Three dimensional non-linear analysis of building pounding during earthquakes
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Abstract

The three dimensional pounding phenomenon of two adjacent buildings during earthquakes with aligned and non aligned rigid horizontal diaphragms is investigated for linear and non linear structural response. The developed formulation takes into account the dynamic contact condition for the velocities and accelerations in three dimensions. The contact points are determined geometrically from the displacements of the diaphragms’ centre of mass. The results of the proposed formulation are compared with those based on the Lagrange multipliers approach. Test results are performed for two adjacent buildings with five storeys subjected to real earthquake motions, with elastic and inelastic structural response.

1 Introduction

The pounding phenomenon of adjacent buildings during earthquakes has been receiving considerable attention in recent years. This is due to the fact that many incidents of seismic pounding have been recorded in many parts of the world. Early work in the idealisation of the pounding phenomenon utilised single-degree-of-freedom (SDOF) oscillators in order to simplify the problem and produce some qualitative results on the behaviour of structures under pounding. It was soon realised that, in order to capture the response of actual structures, multiple-degree-of-freedom (MDOF) idealisations should be used. The majority of the cases the MDOF systems are restricted to two dimensional idealisations while three dimensional model studies are presented under several
restrictions by Liolios\textsuperscript{1} and Maison and Kasai\textsuperscript{2}. A recent work by Papadrakakis et al\textsuperscript{3} employed a three dimensional finite element model which takes into account the dynamic contact condition in space using the Lagrange multipliers method. A state-of-the-art review on earthquake induced pounding was recently presented by Anagnostopoulos\textsuperscript{4}.

In this work the three dimensional pounding of two or more adjacent buildings is viewed as a contact-impact problem between rigid slabs or rigid slab and column in aligned and non aligned floor levels. The simulation scheme developed contains the following salient features:

(i) The structures are modelled as MDOF systems with finite elements assuming rigid slab response at each floor.

(ii) Pounding may occur at different floor levels, allowing the activation of multiple contact locations along the height of the buildings, while the contact points are not known a priori and are located through a searching procedure among the candidate contact surfaces.

(iii) Newton’s laws in their integrated form involving impulse and momentum are applied directly. The interaction process between the two colliding bodies is modelled using the coefficient of restitution $e$ and the ratio $\mu$ of tangential to normal impulses.

(iv) The duration of contact is short. As a result velocity changes are nearly instantaneous and changes in position and orientation during the impact are negligible while effects of other forces can be disregarded.

(v) The proposed scheme can be incorporated with minor modifications into existing computer codes for dynamic analysis of buildings with elastic or inelastic material response.

Test results are performed for two adjacent buildings with five storeys subjected to a real earthquake motion taking into account elastic and inelastic response. A comparison between the present formulation and the Lagrange multipliers approach is also presented.

2 Building Coupling during pounding

The basic conditions of contact along the generic surfaces are that no material overlap can occur and, as a result, contact forces are developed that act on the point of contact.

2.1 Post impact velocities

Taking into account that the duration of impact is very short the unknown rebound velocities (translational and rotational) of the centre of mass of each diaphragm are determined from the velocities prior to impact using the coefficient of restitution $e$ and the coefficient of friction $\mu$. The coefficient of restitution accounts for local energy absorption mechanisms and is defined for the central collision in the classical theory of impact as
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Based on the data provided in Figure 1 the post impact velocities are obtained from the following expressions:

\[
\begin{align*}
\ddot{u}_{rn}^+ &= -e\ddot{u}_{rn} \\
\ddot{u}_{1n}^+ &= \ddot{u}_{1n} - \frac{\bar{m}(1+e)q\ddot{u}_{rn}}{m_1} \\
\ddot{u}_{2n}^+ &= \ddot{u}_{2n} - \frac{\bar{m}(1+e)q\ddot{u}_{rn}}{m_2} \\
\ddot{u}_{1t}^+ &= \ddot{u}_{1t} + \mu\frac{\bar{m}(1+e)q\ddot{u}_{rn}}{m_1} \\
\ddot{u}_{2t}^+ &= \ddot{u}_{2t} - \mu\frac{\bar{m}(1+e)q\ddot{u}_{rn}}{m_2} \\
\omega_1^+ &= \omega_1 - \frac{\bar{m}(1+e)d_{cd}q\ddot{u}_{rn}}{I_1} \\
\omega_2^+ &= \omega_2 + \frac{\bar{m}(1+e)d_{ab}q\ddot{u}_{rn}}{I_2}
\end{align*}
\]

