Earthquake interaction between adjacent buildings under friction effects

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Abstract

A numerical approach is presented for an inequality problem arising in the earthquake analysis of adjacent buildings. This problem concerns the unilateral contact-impact between neighboring buildings, when friction and P-delta effects are taken into account. The proposed numerical approach is based on a double discretization, in space and time, and on optimization methods. First, the Finite Element Method is applied in space. A piecewise linearization for the nonconvex contact laws and for the Coulomb frictional behaviour is used. Next, with the aid of Laplace transform, the linear equality problem conditions are transformed to convolutional ones, involving as unknowns the unilateral quantities only. So the number of unknowns is significantly reduced. Then, by using a time marching scheme, in each time step a nonconvex linear complementarity problem is solved. Finally, the proposed method is illustrated by means of a numerical example and some conclusions useful for the civil engineering praxis are discussed.

1 Introduction

Interaction among adjacent buildings is often a main cause of damages in seismically active regions [1]. Thus a numerical estimation of the interaction effects to earthquake response of such buildings is significant for their earthquake resistant design, construction and repair.

The problem is very difficult from various aspects. Mathematically this problem of pounding of structures belongs to inequality problems of mechanics, where the governing conditions are equalities as well as inequalities. These so-called unilateral problems can be treated mathematically by the variational or hemivariational inequality concept [2,12,16]. As regards the numerical treatment of such inequality problems
in seismic mechanics, some numerical approaches have already been presented, see e.g. [3,4,8] and Liolios [5,6].

In this paper a special case of seismic building interaction is treated numerically. This case concerns the unilateral elastoplastic-softening contact with Coulomb's law of friction between adjacent buildings, when P-Delta effects are also taken into account. The numerical procedure is based on an incremental problem formulation and on a double discretization, in space by the finite element method and in time by using the Laplace transform method combined with a time-marching scheme. The generally nonconvex constitutive contact laws are piece-wise linearized, as has been successfully done by Maier in structural elastoplasticity [9,10]. So, in each time-step a nonconvex linear complementarity problem with reduced number of unknowns is solved. Finally, the method is applied to a civil engineering example and some useful conclusions are discussed.

2 Method of analysis

For simplicity, a system of two adjacent linearly elastic buildings (A) and (B) is considered here. The extension to systems with more than two nonlinear elastic buildings can be done in a straightforward way.

2.1 Uncoupled system analysis

First the system of the two buildings (A) and (B) considered as uncoupled is discretized by the finite element method. Assuming no interaction and no P-Delta effects, the matrix equations of dynamic equilibrium are

\[ M_L \ddot{u}_L + C_L \dot{u}_L + K_L u_L = - M_L \ddot{u}_g, \quad (L = A, B), \]

where \( M_L, C_L, K_L \) are the mass, damping and stiffness matrices, respectively; \( u(t) \) is the sought node displacement (relative to ground) vector corresponding to given ground earthquake excitation \( \ddot{u}_g(t) \) and appropriate initial conditions, and dots over symbols indicate time derivatives. Problem (1) can be solved by wellknown methods of Structural Dynamics.

2.2 Interaction simulation

Let \( j_A \) and \( j_B \) be two associated nodes on the interface (joint), where unilateral frictional contact can take place during an earthquake. These nodes are considered [6] as connected by two fictive unilateral constraints, normal to interface the first and tangential the second one. The
corresponding force-reactions and retirement relative displacements are
denoted by $r_{jn}$, $z_{jn}$ and $y^j_1$, $y^j_2$, respectively. They satisfy in general
nonconvex and nonmonotone constitutive relations of the following type
(2), expressing mathematically the unilateral elastoplastic softening contact
with friction:

$$r_j(d_j) \in \partial R_j(d_j). \quad (2)$$

Here $\partial$ is the generalized gradient of Clarke, $d_j$ the deformation and $R_j(.)$
is the superpotential function, see Panagiotopoulos [2,12]. By definition,
rel. (2) is equivalent to the following hemivariational inequality, where $R^\uparrow$
denotes subderivative and $e_j$ virtual deformation:

$$R^\uparrow_j (d_j, e_j - d_j) \geq r_j(d_j) \cdot (e_j - d_j). \quad (3)$$

By piecewise linearizing these relations as in Liolios [6], Maier [9,10] and
Klarbring [11] we obtain the following linear complementarity conditions:

$$r_{jn} = p_{jn}(z_{jn} - g_j + w_j) + c_j \dot{z}_{jn}, \quad (4a)$$

$$w_j \geq 0, \quad r_{jn} \leq 0, \quad w_j \cdot r_{jn} = 0, \quad (4b,c,d)$$

$$|r_{jt}| \leq f_j \, |r_{jn}|, \quad \dot{z}_{jt} \cdot r_{jt} \leq 0, \quad (5a,b)$$

$$\dot{z}_{jt} \cdot (|r_{jt}| - f_j \, |r_{jn}|) = 0. \quad (5c)$$

In (4a), $c_j$ is damping coefficient, $p_{jn}$ the reaction function for the
normal unilateral constraint, $g_j$ the existing normal gap and $w$ a
non-negative multiplier; in (5), $f_j$ is the Coulomb's friction coefficient. So,
rels (4) and (5) impose that friction phenomenae (slip or adhesion) can
take place only when unilateral contact occurs, i.e. when the compressive
contact force $r_{jn}$ appears.

### 2.3 Coupled system conditions

Taking into account, now, the interaction and the P-delta effects,
we write the incremental dynamic equilibrium conditions for the coupled
system of the interacting buildings (A) and (B):

$$M_A \Delta u_A + C_A \Delta \dot{u}_A + (K_A + G_A) \Delta u_A = -M_A \Delta \ddot{u}_g + \Delta \Gamma, \quad (6a)$$

$$M_B \Delta u_B + C_B \Delta \dot{u}_B + (K_B + G_B) \Delta u_B = -M_B \Delta \ddot{u}_g - \Delta \Gamma, \quad (6b)$$

$$\Gamma = \Gamma_N + \Gamma_T. \quad (7)$$
Here \( G_A \) and \( G_B \) are the geometric stiffness matrices, by which P-Delta effects are taken into account \([10,13,14]\), and \( r \) is the coupling vector of the normal and tangential interaction forces, satisfying (4),(5). To conditions (6) are adjoined the initial conditions, and so the problem consists in finding the time-dependent vectors \( u_A, u_B, g, z, r \) and \( w \) which satisfy rels. (2)-(7) for the given earthquake excitation \( u_g(t) \).

2.4 Convolutional problem formulation and time discretization

Next the procedure of Liolios \([7,19]\) is applied. So, using the Laplace transform for the linear equality problem conditions \([18]\) and a suitable elimination of unknowns, denoting by \( v \) and \( x \) all the unilateral quantities concerning displacements and forces, respectively, and back transforming to time domain, we arrive eventually at

\[
y(t) = A(t) * x(t) + a(t), \quad (8a)
\]

\[
\begin{align*}
  y(t) &> 0, \\
  x(t) &< 0, \\
  v^T \cdot x &= 0. 
\end{align*} \quad (8b,c,d)
\]

Here \( A(t) \) is a Dynamic Flexibility matrix \([25]\), \( a(t) \) a known time function, and \( * \) the convolution operation.

The problem of rels. (8) is called here Convolutional Linear Complementarity Problem (CLCP). In comparison to initial problem of rels. (2)-(7), the CLCP has a significantly reduced number of unknowns. Further, a time discretization scheme is applied as for the convolutional forms in the Boundary Element method \([20,24,25]\). In each time-step we assume that the unilateral constraints remain either active or unactive by adjusting suitably the time-step. To compute what is happening, the procedure of Liolios \([6]\) is applied taking into account discontinuity of the velocities due to contact-impact \([22,23]\). So, a nonconvex linear complementarity problem of the following form is solved for each time moment \( t_n = n \Delta t \), where \( \Delta t \) is the time step:

\[
\begin{align*}
  x_n &< 0, \\
  A_n x_n + a_n &> 0, \quad (9a,b)
\end{align*}
\]

\[
\begin{align*}
  x_n^T \cdot (A_n x_n + a_n) &= 0. \quad (9c)
\end{align*}
\]

