Damage controlled seismic design of quasi-brittle beam structures

G. D. Hatzigeorgiou & D. E. Beskos
Department of Civil Engineering, University of Patras, Greece

Abstract

A new seismic design philosophy based on the concept of damage is presented and applied to the case of plane beam structures exhibiting quasi-brittle material behavior, such as that of plain concrete or masonry. The proposed direct damage controlled seismic design method quantifies and controls damage in a direct and transparent manner much better than any other of the existing methods. The method appropriately combines code based nonlinear stress-strain relations for the material with a simple expression for damage to determine with the aid of the fiber beam model and the finite element method axial force and bending moments as functions of damage. Using this method, the designer can either determine the damage level for a known section and loading, or dimension a section for a target damage level and known loading, or determine the maximum loading for a known section and a target damage level. The method is illustrated by means of two examples. The first one deals with the frequency analysis of a damaged concrete beam, while the second one with the damage control of a masonry arch subjected to vertical and lateral (seismic) load applied statically.

Keywords: damage control, quasi-brittle materials, concrete, masonry, beam structures, seismic design, finite element method.

1 Introduction

In order to satisfy safety requirements, three different seismic design philosophies have been developed to date leading to the following methods: the working stress method or allowable stress method (WSM), the ultimate strength method or force-based method (USM) and the displacement-based method (DBM). In spite of their advantages and wide-spread use, the above methods have various shortcomings. For example, the WSM is associated with elastic
analysis and cannot account for nonlinear phenomena in structures, such as damage. On the other hand, displacements are of secondary concern and treated near the end of the design by both the WSM and the USM. Thus, even the design philosophy of the USM, which works in the inelastic range, does not permit an effective control of damage, which is much better related to displacements. For example, the percentage of the relative inter-story drift of seismically excited buildings is considered a solid basic indicator of the level of damage, as suggested by FEMA [1]. Nonetheless, even the DBM, which holds the displacements at a permissible level (target displacements), does not lead in a direct and transparent control of damage.

To be sure, there are many works in the literature dealing with the determination of damage in concrete members and structures. More specifically, damage determination of framed concrete buildings has been done with the aid of damage indices computed on the basis of deformation and/or energy dissipation, as this is evident, e.g., in the review article of Powell and Alahabadi [2]. In these works the finite element method has been employed in conjunction with a concentrated inelasticity beam element. Damage determination has also been done for two-and three-dimensional concrete and masonry members and structures by employing continuum theories of damage in the framework of the finite element method (Cervera et al [3], Onate [4], Hatzigeorgiou et al [5], Hanganu et al [6]).

In this paper, a new design philosophy is presented, leading to a method named Direct Damage Controlled Method (DDCM) of design. The basic idea of DDCM is the dimensioning of beam members or whole framed structures, so as damage, at local and global level, is directly controlled (target damage). With DDCM, the engineer decides a priori about the level of damage on a structural member or a whole structure. While the knowledge of damage is an end in itself in structural design, inelastic design approaches such as the USM and the DBM are concerned indirectly with damage, while the WSM is evidently irrelevant with respect to the damage-controlled design. This a priori knowledge of damage ensures a controlled safety level, not only in strength but also in deflection terms. In spite of the generality of the presented design philosophy, this paper focuses only on beam type of structures made of concrete or masonry and subjected to statically applied axial forces and bending moments. Thus, seismic design can only be accomplished by the present stage of development of the method on the assumption that seismic forces are equivalent static ones.

More specifically, the present work, unlike all previous works on damage of concrete or masonry structures i) develops for the first time a damage controlled design method compatible with existing codes, which is not just restricted to damage determination, ii) combines the basic concepts of the continuum damage theory with existing concrete code requirements to create a simple and reliable expression for the damage index as a function of the deformation and iii) combines the above damage model with the fiber beam model to create with the aid of the finite element method a versatile design tool.

Two numerical examples are presented here in order to illustrate the method and demonstrate its advantages. The first deals with the frequency analysis of a
damaged concrete beam, while the second with the damage control of a masonry arch under statically applied seismic forces.

