coefficient \( K \) of the pile.

Figure 4 shows the impedance function for translational mode of vibrations of single floating piles. Figure 5 shows the impedance function for translational mode of vibrations of single end-bearing piles. Both solutions obtained by the cone model are compared to rigorous results furnished by literature. Figure 6 shows the impedance function for rocking mode of vibration of single floating or end-bearing piles. For a floating pile, cone model is compared with results taken from Gazetas. In all cases the analysis based on cones demonstrates acceptable.

The dynamic response of the groups of piles, with their heads connected by a rigid cap, are determined using the superposition method and the interaction functions developed by Dobry & Gazetas, testing the influence of the cone model. The method provides an approximate approach as its accuracy depends on how realistically the impedance function of the single piles and the interaction factors can be determined. For the group of \( 2 \times 2 \) floating piles, separated by a distance \( s \) between axes, the vertical dynamic-stiffness \( K_v^G \) of the group is nondimensionalized with its static-stiffness coefficient \( K_v^S \), and the damping \( C_v^G \) with the sum \( 4K_v^S \) of the single pile static-stiffness coefficients. For the group of \( 3 \times 3 \) end-bearing piles, separated by a distance \( s \) between axes, \( K_h^G \) and \( C_h^G \) are normalized by the sum \( 9K_h^S \) of the single pile static-stiffness coefficients. The following results are in good agreement with the direct analysis for the cases of floating (Nogami) and end-bearing (El-Marsafawi et al.) piles in a homogeneous soil layer.
$k_n(a_0)$

$L/d = 25$
$H/d = 40$
$E_r/E_s = 5000$
$\nu = 0.4$
$\zeta = 0.05$

$c_n(a_0)$

$L/d = 25$
$H/d = 40$
$E_r/E_s = 5000$
$\nu = 0.4$
$\zeta = 0.05$

$k_r(a_0)$

$L/d = 37.5$
$H/d = 75$
$E_r/E_s = 2000$
$\nu = 0.4$
$\zeta = 0.025$

$c_r(a_0)$

$L/d = 37.5$
$H/d = 75$
$E_r/E_s = 2000$
$\nu = 0.4$
$\zeta = 0.025$

Figure 4. Impedance functions of floating piles.
Figure 5. Impedance functions of end-bearing piles.
Figure 6. Rocking impedance functions of a single pile.
Figure 7. Impedance functions of pile groups.

$$a_0 = \omega d/V_s$$

- **Nogami (1983)**
  - $L/d = 37.5$
  - $H/L = 2$
  - $s/d = 5$
  - $E_p/E_s = 2000$
  - $\nu = 0.4$
  - $\zeta = 0.025$

- **Cone model**
  - $L/d = 37.5$
  - $H/L = 2$
  - $s/d = 5$
  - $E_p/E_s = 2000$
  - $\nu = 0.4$
  - $\zeta = 0.025$

- **El-Marsafawi et al. (1992)**
  - $L/d = 20$
  - $H/L = 1$
  - $s/d = 5$
  - $E_p/E_s = 100$
  - $\rho_p/\rho_s = 1.25$
  - $\nu = 0.4$
  - $\zeta = 0.05$

- **Cone model**
  - $L/d = 20$
  - $H/L = 1$
  - $s/d = 5$
  - $E_p/E_s = 100$
  - $\rho_p/\rho_s = 1.25$
  - $\nu = 0.4$
  - $\zeta = 0.05$
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References