Analysis of earthquake wave propagation in buildings

H. Kawakami1, M. Oyunchimeg2 & E. A. J. Tingatinga3
1Geosphere Research Institute, Saitama University, Japan
2Graduate School of Science and Engineering, Saitama University, Japan
3Department of Engineering Sciences, University of the Philippines, Philippines

Abstract

This paper presents a wave propagation modelling analysis of strong motion records for actual buildings in comparison with an analysis of simple building models. In previous studies on earthquake response of buildings, vibration approaches were used more often than wave propagation approaches; thus, wave propagation through a building has not been investigated enough. We use the normalized input output minimization (NIOM) method, which can model wave propagation in multiple linear systems by considering the statistical correlation among the strong motions at different observation locations. Output wave models simplified by this method show two clear peaks that correspond to incident and reflected waves propagating through the building in the vertical direction. The obtained wave propagation properties, such as wave travel time and wave amplitude ratio, at each story of the building are found to reflect the structural properties like rigidity and damping ratio at that story. Further, a new method for wave-propagation analysis—called evolutionary normalized input-output minimization—is developed. It models time-variant wave propagation by considering the time-variant statistical correlation between strong motions recorded at different levels in the building. In the case of damaged buildings, the travel time increased during the earthquake; however, in the cases of undamaged buildings and the elastic model, it remained almost constant during the earthquake.

Keywords: strong motion records, buildings, wave propagation, wave travel time, wave amplitude, fundamental period, damping, damage.
1 Introduction

Analyzing the behaviour of a structure during an earthquake involves two problems, vibration and wave propagation, because the vibration of a structure results from seismic-wave propagation in it. Vibration methods are well known and have been developed mainly for structural engineering. In contrast, wave-propagation approaches have been mainly used to investigate ground motions due to earthquakes, and researchers use several methods (such as impulse response and correlation functions) to simplify and clarify the wave propagation in soil layers and to determine soil properties.

One of the wave-propagation modelling methods—the normalized input-output minimization (NIOM) method [1, 2] is used in the present work. The NIOM method models wave propagation by considering the statistical correlation of the earthquake motions at different observation locations. Analysis results of simple building model and actual building records show that wave propagation properties at each story of the building reflect the structural properties at that story.

We developed a so-called evolutionary NIOM method for examining the time-variant properties of buildings. This method can simplify the relationships between waves observed at several levels in the building at different times from the beginning to the end of the earthquake, and it can determine the change in travel time of the wave propagating through the building [3]. The travel time was found to be a very good indicator of damage, and its change was related to the change in the degree of damage, i.e., the change in the structural properties of the building.

2 Methodology

2.1 NIOM method

The normalized input-output minimization (NIOM) method can model wave propagation by considering the statistical correlation between earthquake motions observed at different points. This method can simplify the relationship between the observed waves and give arrival times, as well as their relative amplitudes, of incident and reflected waves. Since the wave-propagation velocity depends much on the characteristics of the materials through which the wave propagates, such a method is applicable not only to ground-motion analysis but also to building-record (i.e., of earthquakes) analysis. A brief review of the NIOM method is given below [1, 2].

2.1.1 NIOM method for a linear system

When a time-invariant linear system is subjected to an earthquake motion, the input and output of the system in the frequency domain can be related by means of the transfer function \( H(\omega) \). In the case of digitized earthquake motions, the output at each frequency is given by
\[ G(\omega_i) = H(\omega_i)F(\omega_i) \]  
\[ (i = 0, \ldots, N-1 \text{ and } \omega_i = i \frac{2\pi}{N\Delta t}) \]

where \( \Delta t \) is the sampling rate in the time domain; \( N \) is the number of samples; and \( F(\omega_i) \) and \( G(\omega_i) \) are the Fourier transforms of the digitized earthquake input and output motions, respectively. Transfer functions depend only on the physical properties of the system. Therefore, the same transfer function as the one that defines the relationship between the observed input \( F(\omega_i) \) and output \( G(\omega_i) \), eqn (1), should satisfy the relationship between the simplified input model \( X(\omega_i) \) and the simplified output model \( Y(\omega_i) \).

\[ Y(\omega_i) = H(\omega_i)X(\omega_i) \]

Minimizing the summation of the squared values of Fourier amplitude spectra of the input and output when a constraint is in existence [1, 2] would result in a simplified input model \( X(\omega_i) \) and a simplified output model \( Y(\omega_i) \). The procedure that gives the simplified input and output models is shown schematically in fig. 1. In the analysis of a feedback system, it should be noted that input cannot be separated from output and that the input motion in this paper is not the incident-wave motion. The motion observed at one arbitrary location (always the building’s roof in this paper) is called the input motion in the following numerical analyses.

