Hydrodynamic pressures on dams during earthquakes - approximate formulation

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Abstract

Whenever carrying out safety inspections on dams under construction in officially declared seismic areas or in any zones of this ilk, one must always take into account not only the normal static effects, but also those triggered by the earthquake. This means one must consider both the inertial effect of the structural mass as well as that of the water in the artificial reservoir.

In this paper, it is demonstrated that the hydrodynamic pressure values for rigid dams and incompressible liquid are lower than those which can be extrapolated from the models based on the assumption of compressible liquid and both rigid and elastic dams (reservoir-dam interaction) and this observation becomes increasingly relevant the greater the height of the dam above 100 m.

Therefore, we propose some simple but technically reliable formulae for the assessment of hydrodynamic pressure both using the assumption of rigid dam and compressible liquid as well as that of dynamic reservoir-dam interaction.

1. Hydrodynamic Pressures on a dam induced by earthquake motions

An evaluation study [1][2][3][4] of dynamic water pressures against the walls of a hydraulic container during an earthquake motion has already been carried out by us (barrages for artificial storage capacity, both subsoil and suspended water reservoirs, head tanks, water clarification tanks, etc.).

For barrages (figure 1), a critical analysis has been carried out on the various methods used to determine the hydrodynamic pressures which can arise on the dam face during earthquakes. These methods are based on frequently differing trends of thought and relate to both dams with vertical upstream face as well as to dams with sloping upstream face (Westergaard [5], von Karman, Bakhmeteff, Zangar [6], Housner [7], Kulmaci [8][9], Chopra
Particular attention has been lent to the various simplified hypotheses introduced by the writers as a basis for their studies (two-dimensional motion, non-viscous liquid, compressible or incompressible liquid, small amplitude motion of the liquid particles, infinite or finite tank length, rigid dam, reservoir-dam interaction). To this end, a sensitivity test was performed on the parameters that the above inertial effects depend on, both in the case of rigid dam (with liquid taken to be both incompressible and compressible) as well as in that of dynamic interaction of coupled reservoir-dam system (elastic structure) [3].

Therefore, we feel it is appropriate to summarise the results of the above study, given that they are used later on in this paper for the simplified formulae put forward by ourselves.

1.1 General findings

a) The hydrodynamic pressure distribution on the upstream face, assumed vertical, of a dam excited by seismic perpendicular horizontal acceleration on the face itself, is approximately of a parabolic type and the peak coincides with the lowest point in the reservoir bed upstream from the barrage. The origin of the resulting thrust, the extent of which is proportional to the square of the water depth and obviously to the seismic coefficient (ratio between earthquake acceleration and gravitational acceleration) can be roughly calculated to be at a distance from the bed equal to 2/5 of the water depth:

b) earthquake acceleration horizontal and perpendicular to the upstream face brings about peaks in the inertial action of the water filling the reservoir. Some methods (Chopra [10] and Nath [17]), given a hypothesis of vertical acceleration and of a conservative system, lead us to assess hydrodynamic effect on an upstream face which can be greater than those due to horizontal movements of the same frequency. But various dissipating phenomena (for example, the compressibility of the bedrock) generally bring about - as noted by Rosenblueth [16] and by Chopra [15] - a significant reduction in the
hydrodynamic effects. Therefore, for practical purposes, in the case of vertical motions, the assumption that the hydrodynamic thrusts are equal to those deducible for acceleration horizontal and perpendicular to the upstream face can be held to be reliable and in many cases can also lead to greater safety levels:

c) the slope of the upstream face still causes a parabolic pressure distribution, but it becomes less so compared to a vertical face, as the angle α widens (figure 1). For α≠0°, however, according to certain methods (for example, the Housner method [7] and the Zangar method [6]), the peak pressure would not be had any more for y=y_o but at relative depths ranging roughly between 0.90 and 0.70 as α increases between 5° and 60°;

d) a sloping of the reservoir bed causes, from a technical viewpoint, a reduction in the hydrodynamic thrust and, moreover, a shifting towards higher values of peak frequencies of the system [18]. The assumption of a horizontal bed, therefore, would appear to ensure better safety standards.

