



Digital Simulation of Non-Stationary Multivariate Earthquake Ground Motion

M. di Paola & M. Zingales

Dipartimento di

Ingegneria Strutturale e Geotecnica

V.le delle Scienze, I-90128 Palermo, Italy,

E-Mail: zingales@stru.diseg.unipa.it

Abstract

The study deals in detail with the digital simulation of non-stationary multivariate vector of seismic ground motions. The ground acceleration consists in an opportune discretization of a random field in different locations. The non-stationarity is considered uniformly modulated in time. The modulating functions accounts for the time-amplitude variation of the seismic acceleration. The digital simulation of the ground motion is performed by means of a new decomposition of the power spectral density matrix based on frequency dependent eigenproperties. Numerical examples show the capabilities of the technique.

1 Introduction

In seismic engineering setting the earthquake accelerations have been always related to a strong temporal variability. The presence of structures with large spatial extension required, lately, the consideration of the spatial fluctuation of the seismic ground motion.

The unknown source mechanism, as well as the complex propagation of the seismic waves in across the soil, suggests a stochastic approach to the problem. The seismic acceleration is considered as a non-stationary two-variate one dimensional (2V-1D) gaussian random field, assuming mono-dimensional propagation. The scattered waves are supposed of constant celerity throughout the region investigated. By collecting the seismic acceleration in specific points of the ground in a vector one obtains a multivariate, non-stationary, gaussian vector process. The digital simulation of multivariate seismic ground motions have been performed by means of several techniques. Shinozuka and Deodatis¹

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(1988) decomposed the power spectral density (PSD) matrix of the multivariate vector process using Cholesky's method. In this setting they obtained two triangular matrices allowing to simulate the multivariate vector component by component. Li and Kareem² (1991) simulated a multivariate vector process as superposition of fully coherent, but mutually uncoherent, vector processes by means of a fast fourier transformation algorithm. Shinozuka³ (1991) proposed a new technique to simulate multivariate vector processes in wind engineering setting. The PSD matrix was decomposed on the basis of its eigenvectors, mutually orthonormal. The wind velocity vector was then obtained superposing totally coherent, but mutually uncoherent, stochastic vector processes. Recently, the present authors^{4,5} have developed a technique to simulate multivariate vectors of stationary seismic accelerations. The proposed technique uses the eigenproperties of the PSD matrix, opportunely factored. The aim was to obtain the multivariate vector as superposition of totally coherent, but mutually uncoherent, random vectors. The digital simulation of non stationary earthquake ground motions has been dealt with the study of Deodatis⁶ (1996) who used the algorithm of Cholesky to decompose the PSD matrix. The disadvantage of the method lies in the great number of decomposition of the time dependent PSD matrix, for each circular frequency and for every time instant in which the vector process is computed.

In this study the authors will use the decomposition of the PSD matrix by means of the technique indicated in their previous study⁴. The theory of evolutionary spectrum will be utilized to model the non-stationarity in time of the seismic acceleration. The time dependent PSD matrix is factored into a circular frequency dependent and a time dependent matrix. Computing the eigenproperties of the frequency dependent matrix, the simulation of the non-stationary multivariate vector is performed multiplying the stationary multivariate vector process by the time dependent matrix. Some numerical results show the application of the proposed methodology.

2 Description of the Problem

Let us assume the seismic waves propagating with deterministic celerity c , fig.1,

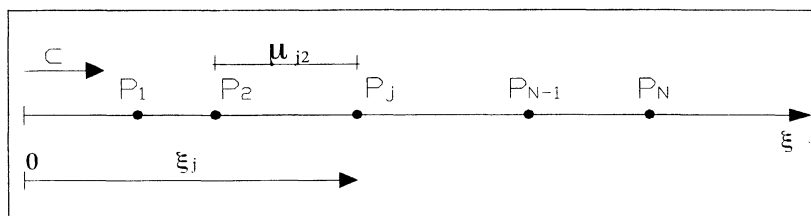


Figure 1: Physical representation of the investigated region and propagation of the seismic waves

