Inelastic seismic response of building-foundation systems
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Abstract

The investigation reported in this paper studies the effects of soil-structure interaction (SSI) on the inelastic seismic response of building-foundation systems. A basic approach in this study is the use of a simple structure for analyzing the overall seismic behavior of multistory building structures. This structure responds as a SDOF in its fixed-base condition. A parametric study is conducted using the EW component of the ground acceleration recorded at the SCT station in Mexico City during the 1985 earthquake. Peak response parameters chosen for this study were the roof displacement relative to the base and the hysteretic energy dissipated by the simple structural model. These parameters are often related to seismic damage in buildings. The results indicated that in most cases of inelastic response SSI effects are not very significant.

1 Introduction

The investigation reported in this paper studies the effects of soil-structure interaction (SSI) on the inelastic seismic response of building foundation systems. Emphasis is made on the case of Mexico City where extensive damage was observed in buildings on soft soil during the 19 September 1985 earthquake. The objectives of this study were to evaluate the importance of SSI effects on the structural response of buildings in Mexico City during the 1985 earthquake and to compare results considering these effects with those assuming an ideal rigid foundation. A simple structural model is used in this study for analyzing the overall seismic behavior of multistory buildings. A parametric study was conducted using the EW component of the ground acceleration recorded at the SCT station in Mexico City during the 1985 earthquake.
2 System and method of analysis

The system considered in this study is shown in Fig 1. It consists of a nonlinear structure of mass m resting on deformable soil. This structure responds as a SDOF with a circular frequency $\omega$ in its fixed-base condition. The equations of motion of this system in coupled horizontal and rocking are

$$\ddot{u} + \dot{v} + \ddot{v}_0 + h\ddot{\theta} + 2\xi\omega u + \frac{r}{m} = 0 \quad (1)$$

$$\ddot{u} + \dot{v} + \ddot{v}_0 + h\ddot{\theta} + \frac{m_0}{m}(\ddot{v} + \ddot{v}_0) + \frac{k_h v_0}{m} = 0 \quad (2)$$

$$\ddot{u} + \dot{v} + \ddot{v}_0 + h\ddot{\theta} + \frac{l_{m_0}}{mh} \ddot{\theta} + \frac{k_m}{mh} \theta = 0 \quad (3)$$

In these equations $u$ is the horizontal displacement of the top mass relative to the base; $v_0$ is the translation of the base mass in addition to the free field motion, $v$ is the free-field horizontal ground displacement; $\theta$ is the rotation of the base mass; $r$ is the resistance function of the structure; $\xi$ is the fraction of critical structural damping; $m_0$ is the mass at the base; $l_{m_0}$ is the centroidal moment of inertia of the top mass; $k_h$ and $k_m$ are the SSI translational and rotational stiffness of the foundation, respectively, which in this study are assumed to have elastic behavior.

In formulating Eqs (1) to (3) it has been assumed that although $\xi$ is the fraction of critical structural damping, it also represents an effective damping of the interacting system, following the approach developed by Veletsos [8] and adopted by the ATC3-06 [1]. In this approach the effective damping of the interacting system, $\bar{\xi}$, is given by

$$\bar{\xi} = \xi_0 + \frac{\xi}{(\bar{T}/T)^3} \quad (4)$$

in which $\xi_0$ represents the contribution of foundation damping, including both radiation and material damping. The second term of (4) represents the contribution of structural damping and depends on $\xi$ and on the ratio $\bar{T}/T$, where $\bar{T}$ is the SSI fundamental period and $T$ is the fixed-base period.

The building-foundation system under study is shown in Fig 2. The building has N floor masses and a constant interstory height equal to $h_0$. The mass at the base and the centroidal moment of inertia of the system are $m_0^e$ and $I_{m_0}^e$, respectively. The SSI translational and rotational stiffness of the foundation are $k_h^e$ and $k_m^e$, respectively. Fig 3 shows the deflected shape of the building-foundation system under study, where $\delta$ is the roof displacement relative to the
base; \( v_0^e \) is the translation of the base mass in addition to the free-field motion; \( v^e \) is the free-field horizontal ground displacement; and \( \theta^e \) is the rotation of the base mass of the system.

