Behavior of rigid bodies in buildings

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Abstract

The behavior of rigid bodies with different shape ratios, located in some levels of buildings of the valley of Mexico and Acapulco, subjected to events of near and distant sources is modeled step by step. A fast estimation of the movement in the base of the content in a floor level of the building comes through a simplified model whose accuracy was proven in a parallel work to this (Arredondo et al [1]). Maps for toppling of contents appear as well as expressions to determine the amplification of the movement in a story of the building.

Keywords: accelerations demands, multistory buildings, earthquakes, dynamics, nonlinearity, rocking, slide, rigid body, seismic response, impact problem.

1 Introduction

The control of losses for damage in contents is important because in most of industrial buildings, stores and offices, it represent most of the total cost of the investment. Also the damage in the contents is associated to forces and smaller levels of deformation that those required to begin the structural damage.

In this work, the response of rigid bodies located in different stories of buildings that are in several sites of Mexico City and Acapulco, is modeled. For this, we estimate the movement in different levels of a building subjected to earthquakes and the value of the amplification of the peak ground acceleration with the height, through a simplified model of the structure whose validity was proven so much in instrumented buildings in Mexico City like United States [1, 2, 3, 4]. A numerical solution is used to solve the nonlinear equations of the dynamic behavior of a rectangular rigid body subjected to movements in its base, considering the energy loss every time that the impact with the support surface happens. Previous works have studied the linear case associated to slender
bodies [5, 6, 7] and the nonlinear [8, 9, 10] of symmetrical rigid bodies; however, they have pointed to model its response in free field, without considering the implications related with the type of structure (walls, frames and dual systems) and the story where the body is located in its interior.

2 Calculation of demands in multistory buildings

2.1 Simplified model

Through to the approximate method of Miranda and Taghavi [2], we estimate floor acceleration demands for regular buildings in plant and height located in different sites of the valley of Mexico and Acapulco, subjected to events of subduction and normal failure. The dynamic characteristics of multistory buildings are approximate by using an equivalent model that consists of a vertical beam that combines lateral deformations of flexion and shear, fig. 1.

![Diagram](Flexural beam - Shear beam - Links axially rigid)

Figure 1: Simplified model of multistory buildings.

The method considers lineal elastic behavior and it combines the application of a modal analysis using the first six modes of vibration of the building with a simplified model of the structure from which it's possible considered modal shapes, modal participation factors and modal frequencies. If it is considered that the distribution of mass and rigidity in height are uniform, the accelerations in any floor level of a structure can be obtained as the combination of the response in all modes of vibration according to the eqn (1).

\[
\ddot{u}^i(x,t) = \left[ I - \sum_{i=1}^{m} \Gamma_i \phi_i(x) \right] \cdot \ddot{u}_g(t) + \sum_{i=1}^{m} \Gamma_i A_i(t)
\]  

(1)

where \(m\) is the number of modes considered in the response, \(\Gamma_i\) is the modal participation factor of the ith mode of vibration, \(\phi_i(x)\) is the amplitude of the ith mode shape of vibration at height \(x\), \(A_i(x)\) is the absolute acceleration of a SDOF with a period and damping corresponding to the ith mode and \(\ddot{u}_g(t)\) is the time history acceleration ground.

Three additionally parameters are required to determine different demands to certain height of a building: the fundamental period of vibration \(T\), the damping \(\xi\) and \(\alpha\) that controls the degree of participation of overall flexural and overall shear deformations in the model. Shear wall and braced frame buildings usually
have values of $\alpha$ between 0 and 2.5, buildings with dual structural systems (moment-resisting frames and shear walls or moment-resisting frames and braced frames) have values between 2.5 and 5 and moment-resisting frame buildings between 5 and 15 (Miranda and Reyes [11]).

2.2 Registered events and studied cases

In the valley of Mexico were selected the sites SCT (Secretaria de Comunicaciones y Transportes) and CDAO (Central de Abastos) in soft soil characterized to evidence important amplification levels regarding firm soil and in the transition zone the place VI (Viveros) was selected. For Acapulco three sites were selected in soft soil: the sites ACAC (Casa de la Cultura) and ACAD (La Diana) located on silts with thickness of 30 m and bigger to 35 m respectively while the place ACAZ (La Zanja) is located on deposits of sand, slime and clay of unknown thickness (Gutiérrez and Singh [12]). These sites correspond to accelerometer stations of Mexico City and Guerrero whose recorded events in EW component, for this work, were considered as the movement in the base of each structure. The events considered in each case and some of their characteristics are indicated next, to see table 1 and fig. 2.