with

\[
qu = \frac{1}{1 + \bar{m}d_{c_c}^2 + \bar{m}d_{a_d}^2 - \mu \left( \frac{\bar{m}d_{c_d}d_{d_c}}{I_1} + \frac{\bar{m}d_{a_c}d_{d_b}}{I_2} \right)}
\]

\[
d_{cd} = d_c - \mu d_b \\
d_{ab} = d_a - \mu d_b \\
\bar{m} = \frac{m_1m_2}{m_1 + m_2}
\]

In the above expressions the subscripts n,t account for the normal and tangential components of the velocities, while the superscripts +,- correspond to post impact and prior to impact velocities, respectively.

When pounding occurs between a column and a diaphragm the post impact velocities are given by

\[
\begin{align*}
\ddot{u}_{1n}^+ &= \ddot{u}_{1n} + \frac{\bar{m}(1+e)q\ddot{u}_{rn}}{m_1} \\
\ddot{u}_{2n}^+ &= \ddot{u}_{2n} - \frac{\bar{m}(1+e)q\ddot{u}_{rn}}{m_2} \\
\ddot{u}_{1t}^+ &= \ddot{u}_{1t} + \mu\frac{\bar{m}(1+e)q\ddot{u}_{rn}}{m_1}
\end{align*}
\]
in this case $d_c = d_d = 0$.

2.2 Equation of motion

The typical equilibrium equation of motion for the centre of mass of the diaphragms for each building prior to impact can be written as

$$[M]\{\ddot{u}^-\} + [C]\{\dot{u}^-\} + \{F(u^-)\} = -[M]\{r\}\ddot{u}_g$$

in which $[M]$ is the mass matrix, $[C]$ is the damping matrix, $\{F(u^-)\}$ is the restoring force vector, $\{\ddot{u}^-\}$, $\{\dot{u}^-\}$ and $\{u\}$ are the relative acceleration, velocity and displacement vectors respectively. $\ddot{u}_g$ is the ground acceleration and $\{r\}$ is the vector relating the input ground motion to the structural degrees of freedom. The time integration of dynamic response is performed with Newmark’s constant acceleration method.

2.3 Post impact accelerations

Based on the assumption that the duration of impact is very short, the displacements and the ground acceleration before and after impact are considered the same. The post impact accelerations are obtained by

$$\{\ddot{u}^+\} = \{\ddot{u}^-\} - [M]^{-1}[C](\{\dot{u}^-\} - \{\ddot{u}^+\})$$

The corrected velocities and accelerations as defined in equations (2-4) and (9) are used as initial conditions for the next integration step.

2.4 Geometric contact conditions

The geometric contact conditions are defined by the relative position of the neighbouring segments AB and CD of the diaphragms as shown in Figure 2. The possible positions of the two segments are the following: (i) intersection inside the segments, (ii) intersection outside the segments, (iii) the two segments are parallel and coincident. The contact point for each case is determined from the known geometric quantities of the diaphragms shown in Figure 2. When the two segments coincide, the contact point is assumed to act at the midpoint of the common interval.
3 Comparison of the proposed method with the Lagrange multipliers approach

The problem of pounding between adjacent buildings during earthquakes was also treated as contact-impact problem in two and three dimensions using the Lagrange multipliers method\(^3\). In this approach the geometric compatibility condition due to contact are enforced with the presence of Lagrange multipliers, while the impulse-momentum relationship is automatically taken into account through the direct integration scheme of the equations of motion. Numerical results for two dimensional building pounding obtained with the Lagrange multipliers method were found to be in good agreement with experimental tests performed on a shaking table\(^7\).

