Methodology for design of earthquake resistant steel liquid storage tanks

M. A. Haroun & M. A. Al-Kashif
University of California, Irvine, and American University in Cairo, Egypt

Abstract

Steel shells of cylindrical liquid storage tanks are initially designed using a factor of safety specified in the prevailing code. When the safety of tanks under seismic loading is assessed, the capacity of the shell against buckling is computed. There are two types of shell buckling: membrane and elastic-plastic. Although the latter is not accounted for in most codes, it may limit the seismic design especially at higher values of total liquid pressure. The elastic-plastic buckling capacity is eventually depleted when the ratio of total-to-hydrostatic pressures is equal to the employed factor of safety. Numerous tank dimensions were selected for an extensive parametric study, and such tanks were evaluated under varying earthquake intensities. Tanks with larger height-to-radius ratio when subjected to high peak earthquake accelerations tend to be unsafe, if originally designed under commonly specified factor of safety. A formula is suggested to modify the factor of safety according to the height-to-radius ratio and the expected peak earthquake acceleration. This may form the core for developing enhanced seismic design code methodology for steel liquid-filled tanks.

Keywords: shell buckling, hydrodynamic pressure, safety factor, storage tanks, seismic design.

1 Introduction

Liquid storage tanks are important elements of lifeline and industrial facilities. The evolution of codes and standards for the seismic design of these structures has relied greatly on observations of tank damages during past earthquakes, yet the time lag between acquiring the information and implementing the findings in practice has remained relatively long. Even though current codes and standards
reflect a mature state of knowledge for tank design, recent earthquakes as well as advanced state-of-the-art analyses continued to point out to a few overlooked issues. Failure modes of ground-based tanks included, among others, shell buckling which is typically characterized by diamond-shaped buckles or "elephant's foot" bulges which appear a short distance above the base.

For steel tanks, demand requirements arise from the hydrodynamic forces and should be less than the capacity. The buckling capacity is the lesser of the membrane buckling stress or the elastic-plastic buckling stress, yet only the former is used in current codes. A knock-down factor is usually employed with the classical buckling stress to obtain a knocked-down buckling stress value. However, the hydrodynamic pressure causes excessive hoop stress in the walls of the tank, which in turn, reduces the capacity of steel to elastic-plastic buckling. This study assesses the role of this buckling stress on tank design.

The main goal of the present study is to conduct a comparative parametric study of the seismic response of anchored steel tanks to determine the effect of the elastic-plastic buckling stress on the allowable buckling stress, and to recommend a systematic procedure for accounting for this capacity reduction early on in the design of the tank shell.

2 Shell buckling capacity

A critical aspect in the earthquake resistant design of steel, cylindrical tanks has been elephant-foot buckling. It occurs near the tank base predominantly due to axial stress in the tank wall, though it is significantly affected by the circumferential membrane stress caused by hydrostatic and hydrodynamic pressures. The axial compressive stress developed at the base of tanks under seismic excitations must be less than an allowable buckling stress to preclude the occurrence of shell buckling. The allowable stress in current codes and standards is basically specified for a uni-directional stress state whereas actual stress state at shell bottom is bi-axial.

2.1 Elastic buckling stress

The axial membrane stress needed to induce buckling in a shell depends on internal pressure, circumferential variation of axial stress, and amplitude of imperfection in the shell. The latter tends to decrease the buckling stress to a fraction of the classical buckling stress. A knock-down factor can be estimated based on the amplitude of imperfection, which in turn depends on the quality of construction, tank radius, \( R \), and shell thickness, \( t \). The internal pressure decreases the effective imperfection amplitude, therefore increasing the buckling stress. Circumferential variation of the axial stress reduces the probability of coincidence of the maximum stress and the maximum imperfection, again increasing the buckling stress. The classical buckling stress, \( f_{cl} \), is defined as

\[
f_{cl} = 0.6E_s \frac{t}{R}
\]
where $E_s$ is the modulus of elasticity of steel, $t$ is the shell thickness, and $R$ is the shell radius. A knock-down factor is usually assumed to estimate the actual stress; in this study, an average conservative knock-down factor of 5 is assumed. Thus, the allowable (knocked-down) membrane buckling stress, $f_{mb}$, is computed.