This study uses a simplified approach for relating the seismic response of a multistory building and the seismic response of a SDOF system (Qi and Moehle [6], Rodriguez [7]). A basic hypothesis in this approach is assuming a constant deflected shape for the multistory building under earthquake excitation. This approach in combination with the equations expressing the equilibrium of the multistory building in coupled horizontal translation and rocking leads to the following three equations:

\[
\ddot{\delta} + \ddot{v}^* + \ddot{v}_0^* + H^* \dot{\theta}^* + 2 \zeta^* \omega^* \dot{\delta} + \frac{R^*}{M^*} = 0
\]  
\[
\ddot{\delta} + \ddot{v}^* + \ddot{v}_0^* + H^* \dot{\theta}^* + \frac{m_0^*}{M^*} (\ddot{v}^* + \ddot{v}_0^*) + \frac{k_h^*}{M^*} v_0^* = 0
\]  
\[
\ddot{\delta} + \ddot{v}^* + \ddot{v}_0^* + H^* \dot{\theta}^* + \frac{I_{m0}^*}{H^* M^*} \ddot{\theta}^* + \frac{k_m^*}{H^* M^*} \theta^* = 0
\]

Equations (5) to (7) can also be viewed as the equations of motion of a system \( Q^* \) (Fig 4), in which \( v_0^* \) and \( \theta^* \) are, respectively, the translation in addition to the free-field motion and rotation of the base mass of the system \( Q^* \), and \( v^e \) is the free-field horizontal ground displacement of the system \( Q^* \). These parameters are defined as

\[
v^* = \gamma v^e
\]
\[
v_0^* = \gamma v_0^e
\]
\[
\theta^* = \theta^e
\]

in which \( \gamma \) relates \( \delta \) and \( u \) by

\[
\delta = \gamma u
\]

In addition, the following parameters of the system \( Q^* \) (Fig 4) are defined as

\[
m_0^* = m_0^e / \gamma^2
\]
\[
k_h^* = k_h^e / \gamma^2
\]
\[
I_{m0}^* = I_{m0}^e
\]
\[
k_m^* = k_m^e
\]

Assuming several hypotheses discussed in the following it can be shown that if \( u \) is a solution of coupled equations (1) to (3), then (11) is a solution of coupled equations (5) to (7). The fixed-base frequency \( \omega^*, \) the fraction of critical damping
ξ^\dagger, and displacement ductility ratio μ^\dagger of the system Q^\dagger are assumed equal, respectively, to the parameters ω, ξ, and μ of the system of mass m (Fig 1). In addition, the parameters \vec{v}, \vec{v}_0, and θ of the building-foundation system, are assumed to be equal to the parameters v, v_0, and θ of the system of mass m. It is also assumed that the following parameters of the system Q^\dagger are related to the corresponding parameters of the system of mass m by:

\[ H^* = \gamma h \]  

\[ m_o^\dagger/M^\dagger = m_o/m \]  

\[ k_h^\dagger/M^\dagger = k_h/m \]  

\[ l_m^\dagger/H^\dagger M^\dagger = \gamma l_m/\gamma \]  

\[ k_m^\dagger/H^\dagger M^\dagger = \gamma k_m/\gamma \]  

Analyses of strong-motion and low-amplitude test data are commonly performed for investigating the dynamic characteristics of buildings. In this approach, the fundamental frequency of the building-foundation system, ω^\dagger, is related to the fixed-base fundamental frequency of the superstructure, ω, through the equation

\[ \frac{1}{\omega^2} = \frac{1}{\omega^2} + \frac{1}{\omega_h^2} + \frac{1}{\omega_m^2} \]  

where ω_h and ω_m are the translational and rotational frequencies of the soil-foundation system, respectively. According to the definitions of these frequencies in a building-foundation system, and following some approximations suggested by Luco et al [4], these frequencies can be approximately evaluated by:

\[ \omega_h^2 = \frac{k_h}{\gamma^2 M^\dagger} \]  

\[ \omega_m^2 = \frac{k_m}{M^\dagger H^\dagger} \]  