2. The Rigid Models

2.1 Dams with vertical upstream face

In the case of a rigid structure firmly locked to the foundation (so the horizontal earthquake acceleration, taken to be perpendicular to the dam axis, is assumed to be evenly spread out over the whole barrage) and of incompressible liquid (as, for example in Zangar’s methods [6] - which current Italian Regulations refer to [19] - and Housner’s methods [7]), the hydrodynamic pressures of seismic origin are not connected to the exciting frequency and therefore, to the period of the earthquake T, but only to the water depth y_o, to the relative depth y/y_o, to the sloping angle α, and obviously, to the seismic coefficient C.

Consequently, the ratio between the inertial thrust S_o of the water on the upstream face and the hydrostatic thrust can be considered independent from the water depth and only a function of C and α.

In the case of a rigid dam and compressible liquid (for example, in the methods of Westergaard and Chopra, which only make reference to barrages with a vertical upstream face), the hydrodynamic pressure is dependent not only on y_o, y/y_o and C, but also on the ratio between the natural period of the reservoir T_i and the earthquake period T and therefore on the coefficient Ω:

\[ \Omega = \frac{T_i}{T} \]  

(1)

The period of the reservoir T_i, equal to 4y_o/c_p, c_p being the sound velocity in water, can be computed with good accuracy (expressing T_i in seconds and y_o in metres) with the relationship [2] [13] [20]:


Consequently, the hydrodynamic pressure value is no longer found to be a linear function of the water depth. Therefore, the ratio $S_o/S_i$ depends not only on the seismic coefficient, but also on the above-mentioned ratio $\Omega$. If we take into account the liquid compressibility, the extent of the hydrodynamic thrust is found to increase with $\Omega$ in the range $1>\Omega>0$ and therefore, given the same earthquake period, to increase with the depth of the reservoir and, obviously, given the same $y_0$, with the reduction of the earthquake period.

To this end, having computed numerous calculations carried out with Westergaard’s and Chopra’s formulae, we propose the following relationship for the calculation of the hydrodynamic thrust on the upstream vertical face of a barrage:

$$K = \frac{S_o}{S_i} = \frac{C}{(1-\Omega)^{0.45}}$$

So, as can be gathered from figure 2, where we can see as a function of $\Omega$ the K ratio between the $S_o$ thrust and the $S_i$ hydrostatic thrust, divided by the seismic coefficient C, this relationship allows us to assess $S_o$ with deviations of only a few percentage units compared to the values deduced from Westergaard’s method.

Especially, from (3), we can see that $S_o$ increases with $\Omega$ in a hyperbolic ratio, taking, for $\Omega>0.25$, values which increase progressively as $\Omega$ grows compared to the values gathered from methods which assume the liquid to be incompressible.
If we can assume the pressure distribution to be significantly parabolic, it is straightforward enough to deduce from (3) the extent of the hydrodynamic pressures at a general point of the face at depth $y$:

$$P_y = \frac{3}{4} C\gamma \frac{\sqrt{y_0}y}{(1-\Omega)^{\alpha/4}} \quad (3')$$

as well as the thrust at depth $y$:

$$S_y = 0.5C\gamma \frac{\sqrt{y_0}y}{(1-\Omega)^{\alpha/4}} \quad (3'')$$

$\gamma$ being the water specific weight.

### 2.2 Dams with sloping upstream face

In order to take into account the slope of the upstream face of the barrage, and therefore of the consequent reduction in the values of the hydrodynamic pressure and of the resulting thrust, many writers suggest introducing into the expressions deduced for the vertical face, a derating coefficient, to be determined as a function of the angle $\alpha$ formed with the vertical axis.

To this intent and purpose, Housner [7], in the case of rigid dam and incompressible liquid, for $35^\circ \geq \alpha \geq 0^\circ$, suggests the following relationship:

$$\beta = \frac{1}{\cos^2 \alpha} \left( 1 - \frac{\sqrt{3}}{2} \sin \alpha \right) \quad (4)$$

whilst for values of $\alpha$ greater than $35^\circ$ the expression of $\beta$ turns out to be extremely complex and has not been listed here for brevity’s sake.