2.2 Elastic-plastic buckling stress

The bottom of the shell is normally subjected to a bi-axial stress state consisting of hoop tension and axial compression. Radial deformations under the internal pressure create additional eccentricity, tending to induce the commonly observed “elephant foot” buckling as shown in Figure 1.

![Figure 1: Elephant's foot buckling.](image)

Elastic-plastic collapse adjacent to the boundary may be assessed by using the relation presented in the New Zealand guidelines [7]. It assumes a quadratic reduction of the classical buckling stress depending on the factor $(PR/t \sigma_y)$ where $P$ is the internal pressure and $\sigma_y$ is the yield stress of steel. This factor essentially represents the effect of shell yielding in the circumferential direction due to internal pressures on the buckling capacity of the shell. Although this relation was developed from predictions of collapse adjacent to pinned-base details in ground silos, the fixed-base details are slightly stronger.

2.3 Buckling in current standards

The allowable buckling stress in AWWA (American Water Works Association) [3] and API (American Petroleum Institution) [1] standards is based on classical value of buckling stress under axial load, significantly reduced by a large knock down factor due to shell imperfections and also increased to account for the effects of internal liquid pressure. In the New Zealand guidelines [7], classical buckling stress in membrane compression is calculated and corrected according
to imperfection amplitude and internal pressure. In addition, an elastic-plastic collapse stress is also calculated. The lower of the two stresses is used in the computation of the margin of safety.

### 2.4 Advanced finite element analysis of shell buckling

An advanced nonlinear finite element analysis was performed by Haroun and Bhatia [6] to evaluate the buckling capacity of a variety of liquid-filled shells. Hydrostatic and hydrodynamic pressures on the tank wall cause circumferential (hoop) tensile stresses in the shell. Earthquake data have always indicated that no seismically induced failures due to hoop stress have occurred, but the primary effect of the circumferential hoop stress is to influence the axial buckling capacity of the tank wall. The finite element model had a sufficient mesh density to correctly capture the buckling behavior. The program “MARC” was used for the analysis with a high mesh density, and large displacement analysis was carried out. The shell was loaded incrementally until radial displacements increased rapidly. The hydrostatic and hydrodynamic pressures were applied as pressure values at the element integration points through a user subroutine, and were integrated using the element shape functions to obtain nodal forces. The tank model was first subjected to hydrostatic pressures, and then pseudo-hydrodynamic loads were incrementally added until buckling was initiated. The axial stress necessary to cause buckling for various height-radius ratios and for different shell thickness was compared to that obtained using the following simplified formula

$$f_{pb} = f_{el} \left(1 - \left(\frac{PR}{t\sigma_y}\right)^2\right)$$

(2)

Throughout this study a yield stress of steel $\sigma_y = 36$ Ksi was used. Sample values of the buckling stress obtained by the finite element analysis are shown in the last column of Table 1.

<table>
<thead>
<tr>
<th>(H/R)</th>
<th>(t/R)</th>
<th>$f_{pb}$ (ksi) Equation (2)</th>
<th>$f_{pb}$ (ksi) FEA [6]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.001</td>
<td>0</td>
<td>2.773</td>
</tr>
<tr>
<td>1.5</td>
<td>0.001</td>
<td>0</td>
<td>3.510</td>
</tr>
<tr>
<td>2.0</td>
<td>0.001</td>
<td>0</td>
<td>2.253</td>
</tr>
<tr>
<td>1.5</td>
<td>0.002</td>
<td>4.957</td>
<td>5.070</td>
</tr>
<tr>
<td>2.0</td>
<td>0.002</td>
<td>5.383</td>
<td>6.846</td>
</tr>
<tr>
<td>2.5</td>
<td>0.002</td>
<td>5.552</td>
<td>7.258</td>
</tr>
<tr>
<td>2.5</td>
<td>0.003</td>
<td>11.349</td>
<td>9.389</td>
</tr>
</tbody>
</table>

It should be noted that elastic buckling is critical only for low values of membrane circumferential stress ($PR/t$) coupled with high compression stress in
the shell. This combination is expected to be rare. With increase in the membrane circumferential stress, elastic-plastic buckling becomes the dominant mode. The stresses obtained from the finite element analysis clearly point towards elastic-plastic buckling in the analyzed cases. A comparison between the finite element results and those of the simple formula shows a close correlation, with the values of the elastic-plastic buckling stress well below the elastic buckling stress. It should be noted that the pressure used in the elastic-plastic buckling equation is the sum of the static and pseudo-dynamic pressures.

