The seismic energy dissipation mechanism of rocking structural systems with yielding base plates

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Abstract

The energy dissipation mechanism of the rocking structural system with yielding base plates is investigated based on seismic response analyses of four steel planar frame models with five stories and one bay. The potential energy of the self weight increasing with uplifting and the hysteresis damping capacity of the yielding base plates are evaluated. Effects of vertical inertia force on the energy dissipation mechanism including impact effects are clarified. The energy dissipation of the system corresponding to the maximum momentary input energy is expressed using the maximum rigid body rotational angle. The maximum response displacements are predicted considering the energy balance condition of the system.

Keywords: uplift, rocking, base plate yielding, energy dissipation mechanism, momentary input energy, response displacement prediction.

1 Introduction

It has been pointed out that the effects of rocking vibration (uplift response) reduce seismic damage of buildings subjected to strong earthquake ground motions [for example, 1-4]. Based on this knowledge, we are now developing the rocking structural systems with yielding base plates (BPY systems) [5-7]. These systems can cause rocking vibration, when the base plate attached at the bottom of each column on their first story yields. The basic idea of the systems is illustrated in fig. 1. According to our previous studies based on shaking table tests, it was cleared the BPY systems can effectively reduce seismic responses of buildings. And we proposed a simplified prediction method for seismic
responses of the systems evaluating the energy dissipation capacity [7]. However, we used 1/3 scale models [5] or 1/2 scale models [6, 7] in these studies. Thus we need further studies using real scale building structural models.

In this paper, the energy dissipation mechanism of the BPY systems is investigated based on numerical analyses using four real scale steel planar frame models with five stories and one bay. And the maximum response displacements are predicted considering the energy balance condition of the systems.

![Figure 1: The basic idea of the base plate yielding system.](image)

2 Seismic responses of base plate yielding systems

2.1 Analysis models

The elevations of analysis models are shown in fig. 2. These models are categorized into two types. One is the pure frame type without a brace. And the other is the braced frame type. The height and width of all models are 18m and 6m respectively. Cross sections of the models are shown in table 1. There are two models in each type of analysis model. Thus we have four analysis models for the case studies in all. In the following analyses, it is supposed that the yield point of steel used for all members is 235.2 N/mm$^2$ and that each floor mass of all models is 28.8 t. The first natural period of each frame which bases are fixed is shown in table 2. The base plate shown in fig. 3 is attached at the column bases of each analysis model to cause rocking vibration during an earthquake.

2.2 Analysis method

Mathematical models of the structural systems are shown in fig. 4. The steel frame is modeled as shown in fig. 4 (a). Mass of structures is concentrated on three points on each floor. The column base of the BPY systems is modeled as shown in fig. 4 (b). Two types of springs are attached at the column bases. One of them represents the restoring force characteristic of base plates as shown in.
The physical values of the springs for the base plate are shown in table 3. The tension yield strength of the base plate is calculated by regarding each wing part of them as a beam with fixed ends. The other physical values are evaluated based on the past test results as well as the force-deformation relationship [8].

Figure 2: The elevations of analysis models for case studies.

Table 1: Cross sections of each member.

<table>
<thead>
<tr>
<th>Model Type</th>
<th>Column Size</th>
<th>Beam Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pure frame</td>
<td>H-458x417x30x50</td>
<td>H-692x300x13x20</td>
</tr>
<tr>
<td>Pure frame</td>
<td>H-414x405x18x28</td>
<td>H-606x201x12x20</td>
</tr>
<tr>
<td>Braced frame</td>
<td>H-414x405x18x28</td>
<td>H-606x201x12x20</td>
</tr>
<tr>
<td>Braced frame</td>
<td>H-250x250x9x14</td>
<td>H-446x199x8x12</td>
</tr>
</tbody>
</table>
Table 2: The first natural period of each model(s).

<table>
<thead>
<tr>
<th>Model</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model I</td>
<td>0.523</td>
</tr>
<tr>
<td>Model II</td>
<td>0.729</td>
</tr>
<tr>
<td>Model III</td>
<td>0.384</td>
</tr>
<tr>
<td>Model IV</td>
<td>0.490</td>
</tr>
</tbody>
</table>

Figure 3: Base plate attached at each column base.

Figure 4: Mathematical model.

(a) Frame model
(b) Column base with two types of spring
(c) Ground contact
(d) Restoring force characteristics of base plates based on the test results [8]
For the response analyses, a step-by-step time integration method (linear acceleration method) is used. Damping is assumed to be proportional to the initial stiffness with 3% damping ratio. This damping is constant even after yielding. Time interval of analyses is 0.001 s. The input ground motion is 1940 El Centro NS. The linear response spectrum for 1-DoF systems with damping ratio $\eta=0.03$ is shown in fig. 5 comparing with design spectrum. The design spectrum will be used to predict base shear of each model in chapter 4.

Table 3: Physical values of the base plate.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial stiffness (kN/mm)</td>
<td>10.67</td>
</tr>
<tr>
<td>Tension yield strength (kN)</td>
<td>33.87</td>
</tr>
<tr>
<td>Yield deformation (mm)</td>
<td>2.72</td>
</tr>
<tr>
<td>Stiffness ratio (K2/K1)</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Figure 5: Response acceleration of 1-DOF systems.

