Non-linear seismic sea waves induced harbour resonance and loading on breakwaters
Th. V. Karambas, C. Koutitas

Department of Civil Engineering, Division of Hydraulics and Environmental Engineering, Aristotle University of Thessaloniki, Thessaloniki 54006, Greece

Abstract

In the present work a non-linear dispersive wave model is developed for the description of the propagation of seismic origin water waves inside harbours. The model is based on the Boussinesq type of equations. As input at the offshore boundary an existing linear dispersive wave model for tsunami generation and propagation in deep water is used. The Smagorinsky approximation is used for the representation of the damping by eddies smaller than the computational grid size. Energy loss due to bottom friction and wave breaking are also included. The model is used for the calculation of the resonant response of harbours. The results are compared with analytical solutions from linear wave theory. Seismic sea wave loading on vertical wall breakwaters of arbitrary shape are also examined in this work. The wave forces are estimated from the pressure distribution along the large structure.

1. Introduction

Seismic sea waves, known as tsunamis, are long period waves created by the earthquakes occurred at the sea bottom or by large scale landslides on the coast. Since probable tsunamis sources are widely spread not only along Pacific ocean coast but also along the Mediterranean one [4], it is recommended that large harbour works be calculated taking into account tsunamis effects.

These waves are often cause damages to coastal structures, by attacking directly a (vertical wall) breakwater or exciting the natural mode water oscillations in a harbour.
Usually linear wave theory is used for the estimation of the wave loads on a vertical structure considering a standing wave in front of the structure. However, that is not valid in the case of a tsunami (a very long wave) since the ratio of the structure characteristic length to the wave length is generally less than unity. Also Morison equation can not be used since the presence of the coastal boundary result to a complicated velocity field near the body.

Finite Element and Finite Differences numerical models for the diffraction of linear waves are generally applied for calculating resonant response of harbours [1]. However, only linear periodic waves can be used as input at the harbour mouth, while a tsunami is a series of non linear waves of variable frequency.

From the above it becomes clear that a non linear dispersive wave model based on the Boussinesq equations is expected to give better results since:
- A more realistic hydrodynamic field near a structure of arbitrary shape is reproduced.
- Non linear waves of different frequencies and heights, resulting to a compound wave field inside the harbour are modelled.
- Non linear bottom friction and wave breaking effects [6] are introduced.

2. Numerical Model

Boussinesq equations are widely used for the simulation of the propagation of the non linear dispersive irregular waves in the nearshore region. The present model for the tsunami propagation near coastal structures is based on an existing model presented by Karambas and Koutitas [6]. As input at the offshore boundary the results obtained by the ‘deep’ water linear tsunami propagation model by Karambas et al. [5] are used.

In flow problems dominated by wave motion an open boundary condition must allow waves generated in the domain of interest to pass through the boundary without undergoing significant distortion and without influencing the interior solution. Thus, in the present problem, the outgoing waves at the offshore boundary can be described using the Orlanski [10] condition at the time \((n+1)\Delta t\), at the boundary point \(JM\):

\[
f^{n+1}(JM) = \frac{f^n(JM - 1) - f^{n-1}(JM - 2)}{f^{n-2}(JM - 1) - f^{n-1}(JM - 2)} \cdot f^{n-1}(JM) - \frac{f^n(JM - 1) - f^{n-2}(JM - 1)}{f^{n-2}(JM - 1) - f^{n-1}(JM - 2)} \cdot f^n(JM - 1)
\]

(1)
in which \( f = (U, V, \zeta) \), \( U, V \) are the depth averaged velocities in \( x \) and \( y \) direction respectively and \( \zeta \) the surface elevation.

The above relation is also used at the boundaries along which incident waves are introduced.

In the numerical integration of a 3-D open channel turbulent flow it is necessary to introduce an eddy viscosity, which represents physically the damping by eddies smaller than the computational grid size. After the depth averaging, to filter out the vertical velocity profile, the residual stress, have the following form in a 2-D horizontal model [8], [9]:

\[
\frac{\partial}{\partial x} \left( E \frac{\partial U}{\partial x} \right) + \frac{1}{2} \frac{\partial}{\partial y} \left( E \left( \frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} \right) \right)
\]

(2)

where \( E \) is the sub-grid eddy viscosity having the isotropic shear-dependent form proposed by Smagorinsky:

\[
E = \frac{1}{2} \left[ \left( \frac{\partial U}{\partial x} \right)^2 + \left( \frac{\partial V}{\partial y} \right)^2 + \frac{1}{2} \left( \frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} \right)^2 \right]^{1/2}
\]

(3)

in which the mixing length \( l \) is determined by:

\[ l = c \Delta x \]

(4)

where \( \Delta x \) is the grid size and \( c \) an empirical constant.