For most practical applications in structural mechanics, matrix \( A_n \) is a P-matrix \([10,21]\) and thus a unique solution of problem (8) can be assured.
3 Numerical example

The two one-storey buildings (A) and (B) of Fig. 1a,b are of reinforced concrete with elasticity modulus $E_b = 3 \times 10^7$ KN/m$^2$, slab thickness 0.25 m, damping ratio 5% and beams 30/80 cm connecting the columns tops perimetrically. The columns section is 30/30 cm for (A) and 40/60 cm for (B). The stress-deformation law for the unilateral constraints normal to the interface J-J is estimated by experimental results to be given as in Fig. 1d, where $\sigma_c = 18$ MPa, and the friction coefficient in Fig. 1e for the tangential constraints is estimated as $f_j = 0.50$. The system is subjected to the horizontal ground seismic excitation along the axis x-x, as depicted in Fig. 1c and mathematically expressed by the relation: $u_e(t) = \mu e^{-2t} \sin(4\pi t)$, with $u_0 = 10$ mm.

Assuming no interaction, the building (A) is symmetric from the seismic point of view and so appears transitional vibrations only along axis x-x. On the contrary, building (B) is an asymmetric one, and appears transitional as well as torsional vibrations [15]. So, when a seismic interaction takes place, building (A) will also appear an asymmetric response.

<table>
<thead>
<tr>
<th>BUILDING</th>
<th>QUANTITY</th>
<th>UNCOUPLED SYSTEM</th>
<th>COUPLED SYSTEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>$H_x$</td>
<td>905.8 KN</td>
<td>778.4 KN</td>
</tr>
<tr>
<td></td>
<td>$H_y$</td>
<td>0</td>
<td>86.7 KN</td>
</tr>
<tr>
<td></td>
<td>$M_z$</td>
<td>0</td>
<td>431.5 KNm</td>
</tr>
<tr>
<td>(B)</td>
<td>$H_x$</td>
<td>3074.7 KN</td>
<td>3291.2 KN</td>
</tr>
<tr>
<td></td>
<td>$H_y$</td>
<td>591.8 KN</td>
<td>401.4 KN</td>
</tr>
<tr>
<td></td>
<td>$M_z$</td>
<td>12188.0 KNm</td>
<td>11448.3 KNm</td>
</tr>
</tbody>
</table>

In Table 1. the absolutely extremum values of some response quantities, occurred during the earthquake excitation and computed by the herein presented method, are shown indicatively. These quantities, necessary for the usual aseismic design, are the horizontal forces (base shear forces) $H_x$ and $H_y$ and the torsional moment $M_z$ in the mass centers of the buildings (A) and (B). The relative values are given for no interaction (uncoupled system) and for the case when interaction and P-delta effects are taken into account (coupled system). As the table values show, the interaction effects in the second case are remarkable, especially as regards $H_y$ and $M_z$. 
Figure 1: Numerical example:

- a. Plan view of the system of buildings (A) and (B)
- b. Vertical section view
- c. Seismic ground displacement.
- d. Stress-deformation law for the normal unilateral constraints
- e. Stress-deformation law for the tangential unilateral constraints
4 Concluding remarks

As in the numerical example has been shown, seismic interaction under P-delta effects, which is not taken into account in the usual Civil Engineering design of adjacent buildings, can change significantly the earthquake response of seismically symmetric buildings in contact with asymmetric ones. A numerical estimation of the so caused seismic interaction effects can be obtained by the herein presented approach. This numerical procedure is realized by using available computer codes of the finite element method and the nonlinear mathematical programming (optimization algorithms).

Certainly the most complicated task in all the above cases is the realistic simulation of the unilateral contact. To overcome this difficulty, experimental results can be used for rational estimation of parameters involved to simulate the interface behaviour between adjacent space structures. Finally, the herein presented numerical approach can be used effectively to estimate numerically and to control actively the influence of the frictional interaction and the instability effects on the seismic response of the adjacent structures. This can be obtained by a parametric use of the method to adjust the gap between the buildings and/or the contact material behaviour (hardening or softening).

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