Effect of rate on strength and energy dissipation of concrete materials

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Abstract

The effect of rate of loading on the entire deformation history of concrete materials, including plastic damage, is considered. Energy dissipation in the linear (viscoelastic) range is described. One-dimensional results are presented and comparison is made with experimental results in tension, at different rates, from quasi-static to impact.

Introduction

A viscoelastic-plastic model in series (i.e. with an additive strain decomposition) was proposed by Panoskaltsis et al. [1], [2] for the description of concrete materials up to and beyond failure, including the effect of rate. The additive decomposition of the strain (as opposed to a viscoplastic formulation) serves to describe time effects during the entire history of the material. As is well known, concrete creeps at very low levels of stress and dissipates energy before yield. Elasto/viscoplasticity provides an alternative framework for the description of rate effects, see for example Bicanic and Zienkiewicz [3]. Although a viscoplastic model can describe rate effects, it cannot account for hysteretic behavior (energy dissipation) before yield. The additive decomposition of strain is also consistent with the observation that
energy dissipation in concrete materials is due both to the creep mechanism, attributed to the movement of water in the calcium silicate hydrate and of the water held in small capillaries, as well as to the propagation and coalescence of microcracks in the transition zone, the latter manifesting itself in later stages of deformation.

The viscoelastic component of strain is described by the modified Kuhn model of Lubliner & Panoskaltsis [4]. The model is capable of representing concrete behavior over a broad range of rates, from quasistatic to dynamic, permitting a calculation of damping. Motivation for the use of the modified Kuhn model comes from its accurate prediction of the effect of frequency on energy dissipation. The behavior of the model under cyclic loads has been studied in detail in Panoskaltsis & Bahuguna [5], Panoskaltsis et al. [6]. The model also accounts for instantaneous elasticity, logarithmic creep in the long-time limit and elastic response in the high-frequency limit. With the help of the viscoelastic model, the viscoelastic characteristics of the material - such as creep and energy dissipation (damping) in the linear range - are mathematically tied together. Creep and damping have traditionally been treated as two different phenomena, though they both are a manifestation of the rate dependence of this class of materials.

The plastic component of strain, which includes damage in the sense of loss of cohesion, is described by the model of Lubliner et al. [7], suitably modified to allow for the prediction of the effect of stress rate on the strength of the material. The plastic-damage model of Lubliner et al. [7] contains the uniaxial stress at initial yield (compressive and tensile) as a material parameter. By introducing, as was originally done in [1], the additive decomposition

\[ \varepsilon = \varepsilon^c + \varepsilon^c + \varepsilon^p \]  

without suitable modification, the resulting model will not, therefore, predict the effect of rate on ultimate strength. The important issues of the description of permanent deformation as well as of the concept "plasticity" of concrete has been discussed in detail in Panoskaltsis & Bahuguna [5]. Both hardening and softening are accounted for and are described by the evolution equations of physically motivated internal variables, contained in the yield criterion.

**The viscoelastic model**

The viscoelastic modified Kuhn model [4] has been described in both a functional form, through its creep or relaxation function, and in rate form, by means of evolution (rate) equations of the viscoelastic internal variables. For linear viscoelasticity the two descriptions are equivalent. The internal variable representation, however, is compatible with the rate theory of plasticity and is more easily extended to accommodate nonlinear phenomena and multi-dimensional load histories. The nonlinear viscoelastic rate equations
for a material, which does not exhibit viscoelasticity in its volumetric response are as follows:

\[
\dot{q}_m + \frac{C}{r^m} q_m = \frac{C}{r^m} \frac{\Gamma}{\sqrt{3} J_2} \left( \sinh \frac{\sqrt{3} J_2}{\Gamma} \right) s B \ln r
\]

(2)

\[
\epsilon^v = \sum_{m=0}^{N} q_m ,
\]

(3)

where \( J_2 \) is the second invariant of the stress deviator, i.e.

\[
J_2 = \frac{1}{2} s : s
\]

(4)

The evolution equation for each component of the viscoelastic strain is a nonlinear and three-dimensional generalization of the rate equation for the strain in each Kelvin element of a rheological model, shown in Figure 1, with spring compliance in each element \( B \ln r \), where \( r > 1 \), distributed relaxation times \( r^m / C \), and viscosity coefficients \( \eta_m = \tau_m G_m = \frac{r^m}{CB \ln r} \). The parameter \( \Gamma \) may be defined as the stress, at which nonlinear creep becomes noticeable.

\[
\eta_m = \frac{1}{B \ln r} \quad G_m = \frac{r^m}{C}
\]

\[
\tau_m = \frac{r^m}{C} \quad m = 0, \ldots, N
\]

\[
\eta_m = \tau_m G_m = \frac{r^m}{CB \ln r}
\]