The location of a generic point P_j , $j = 1, 2, \dots, N$ along the ξ axis will be identified by means of its abscissa ξ_j as shown in fig.1. The seismic acceleration in the point P_j will be denoted with capital letter $\ddot{Z}(\xi_j, t)$, $j = 1, 2, \dots, N$. The ground motion is modeled as a zero mean gaussian random field. Henceforth its complete probabilistic characterization is provided by means of its correlation function,

$$R_{\ddot{Z}}(\xi_j, \xi_k; t_1, t_2) = E[\ddot{Z}(\xi_j, t_1) \ddot{Z}(\xi_k, t_2)] \quad (1)$$

with $E[\bullet]$ indicating the mathematical expectation. The seismic acceleration random field is assumed homogenous in space, i.e., the correlation function in eqn (1) depends, only, of the distance $\mu_{jk} = \xi_k - \xi_j$,

$$R_{\ddot{Z}}(\xi_j, \xi_k; t_1, t_2) = R_{\ddot{Z}}(\mu_{jk}; t_1, t_1 + \tau) \quad (2)$$

with $\tau = t_2 - t_1$. For the sake of brevity, we will denote, hereinafter, the correlation function in eqn (2) as $R_{jk}(t_1, t_1 + \tau)$.

The application of the Fourier transform operator to the correlation function in eqn (2) yields

$$S_{jk}(t_1, \omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} R_{jk}(t_1, t_1 + \tau) \exp(i\omega\tau) d\tau \quad (3)$$

with $i = \sqrt{-1}$ the imaginary unity, and it represents the time dependent cross spectral density of the seismic accelerations in P_j and P_k . Collecting the seismic acceleration evaluated in the points $P_1, P_2, \dots, P_N$ in a vector $\ddot{\mathbf{Z}}(t)$, a multivariate vector process is obtained. The vector $\ddot{\mathbf{Z}}(t)$ is probabilistically characterized by the knowledge of its time dependent power spectral density matrix, which reads

$$\mathbf{S}_{\ddot{\mathbf{Z}}}(t_1, \omega) = \begin{bmatrix} S_{11}(t_1, \omega) & \dots & S_{1N}(t_1, \omega) \\ \dots & \dots & \dots \\ S_{1N}(t_1, \omega) & \dots & S_{NN}(t_1, \omega) \end{bmatrix} \quad (4)$$

Taking into account the assumption of spatially homogeneous random field, the time dependent PSD matrix in eqn (4) is Hermitian, that is

$$S_{jk}(t_1, \omega) = [S_{kj}(t_1, \omega)]^* \quad (5)$$

where the $[\bullet]^*$ indicates complex conjugate. Modeling the seismic acceleration $\ddot{Z}_j(t)$ by means of the theory of the evolutionary spectrum the following relationship holds, (Priestley⁷, 1965)

$$\ddot{Z}_j(t) = \int_{-\infty}^{+\infty} A(t, \omega) \exp(i\omega\tau) dU_j(\omega) \quad (6)$$

where $U_j(\omega)$ is an orthogonal increment process, such that

$$E \left[dU_j(\omega_r) dU_k^*(\omega_s) \right] = \delta_{rs} \delta_{ij} S_{jk}^{(0)}(\omega_r) d\omega_r \quad (7)$$

with δ_{rs} and δ_{ij} the Kronecker delta ($\delta_{rs} = 0$ if $r \neq s$, $\delta_{rs} = 1$ if $r = s$). The function $A_j(\omega, t)$ in eqn (6) represents a deterministic factor, generally complex, necessary to model the time and frequency variation of the stationary, time independent PSD function $S_{jk}(\omega)$. By means of the representation in eqn (6) the correlation matrix reads

$$\mathbf{R}_z(t, t + \tau) = \int_{-\infty}^{+\infty} \mathbf{S}_z(t, \omega) \exp(i\omega \tau) d\omega \quad (8)$$

with the time dependent PSD matrix of the form

$$\mathbf{S}_z(t, \omega) = \begin{bmatrix} |A_1(t, \omega)|^2 S_{11}^{(0)}(\omega) & \dots & A_1(t, \omega) A_N(t, \omega) S_{1N}^{(0)}(\omega) \\ \vdots & \ddots & \vdots \\ [A_1(t, \omega)]^* [A_2(t, \omega)]^* [S_{1N}^{(0)}(\omega)]^* & \dots & |A_N(t, \omega)|^2 S_{NN}^{(0)}(\omega) \end{bmatrix} \quad (9)$$

The next section will concern with the proposed technique to simulate non-stationary random vector process by means of the eigenproperties of the PSD matrix in eqn (9).