The proposed DDCM of design has been applied in this work only to plain concrete or masonry beam structures under static loads. However, this is a general method, which can be extended to other types of material behavior, such as reinforced concrete or steel, other types of structures, such as two-and three-dimensional ones and other types of loading, such as dynamic or seismic loading. These extensions are presently under investigation by the present authors.

2 Stress-strain and local damage for plain concrete

In this work, a good compromise between simplicity, practicality and accuracy is accomplished by adopting a code compatible (Eurocode 2 [7]) stress-strain (\(\varepsilon, \sigma\)) relation of the form

\[
\begin{align*}
\sigma &= 887.3f_c \varepsilon &\text{for } -4.508 \cdot 10^{-4} \leq \varepsilon \leq 0 \\
\sigma &= 1000f_c(1 + 250 \varepsilon) &\text{for } -2.000 \cdot 10^{-3} < \varepsilon < -4.508 \cdot 10^{-4} \\
\sigma &= -f_c &\text{for } -3.500 \cdot 10^{-3} < \varepsilon < -2.000 \cdot 10^{-3} \\
\sigma &= 0 &\text{for } 0 < \varepsilon \leq 10.0 \cdot 10^{-3}
\end{align*}
\]

(1)

where \(f_c\) represents the concrete strength in compression.

According to the continuum damage mechanics theory for elastic quasi-brittle solids (Lemaitre [8]), the effective (inelastic) stress tensor, \(\sigma_{\text{eff}}\), can be expressed as

\[
\sigma_{\text{eff}} = \sigma (1 - d)
\]

(2)

where \(\sigma\) is the elastic stress tensor for the undamaged material. The main scope of continuum damage mechanics theories is the determination (initiation and evolution) of the damage index \(d\) during the deformation process.

Combining appropriately the relations (1) and (2), one can derive an expression for the local damage \(d\) of plain concrete in terms of its deformation \(\varepsilon\) in the form

\[
\begin{align*}
d &= 0.99 &\text{for } 0 < \varepsilon \leq 10.0 \cdot 10^{-3} \\
d &= 0 &\text{for } -4.508 \cdot 10^{-4} \leq \varepsilon \leq 0 \\
d &= -0.1854 - 411.41 \varepsilon &\text{for } -2.000 \cdot 10^{-3} < \varepsilon < -4.508 \cdot 10^{-4}
\end{align*}
\]

(3)
The above damage – strain relation for plain concrete can also be used for other quasi-brittle materials such as masonry.

3 Global damage and global damage levels

The global damage index \( D \) can be determined as a weighted average of local damage indices. The term “local” is associated with damage indices describing the state of the material at particular points of the structure, while the term “global” with damage indices describing the state of any finite material volume of the structure. Thus, global damage indices can be referred to any individual section, member, substructure or the whole structure.

Thus, the total damage index \( D \) for a structural member or a whole structure with total volume \( \Omega \), is proposed here to have the general form

\[
D = q \sqrt{\frac{\int_{\Omega} (d)^q d\Omega}{\int_{\Omega} d\Omega}}
\]

where \( q \) is a damage parameter which specifies the rule of global damage. It is evident that the adoption of the value \( q=1 \) sets the global damage equal to the mean damage. On the other hand, for \( q=2 \), the above relation leads to a more rational definition of global damage index \( D \) where the most critical (damaged) regions influence the global damage.

Many damage classification systems can be proposed in order to evaluate the strength degradation and capacity of a structure to resist further loadings. The simplest system seems to be the one based on the division of the feasible damage range in equal parts (0%, 25%, 50%, 75%).

The proposed design methodology considers a five-value system of global damage levels. This system is named M-scale and is more conservative than the above of equal partitions. The basis level M0 corresponds to zero or almost zero damage (\( D = 0.0 \sim 0.1\% \)), which is associated with elastic behavior, level M1 corresponds to \( D = 0.1 \sim 10.0\% \) and characterizes minor damage, while level M2 corresponds to \( D = 10.0 \sim 30.0\% \) and characterizes moderate damage. Finally, level M3 with \( D=30\% \sim 60.0\% \) can be considered as major damage, while damage level M4 with \( D = 60.0 \sim 99.0\% \) characterizes the state of collapse.