Zienkiewicz and Nath [21] suggest, in the same cases, with reference to Zangar’s experiments, the approximate relationship:

$$\Gamma = \frac{\pi - \alpha}{\pi} \frac{\pi}{2} \quad (5)$$

From Kulmaci’s method [8][9] (rigid dam and compressible liquid), we can deduce the following expressions:

$$\delta = 1 - \frac{\alpha}{153.6} \quad \text{for } 45^\circ \geq \alpha > 0^\circ \quad (6)$$
In the following table we have written out, as a function of the angle $\alpha$, the values of the derating coefficient deducible from the relationships (4), (5), (6) and (6') as well as those recommended by the Italian Regulations.

<table>
<thead>
<tr>
<th>$\alpha(\degree)$</th>
<th>Housner ($\beta$)</th>
<th>Nath ($\Gamma$)</th>
<th>Kulmaci ($\delta$)</th>
<th>Ital. Reg.</th>
<th>$\varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>5.00</td>
<td>0.932</td>
<td>0.944</td>
<td>0.967</td>
<td>0.945</td>
<td>0.950</td>
</tr>
<tr>
<td>10.00</td>
<td>0.876</td>
<td>0.889</td>
<td>0.935</td>
<td>0.905</td>
<td>0.900</td>
</tr>
<tr>
<td>15.00</td>
<td>0.832</td>
<td>0.833</td>
<td>0.902</td>
<td>0.811</td>
<td>0.800</td>
</tr>
<tr>
<td>20.00</td>
<td>0.797</td>
<td>0.778</td>
<td>0.870</td>
<td>0.722</td>
<td>0.750</td>
</tr>
<tr>
<td>25.00</td>
<td>0.772</td>
<td>0.722</td>
<td>0.837</td>
<td>0.667</td>
<td>0.700</td>
</tr>
<tr>
<td>30.00</td>
<td>0.756</td>
<td>0.667</td>
<td>0.805</td>
<td>0.611</td>
<td>0.650</td>
</tr>
<tr>
<td>35.00</td>
<td>0.750</td>
<td>0.611</td>
<td>0.772</td>
<td>0.556</td>
<td>0.600</td>
</tr>
<tr>
<td>40.00</td>
<td>0.553</td>
<td>0.556</td>
<td>0.740</td>
<td>0.608</td>
<td>0.600</td>
</tr>
<tr>
<td>45.00</td>
<td>0.503</td>
<td>0.500</td>
<td>0.707</td>
<td>0.550</td>
<td>0.550</td>
</tr>
<tr>
<td>50.00</td>
<td>0.461</td>
<td>0.444</td>
<td>0.643</td>
<td>0.500</td>
<td>0.500</td>
</tr>
<tr>
<td>55.00</td>
<td>0.423</td>
<td>0.389</td>
<td>0.574</td>
<td>0.450</td>
<td>0.450</td>
</tr>
<tr>
<td>60.00</td>
<td>0.383</td>
<td>0.333</td>
<td>0.500</td>
<td>0.405</td>
<td>0.400</td>
</tr>
</tbody>
</table>

From the data in the table above, we can easily see that when the angle of the upstream face is not greater than approximately 15°, the deviations between the values of the derating factor provided by the methods quoted here as well as those contained in the Italian Regulations are quite small. On the other hand, for $\alpha>15°$, Kulmaci provides derating coefficient values which are still greater than those which may be inferred from the formulae (4) and (5).

To this end, on the basis of the values given by the derating coefficient as a function of $\alpha$ and written out in the table, we propose the following simple expression for the assessment of the above coefficient which is designated by $\varepsilon$:

$$\varepsilon = 1 - 0.01\alpha$$

whose values as a function of $\alpha$ can be also found in the table. If compared, we can see that the deviations between $\beta$, $\Gamma$, $\delta$, the Italian Regulations and $\varepsilon$ are quite minimal, apart from those proposed by Kulmaci when $\alpha$ is greater than 15°.