3 Digital Simulation by Means of the Eigenproperties of the Time-Dependent PSD Matrix

This section is restricted to the form of the uniformly modulated process, i.e. the modulating function is independent from the frequency ω .

$$A_j(t, \omega) = A_j(t) \quad (10)$$

The specific form of the cross-spectral density function $S_{jk}^{(0)}(\omega)$ is represented in the form given by Hanrichandran and Vanmarcke⁸ (1986) as

$$S_{jk}^{(0)}(\omega) = S(\omega) \rho(\omega, |\mu_{jk}|) \exp[i\omega \mu_{jk}/c] \quad (11)$$

where ω represents the circular frequency. The function $\rho(\omega, |\mu_{jk}|)$ in eqn (11) is the coherency spectra of the isotropic random field obtained letting $c \rightarrow \infty$. The use of eqn (11) to model the cross PSD function of the field, implies that the PSD function $S(\omega)$ does not change in the various locations of the region investigated. The eigenvalues and the eigenvectors of the PSD matrix $\mathbf{S}_z(t, \omega)$ are functions of the circular frequency ω and of the time t .

The matrix $\mathbf{S}_z(t, \omega)$ is Hermitian, therefore it possesses real eigenvalues $\lambda_j(t, \omega)$ and orthogonal eigenvectors $\psi_j(t, \omega)$, $j = 1, 2, \dots, N$, so that the following relationships hold:

$$[\Psi(t, \omega)]^T [\Psi(t, \omega)] = \mathbf{I} ; [\Psi(t, \omega)]^T \Lambda(t, \omega) [\Psi(t, \omega)]^* = \mathbf{S}_z(t, \omega) \quad (12)$$

where $\mathbf{I}$ is the identity matrix, the apex T indicates transpose. $\Lambda(t, \omega)$ is the diagonal matrix of the eigenvalues $\lambda_j(t, \omega)$ and $\Psi(t, \omega)$ is the matrix of the eigenvectors $\psi_j(t, \omega)$. We introduce a representation of the multivariate vector $\ddot{\mathbf{Z}}(t)$ by the representation,

$$\ddot{\mathbf{Z}}(t) = \sum_{j=1}^N \int_{-\infty}^{+\infty} \psi_j(t, \omega) [\lambda_j(t, \omega)]^{1/2} \exp[i\omega t] dU_j(\omega) \quad (13)$$

being $dU_j(\omega)$ a zero mean stochastic process calculated by means of eqn (7) where $S(\omega)$ is assumed unitary. Calculating the correlation matrix $\mathbf{R}_{\ddot{\mathbf{Z}}}(t, t + \tau)$ of the multivariate vector $\ddot{\mathbf{Z}}(t)$, as in eqn (13) yields

$$E \left[\ddot{\mathbf{Z}}(t) \left\{ \ddot{\mathbf{Z}}(t, t + \tau) \right\}^{*T} \right] = \int_{-\infty}^{+\infty} [\Psi(t, \omega)]^T \Lambda(t, \omega) [\Psi(t, \omega)]^* \exp[i\omega \tau] d\omega \quad (14)$$

Accounting for eqn (12) the integrand in eqn (14) as the time dependent PSD matrix of the multivariate vector process $\ddot{\mathbf{Z}}(t)$ is identified.

The digital simulation of the multivariate seismic acceleration vector is performed using a discrete form of eqn (13),

$$\ddot{\mathbf{Z}} \equiv \sum_{j=1}^N \sum_{k=1}^{K_c} \psi_j(t, \omega_k) P_j^{(k)} \sqrt{\lambda_j(\omega_k)} \Delta\omega \exp[i\omega_k t] \quad (15)$$

with $\omega_k = k\Delta\omega$ and $K_c \Delta\omega = \omega_c$ the cut-off frequency and $P_j^{(k)}$ independent random variables uniformly distributed such that

$$E \left[P_j^{(k)} \right] = 0 \quad ; \quad E \left[P_j^{(k)} \left(P_l^{(r)} \right)^* \right] = \delta_{jl} \delta_{kr} \quad (16)$$

The main disadvantage of the formulation proposed in eqn (15) consists in the evaluation of the eigenproperties of the PSD matrix $\mathbf{S}_{\ddot{\mathbf{Z}}}(t, \omega)$ for each time instant t and circular frequency ω . Moreover the power spectral density matrix $\mathbf{S}_{\ddot{\mathbf{Z}}}(t, \omega)$ is complex needing more computational efforts to compute eigenvalues and eigenvectors. These latter disadvantages are avoided factoring the PSD matrix $\mathbf{S}_{\ddot{\mathbf{Z}}}(t, \omega)$ in the form

$$\mathbf{S}_{\ddot{\mathbf{Z}}}(t, \omega) = \mathbf{A}(t) \mathbf{S}_{\ddot{\mathbf{Z}}}^{(0)}(\omega) \mathbf{A}^T(t) = \mathbf{A}(t) \mathbf{N}(\omega) \mathbf{G}(\omega) [\mathbf{N}(\omega)]^* \mathbf{A}^T(t) \quad (17)$$

where the matrices $\mathbf{N}(\omega)$, $\mathbf{A}(t)$ are, respectively, associated with the temporal lag due to the passage of the seismic waves and the time dependent modulating functions $A_i(t)$ given by:

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$$\mathbf{N}(\omega) = \begin{bmatrix} 1 & \dots & 0 \\ \dots & \dots & \dots \\ 0 & \dots & \exp[-i\omega \mu_{1N}/c] \end{bmatrix}; \quad \mathbf{A}(t) = \begin{bmatrix} A_1(t) & \dots & 0 \\ \dots & \dots & \dots \\ 0 & \dots & A_N(t) \end{bmatrix} \quad (18)$$

The matrix $\mathbf{G}(\omega)$ contains the PSD functions of a stationary, gaussian, multivariate vector process, which does not account for the time shift of the travelling waves. The real and symmetric matrix $\mathbf{G}(\omega)$ reads

$$\mathbf{G}(\omega) = S(\omega) \begin{bmatrix} 1 & \dots & \rho(\omega, |\mu_{1N}|) \\ \dots & \dots & \dots \\ \rho(\omega, |\mu_{1N}|) & \dots & 1 \end{bmatrix} \quad (19)$$

It has been shown by the present authors⁴ that the eigenvalues of the matrix $\mathbf{G}(\omega)$ and $\mathbf{S}_{\tilde{z}}^{(0)}(\omega)$ are equals, whereas their eigenvectors, respectively, $\phi_j(\omega)$ and $\psi_j^{(0)}(\omega)$ are related by the relationships

$$\psi_j^{(0)}(\omega) = \mathbf{N}(\omega) \phi_j(\omega); \quad \Psi^{(0)}(\omega) = \mathbf{N}(\omega) \Phi(\omega) \quad (20a,b)$$

By means of eqns (19 a,b), accounting for eqn (16), one can write $\psi_j(\omega, t)$ as:

$$\psi_j(\omega, t) = \mathbf{A}(t) \mathbf{N}(\omega) \phi_j(\omega); \quad \Psi(\omega, t) = \mathbf{A}(t) \mathbf{N}(\omega) \Phi(\omega) \quad (21a,b)$$

that is the eigenvectors of the time dependent PSD matrix $\mathbf{S}_{\tilde{z}}(t, \omega)$ are the product of the matrix containing the time dependence by the eigenvector of the matrix $\mathbf{S}_{\tilde{z}}^{(0)}(\omega)$.

Comparisons of eqns (21 a,b) and eqn (17) yields that the eigenvalue $\lambda_j(t, \omega)$ of the time dependent PSD matrix $\mathbf{S}_{\tilde{z}}(t, \omega)$ coincides with the j^{th} eigenvalue $\lambda_j(\omega)$ of the matrix $\mathbf{G}(\omega)$. Substituting eqns (21 a,b) into eqn (15) the following relationships hold,

$$\begin{aligned} \ddot{\mathbf{Z}}(t) &= \sum_{j=1}^N \sum_{k=1}^{N_k} \mathbf{A}(t) \mathbf{N}(\omega_k) \phi_j(\omega_k) P_j^{(k)} \sqrt{\lambda_j(\omega_k) \Delta \omega} \exp[i\omega_k t] \\ &= \mathbf{A}(t) \sum_{j=1}^N Y_j^{(0)}(t) = \sum_{j=1}^N Y_j(t) \end{aligned} \quad (22)$$

which shows that the multivariate vector $\ddot{\mathbf{Z}}(t)$ is obtained by means of the superposition of N monovariate, non stationary stochastic processes $Y_j(t)$. Each of the process $Y_j(t)$ is obtained multiplying its stationary counterpart $Y_j^{(0)}(t)$ by the time modulating matrix $\mathbf{A}(t)$.

The next section will deal with the simulation of a trivariate, non-stationary, vector process by means of eqn (22).