In this section the pounding response of two buildings (flexible-stiff), shown in Figure 3, with two floor diaphragms, is compared using the proposed procedure and the Lagrange multipliers approach. The acceleration record of Kalamata earthquake of Figure 4 was used as input motion. The comparative results depicted in Figures 5 and 6 show that the two methods produce almost identical results.

4 Test Example

4.1 Pounding of two buildings with five storeys

The two buildings shown in Figure 7 with aligned floor levels are studied under Kalamata earthquake. The following cases are investigated: (i) dynamic linear behaviour without pounding, (ii) dynamic non-linear behaviour without pounding, (iii) dynamic linear behaviour with pounding, (iv) dynamic non-linear behaviour with pounding.

4.2 Design Concept of the buildings

Two reinforced-concrete frames having five storeys are considered as structural models. Figure 8 shows the dimensions of these frames in which the beam and column sizes as well as reinforcement details are depicted. The two buildings were designed for combined gravity and seismic effects in accordance with the Eurocode-8. The buildings were designed for peak ground acceleration equal to 0.25g. The general program DRAIN-TABS\(^8\) was used to perform elastic and inelastic dynamic analysis. The dual-component element was used to model the beams and columns. The effect of axial force on yield moment was considered for each column. Axial deformation was not considered for the beams due to the assumption of rigid diaphragms in their own planes. A mass proportional damping is assumed of the form

\[
[C] = \alpha[M]
\]
where \( \alpha = \frac{2\pi \xi}{\omega} \). The numerical simulation of the pounding phenomenon, as described previously, has been incorporated into the DRAIN-TABS program.

### 4.3 Analytical results

Figure 9 shows the displacement response of the centre of masses of impacting 5th floor levels of the buildings for excitation in x-x direction by the N-S component of the Kalamata earthquake, while Figure 10 shows the corresponding input energy demands of the two buildings. Figure 11 shows the rotation responses of the diaphragms of the 5th floor levels of the buildings when both components N-S and E-W of the Kalamata earthquake are considered. It can be seen that the response of the stiff building with pounding suffers large rotations. When pounding is occurring the rotations of both buildings with inelastic response are increased by a factor of 3 compared to those without pounding. In the case of elastic response, however, the rotation of the flexible building are increased by a factor of 15 compared to the inelastic response. It has to be noticed that in the case of no pounding the buildings have zero rotation.

### 5 Conclusions

(i) The proposed formulation provides with an efficient and reliable tool for studying the pounding phenomenon of adjacent buildings during earthquakes taking into account three dimensional simulation and non-linear structural response.

(ii) When eccentric pounding occurs between two buildings the structures may suffer large rotations.

(iii) The induced input energy in both structures with pounding and non-linear response is less than the corresponding induced energy with pounding and linear response.

### 6 References


3. Papadrakakis M., Apostolopoulou C., Zahraropoulos A. & Bitzarakis S., A three dimensional simulation of structural pounding during


Figure 1. Free body diagrams of bodies colliding in a plane.

Figure 2. Geometric representation of two floor diaphragms.
Figure 3. General arrangement of adjacent two storey buildings: (A) stiff, (B) flexible.

Figure 4. Kalamata earthquake acceleration records: (a) E-W, (b) N-S.
Figure 5.
Stiff building. Displacement response: (a) 1st floor, (b) 2nd floor.
Figure 6. Flexible building. Displacement response: (a) 1st floor, (b) 2nd floor.
Figure 7. General arrangement of adjacent five storey buildings: (A) stiff, (B) flexible.

Figure 8. General plan of a typical storey and reinforcement details; (a) stiff, (b) flexible.
Figure 9. Fifth floor displacement response: (a) stiff, (b) flexible.
Figure 10. Input energy demands: (a) stiff, (b) flexible.
Figure 11. Fifth floor rotation of the centre of mass: (a) stiff, (b) flexible.