It is obvious that there is a great flexibility on the selection of damage level restrictions at the local and global sense. Thus, one can select a certain upper global damage level (e.g. \( D=30\% \)), which is different than the local (compressive) one (e.g. \( d=70\% \) or 95 %). The target damage level depends on various factors, but the most critical of them seems to be the importance factor of the structure, since a large value of this factor, necessitates a conservative upper limit of damage. Additionally, the most critical members of a structure must be
designed using a more conservative damage level than the others. Thus, traditional design principles, such as the principle of “weak beams – strong columns” in seismic design of structures, can be satisfied by using such a damage selection.

4 Direct damage controlled design of concrete structural members

In order to examine the applicability of the DDCM to the design of concrete or masonry structural beam members or even entire framed structures, a three-dimensional nonlinear design program named DAMCON (DAMAGE CONTROL) based on the finite element method in conjunction with the fiber beam model has been created. For a pre-selected value of global damage for a beam member section of arbitrary shape, the moment M and axial load N are obtained by using relationships derived from equilibrium in the framework of the fiber beam-column modeling. The expressions for N and M have the form

\[
\begin{align*}
N &= \int_{x=0}^{x=H} b(x)\sigma(\varepsilon)dx \\
M &= \int_{x=0}^{x=H} b(x)\sigma(\varepsilon)(x-c)dx
\end{align*}
\]

(5)

where H is the section height, \( b=b(x) \) is the section width, \( \sigma(\varepsilon) \) is given by eqn (1), c is the distance of the center of gravity from the bottom of the section and \( \varepsilon = \varepsilon(x) \) is the strain with a linear variation along the height x of the section, \( \varepsilon_2 \) and \( \varepsilon_1 \) its top and bottom values and \( x_0 \) the distance of zero strain from the bottom.

The solution of the equilibrium equations (5) is accomplished with the aid of the fiber model (e.g., Spacone et al [9]). Thus, the concrete section is subdivided in a user-defined number of concrete fibers. The local damage is related directly to strain, as described by eqn (3) and in view of the fact that \( \varepsilon = \varepsilon(x) \) takes the form \( d = d(x) \), while global damage can be obtained from eqn (4). For three-dimensional loading (axial force with biaxial bending), a similar to the above procedure can be followed.

5 Numerical examples

Two numerical examples are presented to illustrate the proposed methodology and demonstrate its direct applicability to structural design.

5.1 Frequency analysis of a damaged concrete beam

Consider a simply supported concrete beam of rectangular cross-section with geometry and finite element discretization as shown in figure 1. This beam, which has a modulus of elasticity \( E=32 \) GPa and a mass density \( \rho = 2500 \) kg/m³,
has been analysed by Ren and De Roeck [10] in order to locate damage by means of determining frequency changes. In addition to the elastic state, four damage situations, summarized in Table 1, are considered. For each damage situation, the first five eigenfrequencies have been calculated with the aid of the finite element program DAMCON supplemented by the inverse iteration method and listed in Table 2. In the same table and in parentheses, the analysis results of [10] are also presented in order to check the DAMCON accuracy. It is evident that there is a very good agreement between the results of the two methods.

Using eqn (4) one can directly determine the global damage of this beam on the basis of local damages at particular beam elements for every damage situation of Table 1. Thus, for the D0, D1, D2, D3 and D4 damage situations, and for damage parameter $q=1$, the global damage is equal to 0%, 1.3%, 4.7%, 5.3% and 16.7%, respectively. The global damage can also be approximately determined by the relative decrease of the fundamental frequency (DiPasquale and Cakmak [11]), i.e., as $D = 1 - \left( \frac{f_D}{f_E} \right)$, where $f_D$ and $f_E$ is the fundamental frequency of the damaged and the undamaged (elastic) structure, respectively. Figure 2 shows the global damage as determined by eqn (4) and [11] for the four damage situations. It is evident that there is a reasonable agreement between the results of the above two approaches.

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**Figure 1:** Geometry and beam element discretization.

**Figure 2:** Global damage for various damage situations.
Table 1: Local damages at beam elements for five damage situations.