Table 1: Some characteristics of the registered events.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Events</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Origin</td>
<td></td>
<td>Subduction</td>
<td>Subduction</td>
<td>Normal</td>
<td>Subduction</td>
<td>Normal</td>
</tr>
<tr>
<td>Magnitude (M)</td>
<td></td>
<td>8.0</td>
<td>6.9</td>
<td>6.2</td>
<td>7.3</td>
<td>7.4</td>
</tr>
<tr>
<td>Longitude (W)</td>
<td></td>
<td>102.71</td>
<td>99.480</td>
<td>100.57</td>
<td>98.540</td>
<td>96.960</td>
</tr>
<tr>
<td>Depth (Km.)</td>
<td></td>
<td>16</td>
<td>17</td>
<td>50</td>
<td>22</td>
<td>47</td>
</tr>
<tr>
<td>Amax Ground (m/s^2)</td>
<td>SCT</td>
<td>1.59</td>
<td>0.36</td>
<td>-</td>
<td>-</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>CDAO</td>
<td>0.76</td>
<td>0.33</td>
<td>-</td>
<td>-</td>
<td>0.29</td>
</tr>
<tr>
<td></td>
<td>VI</td>
<td>0.53</td>
<td>0.16</td>
<td>0.08</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>ACAC</td>
<td>-</td>
<td>0.91</td>
<td>-</td>
<td>0.44</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>ACAD</td>
<td>-</td>
<td>3.23</td>
<td>-</td>
<td>0.61</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>ACAZ</td>
<td>-</td>
<td>1.44</td>
<td>-</td>
<td>0.21</td>
<td>0.19</td>
</tr>
<tr>
<td>Distance (Km.)</td>
<td>SCT</td>
<td>415</td>
<td>312</td>
<td>217</td>
<td>300</td>
<td>441</td>
</tr>
<tr>
<td></td>
<td>CDAO</td>
<td>420</td>
<td>310</td>
<td>219</td>
<td>297</td>
<td>436</td>
</tr>
<tr>
<td></td>
<td>VI</td>
<td>412</td>
<td>307</td>
<td>212</td>
<td>297</td>
<td>440</td>
</tr>
<tr>
<td></td>
<td>ACAC</td>
<td>345</td>
<td>50</td>
<td>152</td>
<td>145</td>
<td>330</td>
</tr>
<tr>
<td></td>
<td>ACAD</td>
<td>341</td>
<td>54</td>
<td>149</td>
<td>148</td>
<td>334</td>
</tr>
<tr>
<td></td>
<td>ACAZ</td>
<td>354</td>
<td>41</td>
<td>161</td>
<td>138</td>
<td>322</td>
</tr>
</tbody>
</table>

Two types of structures were considered: the first one with shear walls ($\alpha = 2.5$) and the second whose resistance is contributed mainly by moment-resisting frames ($\alpha = 25$). In both cases the fundamental structural period was changed between 0.1 and 6 seconds and values of damping of 2% and 5% were adopted.
The fractions of the total height of the building for which the histories of acceleration were obtained are $X_i/H = 0.25, 0.50, 0.75$ and 1 (zero correspond to the basement and one is equal to the roof).

![Diagram](attachment:image.png)

**Figure 2:** Location of the considered stations (stars) and epicenters of the events of normal failure (rhombuses) and subduction (circles).

### 2.3 Maximum intensities and amplification factors ($PFA/PGA$)

It was calculated for each case, the history of accelerations in different levels of the buildings analyzed to know the factor $PFA/PGA$ that indicates how many times the movement was amplified in a level of the building ($X_i/H$) regarding the maximum intensity in its basement; $PFA$ means Peak Floor Acceleration while $PGA$ means Peak Ground Acceleration.

With the results of the different analyses for each combination of parameters ($T$, $\xi$ and $\alpha$) and a place, we obtained maps that consider events coming from different sources and that it allow to interpolate values of the amplification in a story of the building ($X_i/H$) regarding the peak intensity registered in the basement ($A_{max}$) of a structure with certain dynamic properties. Next, some amplification maps to calculate the structural response in different height ratios for moment-resisting frame buildings are shown, to see fig. 3. In these maps, the horizontal axis represents a fraction of the total height of the building ($X_i/H$), the vertical axis indicates the range of peak intensities in the basement of the structure and the different level curves a certain amplification factor ($PFA/PGA$) whose value is associated to the scale of colors that it is presented. Each column corresponds to a building with certain structural period and each line represents the site where the building is located. The relevance of this point is so much to model and to evaluate the behavior of contents and non structural elements located in certain story of a building, like to have an idea of the relationship that can exist between structural stiffness and damage.
Figure 3: Amplification maps in some sites of Mexico City and Acapulco for moment-resisting frame buildings ($\alpha = 25$) $\xi = 2\%$.

