A truss model for the confinement of concrete columns
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Abstract

In order to investigate the dependence of the ductility of a concrete column on its confinement by transverse reinforcement, a part of a column, subjected to axial compression, between two successive confined sections, is simulated by a plane truss, with bars obeying non-linear σ-ε laws of concrete or steel. The equilibrium conditions are written with respect to the deformed structure, so that instability phenomena are taken into account. The proposed model is applied on a column with square section, first unconfined and then with increasing confinement, for various values of spacing of transverse reinforcement. Finally, a method of preliminary design, based on the truss model, is proposed, for the preestimation of minimum required section of transverse reinforcement assuring ductility.

1 Introduction

During the response of a tall reinforced concrete frame to a strong earthquake, the columns of lower stories, particularly those of groundfloor, must have sufficient ductility at their ends, which are potential plastic hinge regions, so that the frame do not collapse. An unconfined concrete column has a very small ductility. However, when it is confined by transverse reinforcement, closely spaced over the plastic hinge region and multiple per section, the ductility is significantly increased.
The last 25 years, a lot of experimental results have been published concerning the confinement of concrete columns [1, 2, 4-7]. Many of them has been performed under concentric axial compression of the column. And it is accepted that the strength and ductility of concrete in a confined column, under concentric compression, are conservative predictions of those under eccentric compression [4]. Based on the above test results, empirical formulae have been developed for the description of stress-strain behavior of confined concrete columns.

In present work, a theoretical model is presented for the confinement of columns subjected to axial compression, based on the simulation of a part of the column by a plane truss. Similar truss models have been presented in previous works [3]. However, the present model is provided with the additional feature of writing the equilibrium conditions with respect to deformed structure, so that instability phenomena are taken into account. As the problem of confinement is mainly a problem of structural instability, local due to spalling, as well as global due to buckling of interior vertical concrete struts.

2 The proposed truss model

A concrete column is considered, with rectangular section bxd, subjected to axial compressive load P (fig. 1a). No longitudinal, but only transverse reinforcement is taken into account, with bars parallel to the principal directions of the section. The s is the spacing between successive confined sections, whereas D is the diameter of transverse bars. No concrete cover, but only the confined core is taken into account.

A part of the column, between two successive confined sections, is simulated by a plane truss. This truss has two axes of symmetry. However, a probable instability destroys the symmetry with respect to vertical axis. So, finally, the half of this truss is studied by exploit only of symmetry with respect to horizontal axis.

In most cases, a simple discretization is used, consisting of a single horizontal series of elementary square trusses, as shown in fig. 1b for the case s/d=1/2. The determination of concrete bar sections of model reduces to the simulation of a square prism of concrete (fig. 1c) by an elementary
square truss, from which the sections $A_h$ of horizontal and vertical concrete bars, as well as $A_d$ of diagonal bars are obtained:

$$A_h = 0.4167 \, \text{ab} \quad \text{and} \quad A_d = 0.2946 \, \text{ab} \quad (1)$$

The bars of model obey nonlinear $\sigma$-$\varepsilon$ laws of concrete and steel (fig. 1d,e). The incremental loading of columns is achieved by successive prescribed displacements downwards of the upper supports of the truss.

3 Applications

The proposed model is applied on a column of square section with side 40 cm (fig. 2a), first unconfined and then confined by 4 transverse steel bars per principal direction of section. The spacing of transverse reinforcement takes the values $s$: 80, 40, 20, 10, 5 cm, whereas the diameter $D$ of bars ranges from 1 mm up to 14 mm. The $\sigma$-$\varepsilon$ curve of concrete bars has significant differences from the experimental curve of unconfined concrete (fig. 2cd): 1. The tensile strength of concrete bars is not zero, but much smaller than the experimental one. 2. The compressive strength of bars is greater and the ductility much larger than the experimental ones.

Now, let us examine the cases of various values of spacing $s$ of transverse reinforcement.

Case $s = 80$ cm ($s/d=2$). First an elementary square truss model is considered (fig. 3a), for which the stress-strain curve $\sigma$-$\varepsilon$ of unconfined column (fig. 3b) is not improved when any strong confinement is used, as this specimen always fails by spalling. Then, a more detailed discretization is used (fig. 3c), which describes spalling more accurately, so that, for increasing transverse reinforcement diameters $D$, the spalling is postponed, thus the strength and ultimate strain of specimen are slightly increased. For $D>11$ mm, no more improvement is observed (fig. 3d).

Case $s = 40$ cm ($s/d=1$). A simple discretization consisting of two elementary square trusses is used. In the unconfined specimen (fig. 4a), the spalling happens at same time with global instability. In the confined specimen (fig. 4b), more the global instability and less the spalling are postponed. Only in this case, unilateral spalling is observed, because the inclination of interior vertical strut destroys the symmetry.

In fig. 4c, for increasing transverse reinforcement diameter $D$ up to 6 mm, the strength and ductility of column are increased. In fig. 4d, greater
diameters are tried. For $D = 7\text{mm}$ up to $10 \text{ mm}$, the strength remains constant, whereas the ductility significantly increases. However, it is still observed that the strength becomes suddenly zero, because of buckling of interior vertical struts.

Finally, for $D > 11 \text{mm}$, the global instability is prevented and the full ductility potential of concrete is exploited. However, this ductility is not unlimited, too. For column strain $\varepsilon = 0.030$, crushing of vertical struts begins. After completing of this crushing, a small portion $\sigma$, of column strength remains. Finally, for a large $\varepsilon = 0.085$, the transverse reinforcement is fractured in tension.