Since the mass at the base and the centroidal moment of inertia of the top mass are not significant factors in the seismic response of SSI systems (Bielak [3]), they are neglected in this study. With this assumption, and from (6), (7), (13) and (15), it can be shown that the frequencies ω_h and ω_m in the system Q^\dagger are equal, respectively, to the frequencies ω_h^\dagger and ω_m^\dagger in the building-foundation system, which were defined in (22) and (23). In addition, inspection of (2) and (3), and the use of (16), (18) and (20) also shows that frequencies ω_h and ω_m in the system of mass m are equal, respectively, to the frequencies ω_h^\dagger and ω_m^\dagger in
the system $Q^*$. A parametric study was conducted in this investigation using these findings, assuming $\bar{\omega}^*$ and $\omega^*$ equal to the fundamental frequencies of the building-foundation and fixed-base systems, respectively, and using (21). Typical values for this parametric study were based on ambient vibration test data of typical buildings on soft soil in Mexico City (Muria et al. [5]), as well as on analytical studies (Bazan et al. [2]). The selected values for the ratios $\omega_m^e/\omega_h^e$ and $\omega^e/\bar{\omega}^e$ were 0.5 and 1.3, respectively.

3 Numerical results

Fig 6 shows displacement demands, $u$, for the SDOF fixed-base and simple SSI systems responding to the SCT record, and considering displacement ductility ratio demands, $\mu$, equal to 1, 2, 4, and 8. The parameter $\xi$ was taken equal to 0.05. The abscissa of Fig 6 represents the period of the SSI system. The results show that in a wide range of periods, the fixed-base case gives a conservative estimation of relative displacement demands in SSI systems. Nevertheless, the results show that for most of the inelastic cases, the differences of results using both systems are not significant, which suggests that inelastic relative displacement demands in SSI systems can be evaluated by using the fixed-base case and the amplified period of the SSI system.

Fig 7 shows hysteretic energy spectra, $E_H^e$, per unit mass, for the SDOF fixed-base and simple SSI systems. To construct hysteretic energy spectra, $E_H^e$ is evaluated at the end of the ground motion and plotted as a function of period, damping and displacement ductility ratio demands. As in the case of Fig 6, the abscissa of Fig 7 represents the period of the SSI system. The parameter $\xi$ was taken equal to 0.05, and $\mu$ was taken equal to 2, 4 and 8. The results show that in a wide range of periods, a reasonable estimation of $E_H^e$ in SSI systems can also be obtained using hysteretic energy spectra for the fixed-base case and the amplified period of the SSI system.

The maximum roof drift ratio in buildings, $D_{rm}$, is defined as the ratio of the maximum value of $\delta$ during an earthquake excitation and the height of the building. This parameter and $E_H^e$ have been related to seismic damage in buildings (Rodriguez [7]). As it has been shown in the literature (Qi and Moehle [6], Rodriguez [7]), by using (11) and typical values for $\gamma$, an approximate estimation of $\delta$ in regular buildings can be obtained. Following this procedure and using results such as those of Fig 6, $D_{rm}$ values for the fixed-base and SSI systems, can be evaluated. The calculated values for $D_{rm}$ and $E_H^e$ obtained in this study for the fixed-base and SSI systems indicate that in most cases of SSI systems seismic damage can be evaluated considering the fixed-base case and the amplified period of the SSI system. Caution should be taken when evaluating pounding in buildings under earthquake excitation, since this type of damage should be considered when analyzing the seismic response of SSI systems.
4 Conclusions

A parametric study of the SSI effects on the inelastic seismic response of building-foundation systems using a simple approach has been presented. The results indicate that in most cases of inelastic response SSI effects are not very significant. The results for both fixed-base and SSI systems show that in most cases of building periods, relative displacement and hysteretic energy demands in SSI systems can be obtained by using results for the fixed-base case and the amplified period of the SSI system. The results also show that a similar procedure can be followed for performing seismic damage analysis of SSI systems. Caution should be taken when evaluating pounding in buildings under earthquake excitation, since this type of damage can occur in SSI systems.

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6 References


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**Figure 1:** System considered

**Figure 2:** Idealized building-foundation system
Figure 3: Components of displacements for building-foundation system

Figure 4: System $Q^*$

Figure 5: Components of displacements for system $Q^*$
Figure 6: Displacement demands for fixed-base and simple SSI systems subjected to the SCT record; $\mu=1,2,4,8$; $\xi=0.05$
Figure 7: Demands of hysteretic energy for fixed-base and simple SSI systems subjected to the SCT record: $\mu=2.4.8; \xi=0.05$