### 3. The Elastic Models

For gravity barrages with a vertical upstream face, excited by seismic acceleration horizontal and perpendicular to the dam axis, some mathematical patterns have been proposed which take into account the elasticity of the structure, imagined to be like a cantilever beam of variable cross-section. In particular, if we make the assumption of interaction of reservoir-dam system...
and of compressible liquid, (as in Chopra’s methods [10][11][12][13][14] and Nath’s method [17]), as pointed out in the paper previously quoted [2], the hydrodynamic pressures depend not only on \( C, \gamma_0, \gamma/\gamma_0, \) and \( \Omega \) but also on the \( \Omega_r \) ratio between the period of the actual vibration of the structure, \( T_\gamma \), and that of the reservoir, \( T_\gamma \):

\[
\Omega_r = \frac{T_\gamma}{T_\gamma}
\]

In (8), with reference to a horizontal vibratory motion, the period \( T_\gamma \), in seconds, can be assessed with fair accuracy with the relationship [2]:

\[
T_\gamma = \frac{H_s}{500}
\]

(\( H_s \) being the height of the dam in metres), which can be considered to be accurate when the ratio between the base and the height of the gravity barrage assumes values ranging from 0.7 and 0.8, as takes place generally in real building processes.

As the two periods are respectively proportional to the height of the dam and to the water depth, the ratio \( \Omega_r \) is therefore an index of the filling ratio of the reservoir.

Given that, for practical purposes we need to work with the most extreme conditions, that is to say, with maximum storage capacity in the reservoir, we will have to make reference to the value \( \Omega_r = 0.68 \), for the assessment of hydrodynamic action, assuming in all accuracy that the height of the dam coincides with the depth of the reservoir (\( \Omega_r = 340/500 = 0.68 \)).

Having carried out numerous calculations, using the quite complex formulae provided by Chopra, we propose the following relationship for a rapid assessment of the \( S_\alpha \), significantly accurate in the range 0.80 \( \leq \Omega \leq 0 \):

\[
K = \frac{S_\alpha}{S_i} = \frac{C}{(1 - \Omega)^{1.5}}
\]

The above, as can be gathered from figure 3, in which the ratio \( K/C \) is written out as a function of \( \Omega \), allows us to assess \( S_\alpha \), with deviations of only a few percentage units compared to the values deducible from Chopra (and to be exact, negative for \( \Omega < 0.14 \) and positive for values of \( \Omega > 0.15 \)).

Especially from (10), we can see that \( S_\alpha \) increases with \( \Omega \) in a hyperbolic ratio taking on, for \( \Omega > 0.25 \), values which become progressively greater than those attributed by Westergaard’s method as the value of \( \Omega \) goes up.
For values of $\Omega$ included in the range 0.80–0.90, we propose the following expression:

$$K = \frac{S_o}{S_r} = \frac{C}{(1 - \Omega)^{0.75}} \quad (11)$$

This allows for the assessment of the $S_o$ with quite small deviations compared to Chopra’s values, and in particular positive for values of $\Omega \leq 0.88$ (figure 3), whilst for $\Omega > 0.90$ - that is to say for barrages higher than 300 m - the extent of the hydrodynamic thrust provided in (11), tends towards lower values than those attributed by Chopra’s formulae.

In figure 4 we have drawn up an abstract of the results of our analytical study. In particular, in the above mentioned graph we have written out, with reference to a gravity dam with a vertical upstream face, the values of the ratio $K/C$ as a function of $\Omega$ (in the range $1 > \Omega > 0$), both with reference to the case of rigid dam and compressible liquid (relationship (3)) and with reference to structure-reservoir interaction (relationship (10) and (11)), as well as the $K/C$ value (which is, obviously, independent from $\Omega$) drawn from the Italian Regulations in force ($K/C = 1.074$) in the case of rigid dam and incompressible liquid.

An examination of this graph will show that the effects of the water compressibility and of the reservoir-dam interaction start to become tangible at values of $\Omega > 0.25$. Therefore, hydrodynamic thrusts much greater could occur for values of $\Omega$ greater than those mentioned above, than those assessable without taking into consideration fluid compressibility and structural elasticity. For example, for $\Omega = 0.75$, the $K/C$ ratio takes on the value of roughly 1.62 according to (3), 2.46 according to (10), and 1.074 in the case of rigid dam and
incompressible liquid.