### 2.3 Analysis results

The maximum values of up-lift displacement, lateral roof displacement and base shear coefficient of all models are listed in table 4. Structural members except base plates of all models keep elastic. Fig. 6 shows time histories of the uplift displacement and roof displacement of the model I. In fig. 6 (b), we can see most of the lateral roof displacement is occupied by the rigid body deformation with the uplift response. This effect can reduce deformation of structural members and prevent them from suffering large damage. Fig. 7 shows time history of energy response of the model I. We can see when the increase of energy input becomes the maximum, the displacement responses almost reach to their maximum [7, 9]. Therefore, we should investigate the energy dissipation mechanism corresponding to this maximum momentary input energy (MIE).
Table 4: The maximum response results.

<table>
<thead>
<tr>
<th>Model</th>
<th>Uplift (mm)</th>
<th>Roof (mm)</th>
<th>Base shear coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>46.7</td>
<td>163.1</td>
<td>0.55</td>
</tr>
<tr>
<td>II</td>
<td>38.9</td>
<td>177.1</td>
<td>0.37</td>
</tr>
<tr>
<td>III</td>
<td>55.1</td>
<td>180.2</td>
<td>0.48</td>
</tr>
<tr>
<td>IV</td>
<td>52.1</td>
<td>188.4</td>
<td>0.45</td>
</tr>
</tbody>
</table>

When building structures are subjected to an earthquake ground motion in the x-direction, the energy balance condition is expressed by Eq. (1).

\[ \int M \dddot x \, dt + \int D_x \ddot x \, dt + \int R_x \ddot x \, dt = - \int M \dddot x \cdot \ddot x \, dt \]

where \( D_x \): lateral damping force and \( R_x \): lateral restoring force.

3 Energy dissipation mechanism

When building structures are subjected to an earthquake ground motion in the x-direction, the energy balance condition is expressed by Eq. (1).
Fig. 8 illustrates acting forces on a base plate yielding system with one story and the corresponding displacements. The energy dissipation, $E_{Dx} + E_{Rx}$, by the lateral damping force and the lateral restoring force of this BPY system can be calculated by the following equation based on eq.(1) and the geometric condition shown in fig.(8).

$$E_{Dx} + E_{Rx} = \int -M(\ddot{x} + \dot{x}_0)\dot{x} dt = \int -M(\ddot{x} + \dot{x}_0)(H\dot{\theta}_r + \dot{x}_f) dt$$

$$= \int M_{OPT} \dot{\theta}_r dt + \int (D_{fx} + R_{fx}) \dot{x}_f dt$$

where, $H$: height of a building, $\theta_r$: rigid body rotational angle, $x_f$, $D_{fx}$ and $R_{fx}$: lateral deformation, lateral damping force and lateral restoring force of a superstructure respectively.

The overturning moment $M_{OPT}$ by earthquake lateral force is calculated by eq.(3) considering the moment balance around the fulcrum on the landing side.
Earthquake Resistant Engineering Structures V

\[ M_{OV} = -M(\ddot{x} + \dot{x}_0)H = \left(0.5Mg + 0.5MB\ddot{\theta} + N_B\right)B \]  \hspace{1cm} (3)

where, \( B \): width of a building, \( N_B \): tensile resistance force of base plates.

In eq.(3), P-\( \Delta \) effect and viscous damping of base plates are neglected. Substitution of eq.(3) into eq.(2) yields eq.(4).

\[
E_{dx} + E_{ri} = \int 0.5Mg\dot{\theta}dt + \int 0.5MB^2\ddot{\theta}_r dt + \int N_B\dot{\theta}_r dt + \int D_{ij}\dddot{x}_i dt + \int F_{ij}\dddot{x}_j dt
\]

\[
= E_G + E_{VZU} + E_B + E_{SD} + E_{SR}
\]

where, \( E_G \): potential energy of self weight, \( E_{VZU} \): kinetic energy by uplift motion, \( E_B \): energy dissipation by hysteresis damping of base plates and \( E_{SD} \) and \( E_{SR} \): energy dissipation by viscous damping and stress energy of the super structure respectively.

Obviously, eq.(4) can be also applied to the multi-story BPY systems. Eqs.(2)-(4) mean the summation of \( E_G, E_{VZU} \) and \( E_B \) is represented as area of the figure surrounded with a relationship curve between overturning moments and rigid body rotational angles (\( M_{OV}-\theta \) curve). To investigate the effect of vertical inertia force, the \( M_{OV}-\theta \) curve of the model I is compared with the corresponding static \( sM_{OV}-\theta \) curve in fig 9. These curves are observed during 2.39-3.48 s including duration when the MIE is input. The corresponding static overturning moment \( sM_{OV} \) is calculated by eq.(5), where the vertical inertia force is ignored.

\[
sM_{OV} = (0.5Mg + N_B)B
\]

The difference between the area of \( M_{OV}-\theta \) curve and that of \( sM_{OV}-\theta \) curve reveals the kinetic energy \( E_{VZU} \) by the uplift motion. This difference becomes larger just after the system starts uplifting and just before the system lands. But the former difference is cancelled as the uplifting becomes larger, because the kinetic energy by the uplifting motion is obviously 0 when the uplifting becomes the maximum. On the other hand, the latter difference is accumulated gradually as the system repeats uplifting. This is the impact effect. Fig. 10 shows the composition of energy dissipation corresponding to the MIE. The summation of \( E_G \) and \( E_B \) occupy the most part of the energy dissipation with about 80% ratio.