Figure 1. Rheological representation of the discrete modified Kuhn model

Energy dissipation is described by the loss tangent, defined as the tangent of the phase angle, by which the strain lags behind the stress in the case of oscillatory excitation. Recent experiments by Panoskaltsis [2] confirm the fact that energy dissipation is much less sensitive to the frequency of excitation, than, say, predicted by the Kelvin model, frequently used in structural dynamics. From the equations of state (2), the loss tangent of the linear model \( (\Gamma \to \infty) \) is found to be
\[
\tan \delta = \frac{\sum_{m=0}^{N} \frac{q r^m}{1 + q^2 r^2 m}}{\beta \ln r + \sum_{m=0}^{N} \frac{1}{1 + q^2 r^2 m}},
\]

where \( \beta = B / A \) (A is the external spring compliance), \( q = \omega / C \) and \( N+1 \) is the number of Kelvin elements in the modified Kuhn model. Details of the derivation are omitted and the reader is referred to Panoskaltsis [2] and Panoskaltsis et al. [8].

Figure 2. Loss tangent versus \( \omega \): experimental and fitted analytical results

Figure 2 shows experimentally obtained values of the loss tangent (Panoskaltsis [2]), under sinusoidal loading, of concrete specimens in compression, at frequencies 0.5 to 16 Hz, typical of earthquake excitation. The fitted curve is obtained from a 16 element (AM5) discrete modified Kuhn model.

**Rate-dependent plasticity**

The evolution of the plastic strain (flow rule) is of the classical Prandtl-Reuss type, i.e.

\[
\dot{\varepsilon}^p = \lambda \frac{\partial F}{\partial \sigma},
\]

in which \( F(\sigma) \) is a homogeneous function of the first degree in the stress components, identified here with the yield function (associated plasticity). The yield criterion is of the general form

\[
F(\sigma) = c,
\]

where \( c \) represents the cohesion. For concrete materials, the progress of damage, through the loss of strength, can be described by the loss of cohesion.
For the cohesion to be unambiguously defined, it is necessary that the yield function be homogeneous of the first degree in the stress components. A discussion of several yield functions for concrete is provided in Chen [9]. Here we use the failure surface developed by Lubliner et al. [7], which is homogeneous of the first degree and represents experimental data for concrete in both compression and tension better than the Mohr-Coulomb and Drucker-Prager criteria. The form of the yield function is

$$F(\sigma) = \frac{1}{1 - \alpha} \left( \sqrt{3J_2} + \alpha I_1 + \beta \sigma_{\text{max}} - \gamma \langle -\sigma_{\text{max}} \rangle \right),$$

where $\alpha, \beta, \gamma$ are dimensionless parameters, $I_1 = \text{tr}\sigma$ is the first invariant of the stress tensor and $\sigma_{\text{max}}$ is the algebraically largest principal stress. The symbol $\langle \rangle$ denotes the MacCauley bracket of the argument, i.e.

$$\langle x \rangle = \begin{cases} x & x > 0 \\ 0 & x \leq 0 \end{cases}.$$  

In this yield function, the algebraically largest principal stress is used instead of the third invariant of the stress deviator $J_3$, commonly found in failure criteria for quasi-brittle materials. The parameters $\alpha, \beta$ and $\gamma$ determine the shape of the yield surface.

Following the developments in Lubliner et al. [7], the cohesion $c$ is determined from a plastic-damage hardening-softening variable $\kappa$, which in one-dimension is defined as the plastic work

$$\kappa = \frac{1}{g_{c,t}} \int_0^{\varepsilon_p} \sigma d\varepsilon_p,$$

normalized by the area of the complete $\sigma - \varepsilon_p$ curve, denoted by $g_c$ in the case of compression and by $g_t$ in the case of tension, so that the non-dimensionalized $\kappa$ ranges from zero to one. A three-dimensional generalization results in the following hardening-softening rule (evolution equation) for $\kappa$:

$$\dot{\kappa} = \frac{c(\kappa)}{g_{c,t}} \left[ f_0^C r(\sigma) \varepsilon_{p,\text{max}} - (1 - r(\sigma)) \varepsilon_{p,\text{min}} \right],$$

where $f_0^C, f_0^T$ denote, respectively, the initial "yield" stress in unidirectional compression and tension,

$$r(\sigma) = \frac{\sum_{i=1}^{3} \langle \sigma_i \rangle}{\sum_{i=1}^{3} |\sigma_i|}$$

and $\sigma_i$ are the principal values of stress. Under the assumption of similarity between the stress-strain curves in tension and compression (Chen [9]), an
appropriate [7] direct (functional) relation for $c(\kappa)$, which can be used in both tension and compression is:

$$c(\kappa) = \frac{f_0^{c,t}}{a} \left[ (1 + a) \sqrt{\varphi(\kappa) - \varphi(\kappa')} - \varphi(\kappa) \right]$$

(13)

$$\varphi(\kappa) = 1 + a(2 + a)\kappa$$

If we consider one-dimensional compression and tension, the yield criterion will simply be $\sigma = c$. The $\sigma - \varepsilon^P$ curve is transformed into a $\sigma - \kappa$ curve through the above definition of $\kappa$. The variable $\kappa$ never decreases and increases only when plastic deformation takes place but, unlike classical plasticity, it only increases up to a limiting value, which here is unity. When $\kappa$ becomes unity, the cohesion $c$ vanishes and at this point total damage has occurred, meaning that no load can be carried further. This occurrence may be intuitively associated with the formation of a macroscopic crack, which cannot transfer loads. Clearly, the adopted plasticity model is isotropically hardening-softening. This is in agreement with the experimental results of Mindess and Diamond [10], that the distribution of microcracks follows discontinuous patterns with random orientations. The normalization of $\kappa$ by "global" quantities, such as $g_c$ and $g_t$, is also meant to describe the nonlocal nature of the softening branch.