### 2.1.2 NIOM method for multiple linear system

Controlling the contribution of high- or low-frequency components in the procedure and generalizing to multiple linear systems gives the following equations:

\[
X(\omega_i) = N\Delta t \frac{1}{\sum_{n=0}^{N-1} \left(1 + \frac{k_0}{c_0} \omega_i^2 \right) \left(\frac{c_0}{M} + \sum_{m=1}^{M} |H_m(\omega_i)|^2 \right)} \]

\[ Y(\omega_i) = N\Delta t \frac{1}{\sum_{n=0}^{N-1} \left(1 + \frac{k_n}{c_0} \omega_i^2 \right) \left(\frac{c_0}{M} + \sum_{m=1}^{M} |H_m(\omega_i)|^2 \right)} \]
where \( l=1,\ldots,M \); \( M \) is the number of outputs; \( c_0 \) to \( c_M \) are weighting constants of the squared Fourier amplitude spectra of the input and outputs, and \( k_0 \) is weighting constant of time derivative of the squared Fourier amplitude spectrum of the input. Effects of these weighting constants on the results are shown in the referred papers [1, 2].

The inverse Fourier transforms of eqns (3) and (4) give simplified input and output models in the time domain. These simplified input and output models illustrate the statistical correlation between the observed motions, and the procedure for obtaining these models is called the NIOM method. It should be noted that the Fourier transform defined in eqn (3) is real and symmetric with respect to the folding frequency. This means that the input model is real and symmetric with respect to time zero as in the case of auto-correlation functions.

\[
\begin{align*}
&f(t_i) \rightarrow F(\omega_i) \quad \text{Observation} \\
g_i(t_i) \rightarrow G_i(\omega_i) \\
&G_i(\omega_i) = H_i(\omega_i) \rightarrow X(\omega_i) \rightarrow x(t_i) \\
&X(\omega_i) \rightarrow x(t_i) \\
&Y_i(\omega_i) \rightarrow y_i(t_i)
\end{align*}
\]

Figure 1: Schematic procedure of the NIOM method.

3 NIOM analysis of simple building model

The earthquake responses of a simple 10-story building model were computed. Floor weight and story stiffness distributions for this model are shown in fig. 2(a), and it is the same model as that in the book edited by M. Paz [4]. The acceleration time history recorded in the basement of Transamerica building during the Loma Prieta earthquake of October 18, 1989 [5] (NS component) was used as a motion at the ground surface (fig. 2(b)). The responses of the assumed building were computed by modal analysis, in which a Rayleigh damping ratio was assumed in the first two modes. The ground motion and the computed response acceleration time histories at the fifth floor and roof (damping ratio is 5%) are shown in fig. 2(b).

We applied the NIOM method to the computed response accelerations of the assumed building, such as shown in fig. 2(b), and obtained the simplified input and outputs as shown in fig. 2(c). Here, the response acceleration at the roof was considered as the input, and the acceleration responses at the other nine floors were considered as outputs. Figure 2(c) shows analysis results when the sampling rate of the time series was 0.005 s and weighting constants were \( k_0=0.01, c_0=1 \) and \( c_1=\ldots=c_9=1 \). As shown in fig. 2(c), the input was modelled such that its amplitude at time \( t=0 \) was unity and the amplitudes at the other times and output approached zero unless correlation existed between them. One may understand simply that the amplitude at the top story is assumed to be 1.0, and relative amplitude compared to the top story is obtained for the other stories. One can see two clear peaks in the simplified output models corresponding to the
incident and reflected waves, which are indicated by arrows (1) and (2) in the
figure, respectively. The incident wave propagates from the basement to the roof,
whereas the reflected wave propagates from the roof to the basement. Arrival
times of the incident and reflected waves were the same: 0.14 s and 0.29 s at the
fifth floor and basement, respectively. The waveforms of the simplified models
changed as we changed the value of $k_0$, which determines the contribution of
high or low frequencies [1, 2], but the obtained wave arrival times were similar.
However, due to the limited space, only figures showing NIOM analysis results
for $k_0=0.01$ are given in this paper.