4 Earthquake response prediction

Based on the energy balance condition while the MIE is being input, the maximum uplift displacement of each model is predicted. The MIE is evaluated by response analyses of elastic one-mass systems with 10\% viscous damping ratio and various natural periods [9]. For the corresponding energy dissipation, only the summation of \( \Delta E_G \) and \( \Delta E_B \) is estimated, because these energies occupy the most of energy dissipation as shown in fig 10.
$\Delta E_G$ is calculated by eq. (6).

$$\Delta E_G = 0.5MG(\theta_{r(max)} - \alpha \theta_{r(max)})B$$  \hspace{1cm} (6)

where $\alpha$: amplification ratio [9], which is judged from displacement responses (in this case, about 0.6).

Considering the force-deformation relationships of base plates shown in fig. 11, $\Delta E_B$ is evaluated by eq. (7). Both $E_G$ and $E_B$ can be evaluated using the maximum rigid body angle $\theta_{r(max)}$.

$$\Delta E_B = \Delta E_{B1}[\theta_{r(max)}] + \Delta E_{B2}[\theta_{r(max)}]$$  \hspace{1cm} (7)

where $\Delta E_{B1}[\theta_{r(max)}]$ and $\Delta E_{B2}[\theta_{r(max)}]$: energy dissipation by base plates on the uplift side and the landing side respectively.

Fig. 12 shows the MIE spectra comparing with ED curves of each model which shows the relation between the equivalent natural period and the energy dissipation. The equivalent natural period $T_E$ is calculated by the following equation [9]. If an arbitrary $\theta_{r(max)}$ is given, we can calculate $T_E$ and the corresponding energy dissipation using it.

$$T_E \approx 0.75 \times 2\pi \sqrt{\frac{M_{Lx}}{K_E}}$$  \hspace{1cm} (8)

Figure 11: Energy dissipation by base plate.
where $M_{we}$: the first effective mass of the system and $K_E$: the effective split stiffness including the uplifting effect [7].

On the intersection of the spectra and the ED curves, the MIE balances the energy dissipation [9]. Thus we can find out the energy dissipation of each model subjected to an earthquake ground motion using fig. 12. Because the energy dissipation is expressed using the maximum rigid body rotational angle $\theta_{r(\text{max})}$, we can predict the maximum uplift displacements $B\theta_{r(\text{max})}$. The prediction results are shown in fig. 13 comparing with the response results.

Roof displacements $\delta_{\text{roof}}$ are predicted by calculating the summation of rigid body deformation and frame deformation using eq.(9).

$$\delta_{\text{roof}} = H\theta_{r(\text{max})} + s\delta_{\text{roof}} \left[\theta_{r(\text{max})}\right]$$

where $s\delta_{\text{roof}} \left[\theta_{r(\text{max})}\right]$: roof displacement by static and linear frame deformation corresponding to $\theta_{r(\text{max})}$, which can be calculated considering the moment balance around the fulcrum on the landing side [7].

The prediction results are shown in fig. 13.

As shown in fig. 9, because the overturning moment or base shear is amplified by the vertical inertia effects or higher mode effects, they become larger than the corresponding static values. For the rocking system without some dampers like the yielding base plates, that is simple rocking system, the maximum base shear $Q_{max}$ can be predicted by eq.(10) [1].

$$Q_{\text{max}} = Q_{cr} \left\{H^2/L^2 + (0.5B)^2\right\} + \sqrt{0.0625B^4\left[H^2 + (0.5B)^2\right]^2 + 0.25B^2\left[H^2 + (0.5B)^2\right]^2 \left[Q_{cr}^2/Q_{cr}^2 - 1\right]}$$

Figure 13: Prediction of the maximum displacements. Figure 14: Prediction of base shear.

We try to apply this equation to the BPY system. $Q_0$ is estimated using the design acceleration spectrum shown in fig 5. The prediction results are shown in fig 14. They almost coincide with the response results.
5 Conclusion

The results of this paper are summarized as the follows.
1) Seismic energy input into the BPY system subjected to lateral earthquake ground motions is dissipated or absorbed by uplifting of the gravity center, vertical inertia force on the uplift side including impact effect, hysteresis damping of base plates and viscous damping and elastic deformation of the super structure. (See eq. (3))
2) The seismic energy dissipation by the uplifting gravity center and the hysteresis damping of base plates occupies the large part of the energy dissipation corresponding to the maximum momentary input energy (MIE).
3) The maximum response displacements can be predicted considering the energy balance condition. (See fig.13)

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References