Cases $s < 20 \text{ cm}$ ($s/d < 1/2$). In fig. 5, for each one of the values of $s/d = 1/2, 1/4, 1/8$, the $\sigma$-$\varepsilon$ curves of column are represented for $D$ ranging from $\varnothing$ up to the minimum required value for ductility.

In fig. 6, for every of ratio $s/d$ under consideration, the envelope $\sigma$-$\varepsilon$ curve of column is drawn, corresponding to the minimum required $D$ for ductility. For the case $s/d = 1$, for safety reasons, bilateral spalling was assumed. In the diagram of fig. 6, it is clearly shown that the two main parameters of confinement, the minimum required diameter $D$ of transverse reinforcement for ductility on one hand and the ultimate compressive strength of column on the other, strongly depend on the ratio $s/d$.

4 Preliminary design

Based on the proposed truss model, a method of preliminary design has been developed for the preestimation of the minimum required section $A_v$ of transverse reinforcement assuring full exploit of ductility potential of concrete.

The failure mode of truss model of fig. 7 is considered: The transverse reinforcement has yield, an extended cracking of diagonal concrete bars exists and an interior vertical concrete strut, bearing its ultimate compressive load $V_u$, tends to buckle, by pushing the whole part of the structure to the direction that the strut buckles.

The interrupted lines, in fig. 7, represent cracked bars. So, the equilibrium of the right part of structure, along the horizontal direction, can be written:

$$\Sigma F_x = V_u \frac{u_v}{a} - S_v = 0, \text{ thus } \frac{u_v}{a} = \frac{S_v}{V_u},$$  \hspace{1cm} (2)
where $u/a$ is the maximum possible inclination of the vertical strut under consideration and $S_u$ is the tensile strength of transverse reinforcement.

If $u$ is the horizontal parallel translation of the right part of the structure, as shown in fig. 7, we can write the stiffness relation for this displacement, within a step of the incremental loading process, in which the equations of the problem are linearized, as follows:

$$(K_{ve} - K_{vg} + K_{sh})u + p = 0.$$  

As shown in this equation, the stiffness coefficient is a sum of three terms, which are described below.

1. $K_{ve} = (E_\infty A_y/a) \cos^2 \theta = (E_\infty x 2 \times 0.417ab/a) (u/a)^2$  
   is the elastic stiffness of the strut, where $E_\infty$ the unloading elasticity modulus of concrete, equal to the initial one, and $A_y$ the section of an interior vertical strut. By substitution of eq. 2 to eq. 4, is obtained

$$K_{ve} = 0.8333 E_\infty b (S_u/V_u)^2$$  

2. $-K_{vg} = -V/a = -\sigma_{\infty} x 2 x 0.4167 ab/a = -0.8333\sigma_{\infty} b$  
   is the negative geometric stiffness of the vertical strut

3. $K_{sh} = E_\infty A/\alpha$  
   is the stiffness of transverse reinforcement, where $A_\alpha$ is its section and $E_\infty$ the elasticity modulus in the strain-hardening region.

Beginning from $A_\alpha=0$ and gradually increasing the $A_\alpha$, we find a value $A_{\alpha m}$ such that, for every $A_\alpha>A_{\alpha m}$, the total stiffness coefficient of eq. 3 becomes positive. This $A_{\alpha m}$ is the sought minimum required $A_\alpha$ assuring the ductility of column.

5 Conclusions

A theoretical model has been proposed for the confinement of concrete columns, subjected to concentric axial compression, based on the simulation of a part of the column by a plane truss. The bars of the truss obey nonlinear uniaxial stress-strain laws of concrete or steel, whereas the equilibrium conditions are written with respect to the deformed structure.

The proposed truss model has been applied on a column with square section, for various values of the ratio s/d of spacing of transverse reinforcement to column section depth. The results of applications showed that, for a relatively weak confinement the full exploit of compressive
strength of concrete is achieved, whereas a large section of transverse reinforcement is required in order to assure the full ductility potential of concrete.

Based on the proposed truss model, a method of preliminary design has been developed, by the use of a hand calculator. This method preestimates, in a simple and clear way, the minimum required section of transverse reinforcement assuring ductility, based on the failure mode of truss model which leads to global instability. In this state, there is an extensive cracking of concrete and the transverse reinforcement is called to prevent the buckling of vertical concrete struts.

A remarkable approximation has been observed between the results of preliminary design and those of computer analysis.

6 References


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Figure 1. a. Confined column under axial compression. b. Truss model. c. Concrete prism corresponding to elementary truss. d. Stress-strain laws of concrete and steel bars.

Figure 2. a. Specimen of applications. b. Steel σ-ε curve of model bar compared to experimental one. b. Concrete σ-ε curve of model bar compared to experimental one of unconfined concrete. d. Tension region of σ-ε curve of concrete bar.
Figure 3. Case s/d=2. a. Force-deformation states of elementary model. b. Column stress-strain curve of elementary model. c. Force-deformation states of detailed model. d. Column stress-strain curves of detailed model.

Figure 4. Case s/d=1. a, b. Force-deformation states of unconfined and confined model, respectively. c. Column $\sigma$-$\epsilon$ curves for D ranging from 0 up to 6 mm. d. Column $\sigma$-$\epsilon$ curves for D ranging up to 11 mm.
Figure 5. Column $\sigma$-$\varepsilon$ curves for $D$ ranging from $0$ up to the minimum required value for ductility. a. Case $s/d=1/2$. b. Case $s/d=1/4$. c. Case $s/d=1/8$.

Figure 6. Envelope $\sigma$-$\varepsilon$ curve of column and minimum required $D$ for ductility, for various values of ratio $s/d$.

Figure 7. Failure mode of truss model.