4 Numerical Example

In this section we use the algorithm obtained in sec.3 to the simulation of a trivariate, uniformly modulated, gaussian, vector process. The real valued function $\rho(\omega, |\mu_{jk}|)$ is set in the form of Hanrichandran and Vanmarcke⁸ (1986),

$$\rho(\omega, |\mu_{jk}|) = A \exp[-2|\mu_{jk}|B/(\alpha \vartheta(\omega))] + (1-A) \exp[-2|\mu_{jk}|B/\vartheta(\omega)] \quad (23)$$

with $B = 1 - A + \alpha A$, A and α experimental parameters, and the function

$$\vartheta(\omega) = l \left[1 + (\omega/\omega_0)^b \right]^{-1/2} \quad (24)$$

is the spatial scale of fluctuation where k and b are parameters depending upon the soil conditions. The stationary power spectral density function is represented in the form (Tajimi⁹, 1960)

$$S(\omega) = \frac{1 + 4\zeta_g^2}{\left(1 - \omega^2/\omega_g^2\right)^2 + 4\zeta_g^2 \omega^2/\omega_g^2} \quad (25)$$

with the parameters ω_g and ζ_g set to satisfy experimental data. The time modulating functions $A_j(t)$ are chosen in the form given by Bogdañoff-GoldBerg-Bernard¹⁰ (1961)

$$A_1(t) = a_1 t \exp(-a_2 t) \text{ for } t > 0$$

$$A_j(t) = \begin{cases} 0 & \text{if } t \in [0, \mu_{j1}/c] \\ a_1 (t - \mu_{j1}/c) \exp[-a_2 (t - \mu_{j1}/c)] & \text{if } t > \mu_{j1}/c \end{cases} \quad (26)$$

with $j = 2, 3, \dots, N$ and the coefficients a_1 and a_2 parameters to specify. Eqn (26) assumes the seismic waves travelling in the direction $P_1 \rightarrow P_N$. The parameters appearing in the expression of the power spectral density function $S(\omega)$ in eqn (26) are set at $\omega_g = 20.7$ rad/sec and $\zeta_g = 0.57$. The other experimental parameter in eqns (23,24) are specified at $A = 0.53$, $b = 0.43$, $l = 1.47$ and $\omega_0 = 10.7$ rad/sec, whereas the mutual distances between the points P_j are set $\mu_{12} = 500$ m, $\mu_{13} = 1000$ m and $\mu_{23} = 500$ m.

The contrast between the proposed algorithm and the target multivariate stochastic process $\ddot{Z}(t)$ is reported comparing the their respective correlation functions. These comparisons are depicted in figs.(2-4). Fig.2 portrays the cross-section of the auto-correlation surface $R_{11}(t, t+\tau)$, with the plane $t = 2$ contrasted with its corresponding ensemble average. The examination of fig.2 shows that the auto-correlation obtained by means of eqn (22) reaches a good approximation of the target. It is worthy to note that the auto-correlation functions possess different peak value in the different point locations, for the selected time t , due to the non-stationarity.

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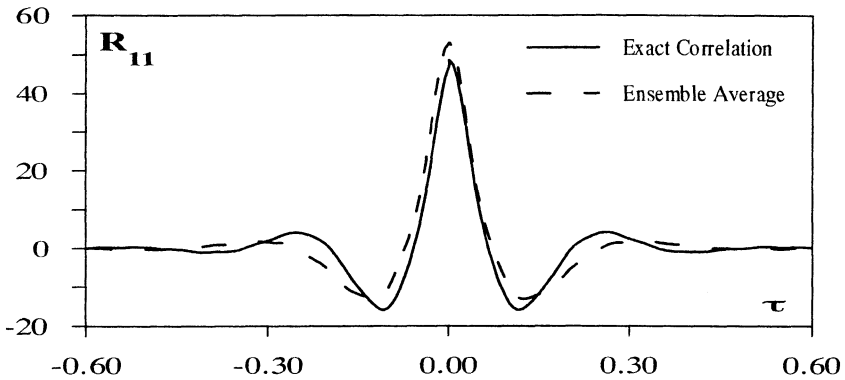


Figure 2: Auto-correlation $R_{11}(t, t + \tau)$ function obtained setting $t = 2$ sec in eqn (8) and ensemble average of eqn (22), $a_1 = 0.906, a_2 = 0.33$

The cross-correlation function $R_{12}(t, t + \tau)$ between the point locations P_1 and P_2 is reported in fig.3. This figure portrays the comparison between the cross-correlation obtained by means of eqn (8) and the corresponding ensemble average of the seismic accelerations computed with the aid of eqn (22). The examination of fig.3 reveals that the maximum correlation of the ground motions in the points P_1 and P_2 is found for a time lag τ corresponding to the time delay of the seismic waves scattering from P_1 to P_2 .