![Figure 2](image)

Figure 2: Assumed 10-story building model: (a) configuration and properties,
(b) computed acceleration response, (c) NIOM method analysis result.

Simplified models obtained from this analysis give the arrival time and
relative amplitude of incident and reflected waves at each level.

Corresponding to the non-uniform building model shown in fig. 2(a) (case 1 in
fig. 3), we assumed a 10-story uniform building with the same stiffness of 230
tf/cm in all stories (case 2 in fig. 3). This value of $K = 230$ tf/cm was chosen by
assuming a series spring system: 

$$\sum_{i=1}^{10} \frac{1}{K_i} = \frac{10}{K},$$

where $K_i =$ stiffness of each story

in case 1 (see fig. 2(a)) and $K =$ story stiffness (same in all stories) in case 2.

Figure 3 (a) shows the wave arrival times at the floors for three cases of story
stiffness distribution for the assumed building. In Case 3 the order of story
stiffness is reversed from that in Case 1. From this figure one can see that how
the wave travel time between floors changes when story stiffness distribution is
changed. It is interesting that the wave travel times at basement are equal to 0.29
s in all three cases. This means that the arrival time obtained from the NIOM
method is precisely the wave travel time from the basement to the roof.

Figure 2(c) shows that the reflected wave amplitude was smaller than the
incident wave amplitude in the output models. The NIOM results for the
assumed building (Case 1) were obtained by changing the damping ratio from 1% to 30%: cases of 1%, 5%, 10% and 20% damping are shown in fig. 3 (b). One can see that the difference between the incident and reflected wave amplitudes increased with increasing damping ratio, and that the arrival times were similar for different damping ratios.

Figure 3: (a) Wave arrival times at the floors of the assumed model for three cases of story stiffness distribution, (b) NIOM results of assumed 10-story building with different damping ratios.

4 NIOM analysis of actual building records

We have applied the NIOM method to a number of earthquake strong motion records observed in actual buildings and fig. 4 shows some results. Here, the record at the roof was considered as input and records at all other floors as outputs of the system.

4.1.1 Los Angeles, 52-story office building

This office building located 31 km from the epicentre of the January 17, 1994 Northridge earthquake and has 52 stories above and 5 levels below the ground. Its lateral force resisting system consists of concentrically braced steel frames at the core with moment resisting connections and outrigger moment frames in both directions [6]. Figure 4 (a) shows NIOM analysis results of the building record in the NS direction. Simplified wave models obtained at each story in the building show two clear peaks that correspond to incident and reflected waves propagating through the building in the vertical direction. Arrival times of the incident and reflected waves were mostly the same, and reflected wave amplitude was always smaller than incident wave amplitude. Lines with circle in fig. 4(a) show wave arrival times obtained at the instrumented floors, and one can see a clear change in wave velocity between floors above and below ground. Quantitatively, obtained wave velocities were about 180 m/s and 430 m/s above
and below ground floors, respectively. Wave velocity difference shows the difference in story property.

Figure 4: NIOM analysis results of actual building records: (a) 52-story office building in Los Angeles, (b) Hollywood storage building in Los Angeles, (c) 3-story commercial building in Los Angeles.

4.1.2 Los Angeles, Hollywood storage building
This building stood in 23 km from the epicentre of the January 17, 1994 Northridge earthquake and has 14 stories above and one level below ground. Its lateral load resisting system consists of reinforced concrete frames in both directions [6]. Figure 4 (b) shows NIOM analysis results in the NS direction of the building, which shows a similar trend with the previous building. There is also a difference in wave velocity: about 90 m/s between roof and 8th floor and about 125 m/s between 8th floor and basement.

4.1.3 Los Angeles, 3-story commercial building
This building located in 20 km from the epicentre of the January 17, 1994 Northridge earthquake and it has three stories above and two parking level below the ground. Its lateral load resisting system consists of steel braced frames in the upper three stories and concrete shear walls in the parking floors [6]. Figure 4 (c) shows NIOM analysis results in the NS direction of the building. There was also wave velocity difference: about 95 m/s above and 170 m/s below ground floors.