For values of $\Omega > 1$, the extent of the thrust tends to diminish, whilst for $\Omega = 1$ (earthquake period which coincides with that of the reservoir) the hydrodynamic actions tend to reach infinitely higher values (resonance). However, bearing in mind that for practical purposes, we can take as a reference $T$ values not lower than one second, excluding especially high dams (roughly 300÷350 m high), the phenomenon of resonance should not occur [22].

From (10) and (11), if we still take the pressure distribution to be significantly parabolic, we can deduce the following formulae:

\[
P_y = \frac{3}{4} C \gamma \sqrt{\frac{Y_o Y}{(1-\Omega)^{1/6}}} \quad (10')
\]

\[
S_y = 0.5 C \gamma \sqrt{\frac{Y_o Y}{(1-\Omega)^{1/6}}} \quad (10'')
\]

for $0 < \Omega \leq 0.80$, and

\[
P_y = \frac{3}{4} C \gamma \sqrt{\frac{Y_o Y}{(1-\Omega)^{1/6}}} \quad (11')
\]

\[
S_y = 0.5 C \gamma \sqrt{\frac{Y_o Y}{(1-\Omega)^{1/6}}} \quad (11'')
\]

for $0.80 \leq \Omega < 0.90$.

These allow us to assess the hydrodynamic pressure and thrust at the general depth $y$. 

Figure 4 - $K/C$ ratio curve versus $\Omega$ deduced from the Italian Regulations and from the approximate formulae proposed
We likewise believe that, for technical purposes and with fair accuracy one can still refer to the relationship (7) in order to take into account the influence of the slope of the structure’s upstream face on the extent of the hydrodynamic pressure.

4. Approximate formulae for the assessment of hydrodynamic pressure

From what has been said above, it is obvious that the extent of the hydrodynamic pressure caused by an earthquake motion, characterized by horizontal acceleration perpendicular to the dam axis, on the upstream face of a barrage, assessed in the cases of rigid structure and incompressible liquid, can be taken to be:

- lower than that deductible in the case of rigid dam and compressible liquid, when the \( \Omega \) ratio takes on values greater than 0.25–0.30;
- much lower than that deductible considering the reservoir-dam interaction.

Therefore, we believe that for the assessment above, reference should be made to Westergaard’s method when a barrage is assumed to be rigid and to Chopra’s when one cannot do anything else but assume that the structure is flexible.

In particular, we believe that for arch dams and earth dams, given their specific characteristics (barrage constraints in the first case where structural elasticity is generally quite limited: material composition in the second case where any interaction effects should be absorbed locally), reservoir interaction can be assumed to be negligible. On the other hand for gravity barrages, it is not a good idea to overlook the elasticity of the structure, and therefore a dynamic analysis of the reservoir-dam interaction should always be carried out.

To this intent, we propose the approximate expressions (3), (3’) and (3’’) drawn up by ourselves in the case of rigid dam and compressible liquid, and (10), (10’) and (10’’) for \( \Omega \leq 0.80 \) as well as (11), (11’) and (11’’) for \( 0.90 \geq \Omega > 0.80 \), in the case of reservoir-dam interaction. These relationships were obtained by assigning values ranging between 0.2 and 2 seconds to the earthquake period \( T \) and values ranging between 10 and 350 m to the height of the barrage [2].

In the above relationships the \( S_0 \) is a function of \( \Omega = T/T \) and therefore, of the predominant earthquake period, which the ground acceleration peak values correspond to. The earthquake period generally has values ranging between 0.5–1.5 seconds, especially as a function of the ground features.