<table>
<thead>
<tr>
<th>Situation</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
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<th>12</th>
<th>13</th>
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<tbody>
<tr>
<td>D0</td>
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<tr>
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<td>0.2</td>
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<tr>
<td>D3</td>
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<td>0.5</td>
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<td>0.2</td>
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<td>0.1</td>
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<tr>
<td>D4</td>
<td>-</td>
<td>-</td>
<td>0.3</td>
<td>-</td>
<td>-</td>
<td>0.3</td>
<td>0.5</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 2: First five frequencies of beam for every damage situation.

<table>
<thead>
<tr>
<th>Situation</th>
<th>F1 (Hz)</th>
<th>F2 (Hz)</th>
<th>F3 (Hz)</th>
<th>F4 (Hz)</th>
<th>F5 (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D0</td>
<td>9.0129 (9.0087)</td>
<td>36.052 (35.986)</td>
<td>81.124 (80.793)</td>
<td>144.25 (143.21)</td>
<td>225.50 (222.97)</td>
</tr>
<tr>
<td>D1</td>
<td>8.8667 (8.8627)</td>
<td>36.043 (35.978)</td>
<td>79.880 (79.554)</td>
<td>144.12 (143.08)</td>
<td>222.32 (219.83)</td>
</tr>
<tr>
<td>D2</td>
<td>8.8108 (8.8068)</td>
<td>35.216 (35.152)</td>
<td>76.705 (76.389)</td>
<td>137.06 (136.06)</td>
<td>212.37 (209.88)</td>
</tr>
<tr>
<td>D3</td>
<td>8.7831 (8.7791)</td>
<td>34.974 (34.910)</td>
<td>76.333 (76.019)</td>
<td>136.94 (135.94)</td>
<td>211.93 (209.54)</td>
</tr>
<tr>
<td>D4</td>
<td>7.3612 (7.3578)</td>
<td>33.467 (33.405)</td>
<td>71.068 (70.777)</td>
<td>126.11 (125.19)</td>
<td>203.62 (201.30)</td>
</tr>
</tbody>
</table>

Figure 3: Geometry and loading of the masonry arch (dimensions in meters).

5.2 Damage control of a masonry arch

In this example, the damage in an ancient masonry (adobe) arch subjected to a horizontal seismic excitation is determined. The geometry of the arch and its rectangular cross-section are shown in figure 3. The material behavior of
masonry is assumed to be the same with that of concrete. The arch is loaded by an equivalent static force applied horizontally, which equals to the product of the equivalent seismic coefficient \( ESC = 8\% \) times the vertical load \( q = 50 \text{ kN/m} \) including self-weight and a possible percentage of live load, as shown in figure 3. The masonry compressive strength is equal to 1200 kPa, while the maximum allowable global damage \( D \) is equal to 40\% in the arch sections, as defined by eqn (4) with \( q = 2 \). Application of the program DAMCON leads to the solution of the problem. Figure 4 shows the damage distribution in the entire structure, while figure 5 the ESC-D diagram, which indicates that the maximum allowable global damage of 40\% is reached for \( ESC \approx 9.7\% \). Thus, the seismic bearing capacity of this arch is adequate to limit damage to \( D \leq 40\% \). It is found from figure 5 that for \( ESC = 8\% \) the global damage is \( D \approx 30\% \).

![Damage Distribution](image1)

**Figure 4:** Damage distribution along the arch.

![ESC-D Diagram](image2)

**Figure 5:** Equivalent seismic coefficient – maximum damage diagram.
6 Conclusions

On the basis of the preceding developments, one can draw the following conclusions:

1) A new method of structural design, the Direct Damage Controlled Method has been developed in this paper. This method quantifies and controls damage in a direct and transparent manner much better than any of the existing methods of structural design.

2) The method has been illustrated here as applied to the case of beam structures made of plain concrete or masonry and subjected to a statically applied axial force-bending moment load combination.

3) The proposed method appropriately combines code based stress-strain relations with a simple expression for damage to determine with the aid of the fiber model and the finite element method axial force and bending moments as functions of damage.

4) Using this method, the designer can either determine the damage level for a known shape section and loading, or dimension a section for a target damage level and known loading, or determine the maximum loading for a known section and a target damage level, with the aid of either a computer program or design charts.

5) The proposed damage controlled method of design has been successfully applied to the cases of a damaged concrete beam undergoing free vibrations and a masonry arch under static seismic forces to illustrate its use and demonstrate its direct applicability to seismic structural design.

References