2.5 Enhancement of buckling stress computation

The nonlinear finite element study performed by Haroun and Bhatia [6] showed that elephant-foot buckling is the most likely mode in a typical tank. It generally confirmed the accuracy of the approximate formula for buckling capacity (2). Neither the API nor the AWWA standards consider elastic-plastic buckling which has been typically observed as elephant-foot buckling at the base of earthquake vulnerable tanks. These codes should adopt the proven simple formula for elastic-plastic buckling to minimize the occurrence of such damage in future seismic events.

3 Seismic behavior of anchored tanks

The seismic response of fluid-filled tanks to earthquake excitations should combine its response to both horizontal and vertical components of ground motion. The horizontal component of the earthquake produces lateral shear force and overturning moments on the tank. However, the vertical component results in an axisymmetric increase or decrease of the hydrodynamic pressure with no additional lateral forces. Thus, unlike other structures, vertical earthquake motion may play a measurable role due to the development of additional hydrodynamic pressures which, in turn, induce hoop stresses in the shell.

3.1 Tank under horizontal excitation

The tank under consideration is a circular cylindrical steel tank of radius R and thickness t, filled with liquid to a height H. Tank response to a horizontal component of an earthquake can be reasonably represented by the mechanical model presented by Haroun [5]. There are three modes of response. The first is a low-frequency mode due to liquid sloshing (convective mode) in which the contained liquid sloshes within the tank with negligible interaction with the deformation of the tank shell. The other two modes are the impulsive-flexible component which represents the interaction between the liquid and tank deformation, and the impulsive-rigid component in which part of the tank and the contained fluid move as rigid body with no contribution from shell deformation. The mechanical analog thus contains three equivalent masses \( m_1 \) (convective mass), \( m_o' \) (impulsive mass), \( m_f \) (impulsive mass associated with wall flexibility) along with their heights, and two equivalent springs with periods representing sloshing and liquid-shell vibrations. Overturning moment exerted
on the tank, neglecting the small contribution of the convective mass, can be expressed as

\[ M_{\text{max}(f)} = m'_o h'_o \ddot{u}_g + m_f h_f S_f \]  

(3)

where \( S_f \) is the spectral acceleration corresponding to the impulsive-flexible vibration mode. The axial stress developed at the base of the shell is therefore calculated from

\[ \sigma_{(f)} = \frac{W_s}{A} + \frac{M_{\text{max}(f)}}{\pi R^2 t} \]  

(4)

### 3.2 Vertical excitation

The vertical component of an earthquake has a negligible contribution to the base shear and overturning moment, which are mainly caused by the two horizontal components. However, the vertical component of an earthquake amplifies the axisymmetric pressure. The peak hydrodynamic pressure occurs near the base of the tank and is approximately equal to a fraction of the peak hydrostatic pressure. This is computed by replacing the acceleration of gravity by the spectral vertical acceleration corresponding to the period of vibration of shell under axisymmetric vibration.

### 3.3 Liquid pressure

The total pressure at the tank base, including the hydrostatic pressure and that resulting from the horizontal and vertical components of an earthquake, is given by

\[ P = P_s + P_d \]  

(5)

The hydrostatic pressure is expressed as

\[ P_s = \gamma_w H \]  

(6)

where \( \gamma_w \) is the unit weight of liquid (water).

The pressure at the base of the tank due to the horizontal earthquake component is made up of both impulsive and sloshing components. Because the maxima of the impulsive pressure, convective pressure, and the pressure due to the vertical components of the earthquake do not occur at the same time, it is estimated that the maximum value of the total dynamic pressure is

\[ P_d = \sqrt{P_i^2 + P_c^2 + P_v^2} \]  

(7)

The second term of Equation 7 is the contribution of the convective mode which is small and may be neglected. The impulsive pressure for flexible tank is proportional to the spectral horizontal acceleration and can be calculated from

\[ P_{if} = q_o \gamma_w H A_h \ddot{u}_g \]  

(8)
where $A_h$ is the normalized spectral horizontal acceleration and the coefficient $q_0$ may be obtained from the chart displayed in Figure 2.