*Rate-dependent initial yield.* As has been experimentally established, concrete is rate-dependent in both tension and compression, the effect being more pronounced in tension than in compression, as noted by Suaris & Shah [11] and by Ross et al. [12]. The multiplicative decomposition (1) can describe rate dependence through the viscoelastic part of the strain, but not the dependence of the strength on rate, since this value is fixed by the yield criterion. The unidirectional stress at initial yield, in either tension or compression, is

$$f_0 = E(\varepsilon_0 - \varepsilon^V_0),$$

(14)

where $\varepsilon_0$ and $\varepsilon^V_0$ are the total and viscous strains, respectively, corresponding to initial "yielding" of the material. Let $f_0^{c,t}$ at a quasistatic rate be a known property of the material and let the corresponding quantity $E\varepsilon_0$, denoted $\bar{f}_0^{c,t}$, be a material parameter, independent of rate. At any given rate the stress is determined from

$$\sigma(t) = \bar{f}_0 - E \varepsilon^V(t).$$

(15)

When the yield criterion is met, this defines the new initial "yield" stress, corresponding to the given rate, from which the cohesion is determined according to equation (13). The above procedure describes the rate dependence of the strength under any conditions (i.e. stress, strain or mixed control).
Figure 3 shows experimental results in direct tension, obtained by Gopalaratnam and Shah [13] at a quasistatic rate and the corresponding model prediction. The effect of rate has been experimentally studied by Albertini et al. [14], who developed a modified Hopkinson bar of a square shape 6cm x 6cm. They used concrete specimens 6cm x 6cm and of several lengths 3-15cm and subjected them to impact rates 1-15 s\(^{-1}\) in tension. At lower (earthquake) rates, the effect of rate on strength is not significant, as shown by several authors, see, for example, Fintel [15]. In Table 1 we show a classification of rates, depending on loading conditions. The unified model is capable of reproducing the experiments of Albertini et al. [14] with satisfactory accuracy. Figure 4 shows fitted and experimental results at three different rates.

![Graph showing experimental and fitted results at a quasistatic rate.](image)

Figure 3. Experimental (Gopalaratnam and Shah [13]) and fitted results at a quasistatic rate.

<table>
<thead>
<tr>
<th>Loading Conditions</th>
<th>Strain Rate (s(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quasi-static</td>
<td>10(^{-5})</td>
</tr>
<tr>
<td>Earthquake</td>
<td>10(^{-5})</td>
</tr>
<tr>
<td></td>
<td>10(^{-4})</td>
</tr>
<tr>
<td>Impact</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>10</td>
</tr>
</tbody>
</table>
Figure 4. Experimental (Albertini et al. [14]) and fitted results at three different rates.
Controversy exists regarding the existence of the falling branch of the stress-strain curve in tension and in compression. Gopalaratnam and Shah [13] report that the curve in the post-peak softening regime in tension is not unique and they proceed to model a stress-crack opening displacement relationship. In their experiments, Albertini and coworkers [14] use the Hopkinson bar formula to determine the stresses, while they obtain the strains from both the Hopkinson bar formula and from direct measurements, by means of an electrical strain gauge glued on the specimen. They report that both methods give results, which are in good agreement in the hardening part of the curves. The softening branch can only be obtained by the Hopkinson bar formula, since the strain gauge breaks when the highest strength is reached. The Hopkinson bar formula for strain, however, holds only for an uncracked specimen. Our approach is that, by means of a stiff testing machine, the softening branch can be obtained (in a displacement control experiment) and that, therefore, a continuum model, capable of describing localization and the associated mesh sensitivity, may be used to describe the macroscopic behavior of the material.

Conclusions

A unified model for concrete in tension and in compression has been presented. The model is capable of describing energy dissipation and the effect of rate, from quasistatic to earthquake to impact, on the complete history of the material. One dimensional results have been obtained. Comparison was made with experiments in compression, for energy dissipation (damping) under harmonic load, and with tensile experiments at a quasistatic and a series of impact rates. The extension to multi-dimensional problems and the numerical algorithms needed in the context of finite element computations will be presented in a forthcoming publication.

References


2. Panoskaltsis, V.P., Rate effects in the constitutive modeling of frictional materials including plasticity and damage, Ph.D. Dissertation, Dept. Civil Engineering, University of California, Berkeley, 1992.