Here, one should notice that shear wave velocity for steel is about 3000 m/s, and even for rubber the wave speed is 980 m/s, way above the values found. Such differences are because the obtained values are not the shear wave velocities in the dense materials but those in the building frame. Clough and Penzien [7] have approximated them as $v = \sqrt{12 \sum EI / (m_i h)}$, where $m_i$ is the mass of each story, $\sum EI$ is the total rigidity of all columns, and $h$ is the height of each story.
5 Damaged buildings

Strong motions recorded in a damaged building, which has deformed into the plastic range, give different information from that expressed by the elastic characteristics of the building. To reveal the damage process of such a building during an earthquake, the change in building characteristics with time should be examined from the beginning to the end of the earthquake. So, we tried to analyze the records segmentally in time and compared the results.

The observed strong motions were divided into short (but long enough to obtain reliable results) portions and NIOM was applied to each portion. We named this method as the "evolutionary NIOM method". The shortest length of each portion was chosen to be from $2\tau$ to $4\tau$ ($\tau$ is the wave travel time from the basement to the roof) so that a reliable correlation between motions at different levels in the building could be obtained. Windows with length of 5.12 to 10.24 s, shifted by 1 or 2 s from the beginning to the end of the record were used. Narrow windows, strong motion records in the EW direction for Van Nyus-7-story hotel building, and analysis results are shown in fig. 5. This 7-story hotel located 7 km from the epicentre of the 1994 Northridge earthquake experienced serious structural and non-structural damage. It was found that incident-wave arrival time changed from 0.24 s to 0.40 s (see fig. 5) at the ground floor in the EW direction. As shown in fig. 5 the first change in the incident-wave arrival time occurred at around 3-4 s, and an additional change occurred at around 7-8 s. These results indicate how the damage developed in the building during the earthquake. The possibility to locate the damage and identify the extent of damage depends on the availability of the strong motion records in the building.

Figure 5: Narrow windows used in evolutionary NIOM method and analysis results for Van Nyus-7-story hotel building.
To test the method thoroughly, it was applied to an assumed elastic seven-degrees-of-freedom shear-spring model, which represents a seven-story building. In the model, all floors have the same mass of 45.3 t and all the stories have equal stiffness of 200 kN/cm, and the fundamental period by modal analysis is 1.43 s. Elastic response of the model was computed by the conventional vibration method. In this calculation, the ground motion was considered as the strong motion (EW component) recorded at the ground floor of the seven-story hotel in Van Nuys during the 1994 Northridge earthquake. Figure 6 shows the computed acceleration responses when the damping ratio in the first mode is assumed to be 10% and wave arrival times obtained by the evolutionary NIOM method. According to the elastic model, there should be no change, theoretically, in wave travel time during the earthquake. Figure 6 clearly proves the above-mentioned theory; namely, differences in the arrival times obtained between windows were only ±0.02 s, which is the same as the sampling rate.

6 Conclusions

Simplified wave models obtained by the NIOM method at each story in the building show two clear peaks that correspond to incident and reflected waves propagating through the building in the vertical direction. Arrival times of the incident and reflected waves were mostly the same, and reflected wave amplitude was always smaller than incident wave amplitude. Wave travel times obtained in each story in actual buildings were found to reflect the structural properties in that story, such as irregularity in height and difference in structural types.

Evolutionary method for normalised input-output minimization (NIOM) enables time-variant wave propagation modelling by taking into account the time-variant statistical correlation among the strong motions recorded at different levels in the building. The NIOM results for actual damaged and undamaged buildings, as well as those from an assumed elastic building model, were
compared. In the case of the damaged buildings, the travel time increased during the earthquake; however, in the case of the elastic model, it remained almost constant during the earthquake. It was found that the change in the travel time is related to the change in the structural properties and to the degree of damage to a building. These results show that evolutionary NIOM is an effective new method for investigating the change in structural properties and the damage to buildings. This method has an advantage of investigating building behaviour during earthquakes without assuming any structural properties, because only strong motion time histories recorded in the building are needed.

References