![Figure 2: Pressure coefficient ($q_0$).](image)

The pressure due to the vertical component of the earthquake can be evaluated from

$$P_{vf} = 0.5\gamma_w HA_v \frac{\ddot{u}}{g}$$

in which it was assumed that the peak vertical ground acceleration is half of the peak horizontal ground acceleration, and $A_v$ is the normalized spectral vertical acceleration.

### 3.4 Reduction factor

The thickness of the tank shell plays a dominant role in determining the shell buckling capacity. It is primarily evaluated from consideration of allowable hoop stress under hydrostatic pressure, and therefore, it is dependent on tank radius, liquid height, yield stress of steel, and the factor of safety. The latter is viewed as a reduction from the yield stress of steel to provide the allowable hoop stress. The equation for calculating the thickness is

$$t = \gamma HR(r) / \sigma_y$$

where $r$ is the reduction factor as defined above.

Shell membrane buckling limits the design at low total liquid pressure but the effect of elastic-plastic buckling becomes apparent with the increase of earthquake acceleration accompanied by higher (H/R) values. High dynamic pressures cause the elastic-plastic buckling stress to dip below the knocked-down membrane buckling stress. Therefore, the problem arising from such a low elastic-plastic buckling stress can be controlled by selecting an appropriate reduction factor that ensures the elastic-plastic buckling does not limit the design. This is satisfied if
where the ratio between total pressure to static pressure equals to
\[
\left(\frac{P}{P_s}\right)^2 \leq 0.8 \tag{11}
\]

For a given reduction factor, one can calculate the above ratio and use (11) to test for elastic-plastic buckling. If (11) is not satisfied, one may need to increase the reduction factor. It should be noted that using (11) does not guarantee a safe design; however, it ensures that elastic-plastic buckling is controlled especially at higher earthquake accelerations.

Figure 3: Variation of elastic-plastic buckling stress with reduction factor (r).

4 Parametric study

Different tank dimensions were chosen and analyzed for varied earthquake loading intensities. The parameters used were H = 20, 30, 40 and 50 ft, R = 20, 30, 40 and 50 ft, and peak horizontal accelerations of 0.2g, 0.3g, 0.4g and 0.5g. The tanks were analyzed, developed stresses were calculated and the margin of safety for each case was inspected. Unsafe tanks were identified and an increase of the reduction factor was introduced to re-design each of those unsafe tanks.

The margin of safety is the ratio between the allowable buckling stress and the maximum actual axial stress. When the margin of safety is less than 1, then the design is unsafe. Several of the tanks investigated in this study showed a margin of safety smaller than 1 when a uniform safety factor was applied initially to determine the shell thickness. Tanks subjected to higher earthquake intensities or had higher (H/R) ratios were mostly unsafe. The redesign process involves an iterative approach by increasing shell thickness through increasing the reduction factor. Figure 4 and 5 show examples of design charts to select the...
appropriate reduction factor for peak ground accelerations of 0.4g and 0.5g, respectively, and for different values of R and H.

Figure 5: Reduction factors to provide safe tanks ($\ddot{u}_g = 0.4g$).

Figure 6: Reduction factors to provide safe tanks ($\ddot{u}_g = 0.5g$).

5 Conclusions

Observations of tank response under hydrostatic and hydrodynamic loadings showed that

- The elastic-plastic buckling stress may limit the seismic design, especially at higher values of total liquid pressure.
- The elastic-plastic buckling capacity is depleted when the ratio of total-to-
hydrostatic pressures is equal to the factor of safety used in computing shell thickness. Yet, elastic-plastic buckling phenomenon is not included in most commonly used standards for liquid storage tanks.

- The factor of safety adopted in current design codes for computing shell thickness must not be applied uniformly to all tanks. Tanks with larger height-to-radius ratio, when subjected to high peak earthquake accelerations, tend to be unsafe, if originally designed under such a factor of safety.
- A formula is suggested to modify the factor of safety (reduction factor) in the initial design phase of the tank to ensure that the elastic-plastic buckling stress does not dip below the knocked-down membrane buckling stress. Such a formula depends primarily on the height-to-radius ratio of the tank and the expected peak earthquake acceleration.
- The recommended procedure in the present study may form the core for developing enhanced seismic design methodology in commonly used codes for steel liquid storage tanks.

Acknowledgement

The support of this research by a grant from University of California Energy Institute (UCEI) is greatly appreciated.

References