For design purposes, many writers (Westergaard [5], Zangar [6], Bratu [22], Chopra [10]) believe that, for the assessment of hydrodynamic actions on dams during earthquakes, one can make accurate reference to a period of roughly 1 second.
Therefore, with reference to the assumption of **rigid dam with a vertical or sloping face and compressible liquid**. (3), (3’) and (3’’) can be written out as:

\[
K = \frac{S_y}{S_i} = 7.7 \frac{C}{(340 - y_o)^{0.45}} (1 - 0.001\alpha) \quad (12)
\]

\[
P_i = 5.8C\gamma \frac{\sqrt{y_o y}}{(340 - y_o)^{0.45}} (1 - 0.001\alpha) \quad (12')
\]

\[
S_i = 3.85C\gamma y \frac{\sqrt{y_o y}}{(340 - y_o)^{0.45}} (1 - 0.001\alpha) \quad (12'')
\]

As can be gathered from figure 5, in which the ratio K/C has been written out as a function of the water depth to the maximum reservoir storage capacity (and therefore, with good accuracy, of the height of the barrage), the deviations contained in the values deducible from Westergaard’s method and those provided by (12) are quite small, and moreover contributes to higher safety standards as already has been made clear in figure 2.

![Figure 5 - K/C ratio curve versus y_o (rigid dam and compressible liquid)](image)

In figure 5, we have also written out the constant value of the ratio K/C deducible from the current Italian Regulations (based, as already mentioned, on the hypothesis of rigid dam and incompressible liquid), equal to 1.074, which for dams of a height greater than 100 m comes to less (and increasingly so as y_o goes up) compared to those provided in Westergaard’s method, and therefore, by the approximate formula proposed. Bearing in mind that for dams (even earth ones) the height goes over 300 m, the hydrodynamic thrust can reach values equal to even 2 to 3 times those deducible from the hypothesis of rigid structure and incompressible liquid.
For gravity dams higher than 100 m, given the sort of construction they are, it is not a good idea to overlook the elasticity of the structure and it would be wise to refer to Chopra’s methods, and therefore to the approximate formulae proposed which can be written, with reference to the assumption of T=1 s:

\[ K = \frac{S_o}{S_i} = 44 \frac{C}{(340 - y_o)^{0.65}} (1 - 0.01\alpha) \]  
\[ P_i = 33C\gamma \frac{\sqrt{y_o y}}{(340 - y_o)^{0.65}} (1 - 0.01\alpha) \]  
\[ S_i = 22C\gamma \frac{\sqrt{y_o y}}{(340 - y_o)^{0.65}} (1 - 0.01\alpha) \]

(13) accurate for dams of a maximum height of 275 m and:

\[ K = \frac{S_o}{S_i} = 79 \frac{C}{(340 - y_o)^{0.75}} (1 - 0.01\alpha) \]  
\[ P_i = 59C\gamma \frac{\sqrt{y_o y}}{(340 - y_o)^{0.75}} (1 - 0.01\alpha) \]  
\[ S_i = 40C\gamma \frac{\sqrt{y_o y}}{(340 - y_o)^{0.75}} (1 - 0.01\alpha) \]

(14) for values of \(y_o\) ranging from 275 to 300 m.

In figure 6 we have written out the ratio K/C as a function of the water depth deduced from formulae (13) and (14), as well as the constant value (K/C=1.074) deducible from Zangar’s method (which the Italian Regulation in force are based on). From this figure, apart from noting the low deviations between the values of the thrust \(S_o\) computed on the basis of approximate formulae and those assessable using Chopra’s methods, starting with dams higher than 100 m, the considerable difference between the values provided by (13) and (14) and those deduced from the hypothesis of rigid dam and incompressible liquid, should become evident to us. For example, for a 300 m high gravity dam, the \(S_o\) obtains a value equal to roughly 5 times that calculated with the Zangar’s method.
5. Conclusions

Even though, in view of the importance of dams, model tests, especially on very tall structures, are always advisable, especially with a view to looking more deeply into the resonance factor, for an assessment of the hydrodynamic pressures which can be generated on the upstream face of a dam in a seismic area, we propose these simplified but technically accurate formulae, both in the case of rigid dam with compressible liquid as well as in the case of reservoir-dam interaction. They allow for a simple determination of the hydrodynamic thrusts which can be much greater than those deducible from the assumption of rigid dam and incompressible liquid, which for example the Italian Regulations in force base themselves on.

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