Of figure 3 we can say that: a) Still for an identical structure, the same value of intensity in the ground or $X_i / H$ doesn't imply similar amplifications of the movement due to the variability in the dynamic properties from the ground site to site, b) bigger values of damping $\xi$ implies reductions in the amplifications in superior levels but not necessarily changes in the form of the maps, c) the maximum amplifications in a place are present for structures with periods of vibration near to the fundamental period of the ground and d) $\alpha$ it is not a definitive parameter in the amplification value in different stories of a building, because in the simplified model its function is to adjust the main peaks of the floor spectrum corresponding to higher modes (Miranda and Taghavi [2]).

2.3.1 Obtained regressions
Once obtained the amplifications in the selected sites, for structures with different dynamic properties, we proceeded to make a regression analysis to outline expressions that in function of the identified variables as important during the first phase of the study, allow estimating the value of the ratio $PFA/PGA$ in any story of a building. After to consider different types of regressions and to evaluate the best adjustment we adopt in soft soil of Mexico City a family of curves from a similar way to the proposal for Arroyo and
Terán [13] in a previous work related with reduction factors of seismic forces in the design of structures with systems of energy dissipation. The eqns (2) and (3) correspond to regressions for soft soil of Mexico City and for the different sites in Acapulco respectively; this expressions should satisfy like frontier condition that \( PFA/PGA = 1 \) as \( T \) tends to zero, independently of \( \xi \) and \( \alpha \) in the model; this is due to that the peak acceleration that experiences an infinitely rigid system during a seismic excitations is similar to the maximum ground intensity. The parameters \( b, \phi, \theta, \chi, \beta, \delta \) are function of \( X_i/H \).

\[
\frac{PFA}{PGA} = \left( \frac{T}{T_s} \right)^{b} \cdot \phi + \left( \frac{T}{T_s} - 1 \right) \cdot \theta
\]

\[
\frac{PFA}{PGA} = \chi \cdot \beta T \cdot T^{\delta} \cdot \theta
\]

In the fig. 4 the amplification factors calculated with the simplified model [2] and the adjustments proposed for different sites of the valley of Mexico and Acapulco are presented; the above-mentioned for a damping of 2%, the two types of studied structures and the fractions of the total height of the building considered. It was estimate correlation factors of 0.8 in soft soil of Mexico City and between 0.6 and 0.9 for Acapulco.

Figure 4: Amplification factors \( PFA/PGA \) and adjustments for buildings located in different sites of Mexico City and Acapulco; \( \xi = 2\% \).
3 Modes of dynamic response for rigid bodies

Following previous works [8, 9], the behavior of a rectangular rigid block that rests on a plane and horizontal surface was modeled step by step for four possible types of movements: rest, slide, rotation and slide rotation. The non linear formulation of the problem in each case is given by the eqns (4) to (7) for the last three types of movements respectively; these expressions describe the movement of a rectangular body rotating and sliding regarding their mass center, in function of the horizontal base excitation $x_g''$, the rotation $\theta(t)$, the mass $m$, the mass moment of inertia $I$, $R^2 = b^2 + h^2$ and the angle $\alpha = \tan^{-1}(b/h)$. The parameters $2b$ and $2h$ correspond to the breadth and height of the body respectively, $\beta = mgR/I$ and $S()$ it represents the signum function. It was only considered horizontal movement in the base of the body, what assures that the body is always in contact with the support surface and the vertical force that it acts in its center of mass is always bigger than zero.

$$x'' + x_g'' = -S(x')\mu_k g$$ (4)

$$(I + mR^2)\theta'' = mR\cos(\alpha - |\theta|)x_g'' - S(\theta)mRg\sin(\alpha - |\theta|)$$ (5)

$$\theta'' + \beta^2 f_1(\theta, \theta'/\beta, x') = 0$$ (6)

$$x'' + \beta^2 Rf_2(\theta, \theta'/\beta, x') = -x_g''$$ (7)