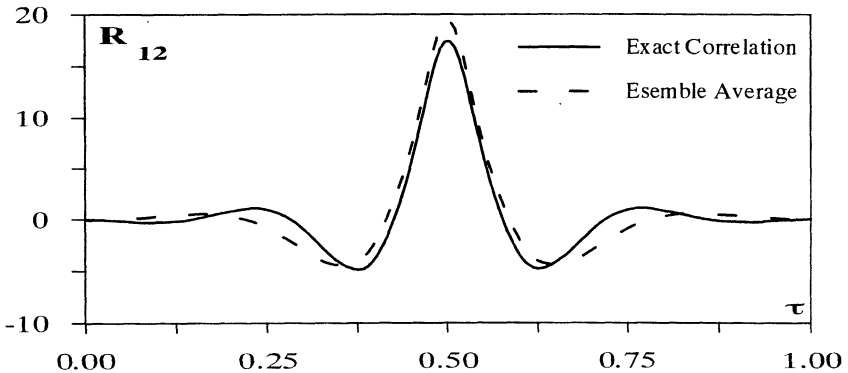


Figure 3: Cross-correlation $R_{12}(t, t + \tau)$ function obtained setting $t = 2$ sec in eqn (8) and ensemble average of eqn (22), $a_1 = 0.906, a_2 = 0.33$

The cross-correlation $R_{13}(t, t + \tau)$ is depicted in fig.4 contrasted with its respective ensemble average. In this figure the greatest correlation is obtained , for a time lag τ equal to $\mu_{13}/c = 1\text{sec}$. The target correlation functions in figs.(2-4) are obtained by means of the numerical fast fourier transform (FFT) of their corresponding PSD function in eqn (8). The figures show a good accordance between the ensemble averaged correlation functions and the target confirming the validity of the proposed algorithm.

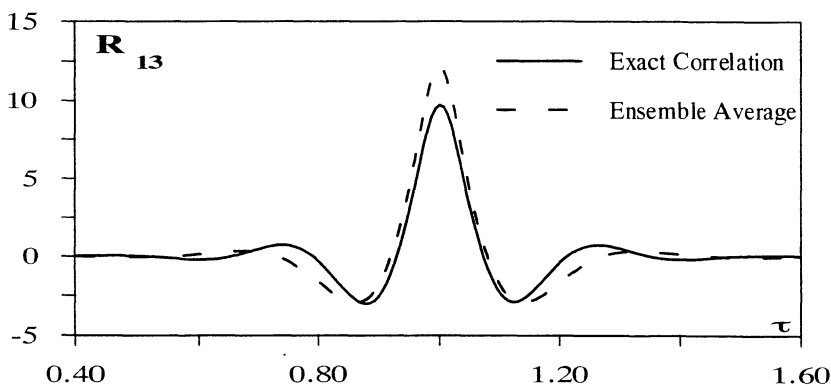


Figure 4: Cross-correlation $R_{13}(t, t + \tau)$ function obtained setting $t = 2\text{ sec}$ in eqn (8) and ensemble average of eqn (22), $a_1 = 0.906, a_2 = 0.33$

5 Conclusions

This study discussed in detail the digital simulation of earthquake ground motion in different points of the ground, assuming mono-dimensional propagation of the seismic waves. The seismic accelerations, recorded in the various points, are considered as realization of stochastic non-stationary and gaussian processes. The scattering of the seismic waves across the soil is considered in the probabilistic relationships of the simulated accelerograms, at different locations of the ground, to obtain a realistic simulation of the seismic ground motions. This aim is pursued collecting the stochastic processes in the different locations in a multivariate vector of seismic ground motion. The non-stationarity of the ground accelerations is considered by means of such modulating functions. They account for the time variation of the peak acceleration during the seismic event. The digital simulation of the non-stationary vector process is performed by a specific factorization of the time dependent power spectral density matrix. This operation allows the present authors to apply, with the maximum efficiency, a previously algorithm proposed for the stationary case. There are some interesting and pertinent questions: Is it possible to apply the obtained formulation to model a wider class of non-stationary vector process? And what is the effect of the dimension of the vector of seismic accelerations in the efficiency of the proposed



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technique? The work of the present authors is underway and it will be reported, with the answer to these questions and further developments, elsewhere.

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