In 3-D modelling the value of \( c = 0.1825 \) is used. However, in 2-D horizontal flow it must be expected a different value for \( c \). Love and Leslie [8], in their study on Burgers' equation (depth averaged non linear shock wave propagation), suggested for \( c \) the value of 0.4, A similar value was also proposed by Madsen et al. [9] for depth integrated flows. This value of \( c \) is also adopted here.

Breaking and bottom friction effects are introduced as in Karambas and Koutitas [6].

3. Horizontal forces on breakwaters

As mentioned in the introduction Morison equation for the forces on vertical cylinders is not valid in this complicated case where the presence of the structures and the shore boundary affects the hydrodynamic field. In addition the values of the drag and the inertia coefficients have not been calibrated for this type of flow. The effects also of the wave diffraction can not be ignored. Thus an estimation of the force from the
hydrodynamic field resulting from the wave model it seems to be more appropriate.

Figure 1. Surface elevation and velocity field around a monolithic breakwater with length $e=100$ m and width $w=10$ m
The total horizontal force on a vertical wall breakwater is estimated by integrating the force due to pressure over the whole body:

$$\mathbf{F} = -\int_{S} \mathbf{Pn} \, dS$$

(5)

where \( \mathbf{n} \) is a unit normal vector into the fluid, \( \mathbf{P} \) the pressure and \( S \) the surface of the structure.

In a Boussinesq model the pressure \( \mathbf{P} \) is given by:

$$\mathbf{P} = \rho g (\zeta - z) + \rho \left( \frac{(d + z)^2 - h^2}{2} \right) \left( \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} \right)$$

(6)

\( \rho \) is the fluid density, \( \zeta \) the surface elevation, \( d \) the water depth, \( h = d + \zeta \).

Figure 2. Calculated horizontal force on a rectangular vertical breakwater.

In Figure 1 the surface elevation and the velocity field around a vertical wall breakwater are shown. The length \( e \) of the structure is 100 m and the width \( w = 10 \) m. Forces on the structure due to waves, at a depth \( d = 10 \) m, with different amplitude and period are shown in Figure 2. For small values of the ratio \( e/L \) (with \( L \) the wave length) Morison equation is valid.
(in open sea only) while for larger values of \( e/L (>0.2) \) diffraction theory is necessary. The present model is valid in both cases.

4. Harbour resonance

Seismic waves arriving within a harbour are reflected seaward by the rear boundary and at the harbour entrance, due to sudden widening, are partially reflected again. This trapping of energy leads to resonance if the phases of the various incident and reflected waves happen to be such that reinforcement occurs. The resonant response of harbours can be calculated by applying a wave propagation model based on the mild-slope equation for periodic linear waves \([1]\). However seismic sea waves are not periodic waves and when a tsunami approach the near shore region non linear effects become important and can not be ignored. On the other hand a Boussinesq model can describe both the amplitude and frequency dispersion. In addition as input at the offshore boundary the realistic results obtained by the ‘deep’ water tsunami propagation model \([5]\) can be used.

![Figure 3. Response curve for a fully open harbour (solid line: analytical solution, symbols: numerical results).](image)

The open boundary condition permits the radiation of the outgoing waves resulting in the formation of a standing wave inside the computational domain.

For verification purpose the model is applied to calculating the resonant response of a fully open rectangular harbour. The amplification
factors at the centre of the backwall are presented in Figure 3. The model results are compared with the analytical solution given by Sandoval and Farreras [3]:

\[ a = \frac{2a_i}{\cos k - \frac{kW}{2} Y_c \frac{\sin k}{i \frac{kW}{2} \sin k}} \]  

(7)

where \( a \) is the wave amplitude at the centre of the backwall of the harbour, \( a_i \) is the incident wave amplitude, \( k \) is the wave number, \( l \) is the basin length, \( W \) the entry width and \( Y_c \) is given by:

\[ Y_c = \ln\left(\frac{kW}{2}\right) - 8.5 \]

The length \( l \) is taken equal to 625 m and the width \( W = 60 \) m.

The model is also applied in Thessaloniki. A fictitious tsunami is generated southern of the port resulting to the oscillation of the area as shown in Figure 4.
References