In this work, we obtained the solution of the eqns (4) to (7) through the mid central difference method that is based on an approach in finite differences of the derived of the displacement regarding the time (velocity and acceleration); this allows to obtain the answer for the non linear behavior of the rigid body in way more efficient considering that other numeric subroutines are iterative processes less practical in terms of time. This way, the answer $\theta(t)$ and $x(t)$ in the instant of time $t = i + 1$ are calculated using the equation of the movement, their derived and the known response in previous instants of time; in our case for the instants $t = i$ and $t = i+1/2$. Because the problem is highly non linear it is possible to use this method only modifying the increments of time in such a way that $\Delta t = \Delta t_{i+1/2} = \Delta t_{i+1} - \Delta t_i$ and the mid steps are redefined for the velocities. It is assumed that the movement is initiated from the rest ($(\theta(i) = 0, \theta'(i) = 0)$ and that the acceleration is constant between intervals of time $\Delta t$, while the velocity and the displacements vary in lineal form and quadratic form respectively. The different solutions considered the energy loss according to the model that governs the impact outlined by Shenton [9]. The solution was proven comparing with results obtained in previous works [8, 9].
3.1 Obtained results

The dynamic response of rigid bodies with different shape ratios \((h/b)\) was evaluated, located in different stories of the studied buildings and subjected to movements calculated in the same ones through of the simplified model \([2]\). In the fig. 5 the critical dimensions to overturning are shown, obtained for contents in a structure of \(T = 2\) seconds located in the different considered sites, \(\xi = 2\%\) and the April 25 of 1989 event. In these figures, the diagonal and horizontal continuous lines represent: the first one, a limit proposed by Housner \([5]\) to classify slender bodies \((h/b > 2.75)\) or small bodies \((h/b < 2.75)\) and the second, the maximum interstory height considered in this work (3 m) and that is a superior limit for the height of the content \((2h)\). The dotted line represents the dimensions to initiate rocking motion from a body subjected to an excitation of certain intensity and in previous researches (Ishiyama \([6]\)) was used as a critical value of the intensity so that the overturn happens.

![Figure 5: Dimensions to overturn rigid bodies in the roof of buildings located in Mexico City and Acapulco; \(T = 2\) seconds and \(\xi = 2\%\).](image)

The results of the non linear model for rigid bodies presented in this research, show that the dotted line gives good results in sites of soft soil and transition of the Valley of Mexico (fig. 5) while in different sites of Acapulco it is observed that, for oneself breadth \(2b\) of the body, this it can underestimate or overestimate the height to overturn. In general, we could observe that slender bodies are more vulnerable and it can collapse in different levels of buildings located as in the center as near the Pacific coast of Mexico. It is probable that some small bodies overturned if: a) they are located in superior stories of structures whose fundamental period is near to the period of the ground, an example of this it was observed in the site SCT to \(\alpha = 25\) or b) they are in structures located near to the...
epicenter of the earthquake, like it is the case of the site ACAD with $\alpha = 2.5$ and 25 at a smaller distance to 50 km of the source of the event 2, table 1.

With the critical dimensions of overturned calculated in the different studied cases we obtained maps that relate, for the sites in Acapulco and Mexico City, the location of a rigid body inside the building $(X_i/H)$ and the peak intensity ($PFA$) to which is subjected with the shape ratio $h/b$, fig. 6. Of this figure, it is possible to interpolate values of the shape ratio for which a rectangular body overturned when it is located in certain level of a structure that presents a certain structural period ($T$). For example, a rigid body in the roof of a shear wall building located in the Lakebed Zone of Mexico City (SCT) with a fundamental period of 1 second and a damping of 2% it requires to overturned values of $h/b$ bigger than 4 when it is subject to intensity of 3 m/s². The figures present different scales in their ordinates to avoid extrapolate the information and indicate the range of values considered in each case.

![Figure 6: Values of the shape ratio ($h/b$) to overturned rigid bodies in different stories of buildings located in the site SCT; $\xi = 2\%$ y $\alpha = 2.5$.](image)

4 Conclusions

Using a simplified method to model the behavior of a building, expressions were obtained that relate the amplification of the movement regarding to the ground and the behavior of rigid bodies located in different levels of the structure was studied.

Later works will allow to obtain complete maps in function of the amplification of the movement and of the critical dimensions of contents if it is considered in the first case sites and buildings with different dynamic properties, subjected to events coming from more sources and in the second case more sophisticated models of so much flexible as rigid bodies, distribution non uniform of mass and subjection systems and anchorage; their calibration will depend from the employment of models to scale and devices designed to reproduce in laboratory intensities and displacements.
References


