Seismic isolation and site effects, an inter-disciplinary investigation at the seismic test site INCERC, Bucharest, Romania

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Abstract

The Collaborative Research Center 461 (CRC 461) Strong Earthquakes: A Challenge for Geosciences and Civil Engineering (Germany) and the Romanian Group for Strong Vrancea Earthquakes (RGVE) work on a multidisciplinary attempt towards the mitigation of the impact of the next strong Vrancea earthquake (Romania). The recently installed Multidisciplinary Seismic Test Site INCERC (MSTS) located in the eastern part of Bucharest serves as focus to study the entire sequence relevant in engineering seismology including structural dynamics, seismology, soil mechanics, and engineering geology.

Within this co-operation, the complete chain for seismic hazard and seismic risk analysis and can be checked and verified.

For this purpose a base isolated test building was constructed, where acceleration will be measured at the base and at the top.

Additionally, free field accelerometers in various depths were installed. Thus, free field accelerations and the response of the test building can be measured simultaneously.

One of mainly objectives, among others, is to investigate the complete dynamic system, incorporating the base isolated test building and the soil, under seismic loading.
Geotechnical parameters which will be available to a depth of 180 m ensure that the soil response incorporating the seismic behaviour of rubber bearings can be computationally studied with Finite Element methods (FEM). These simulations include constitutive laws describing non-linear behaviour of rubber and soil material under cyclic loading.

1 Multidisciplinary seismic test site (MSTS)

The Romanian Vrancea area located in the southern Carpathians is the source region for strong earthquakes. Within the last 20 years Bucharest was affected by 3 events with moment magnitudes larger than 6.9.

With a moment magnitude of 7.5, the 1977 earthquake caused more than 1500 casualties, most of them in Bucharest. The average recurrence rate make another strong event within the next 20 years highly probable.

Under the umbrella of the Collaborative Research Center 461 (CRC 461) Strong earthquakes: A Challenge for Geosciences and Civil Engineering and the Romanian Group for Strong Vrancea Earthquakes (RGVE) German and Romanian scientists from various fields (geology, seismology, civil engineering, operation research) work on a multidisciplinary attempt towards the mitigation of the impact of the next strong Vrancea earthquake.

In this frame a Multidisciplinary Seismic Test Site (MSTS) is installed at the premises of the Building Research Institute INCERC which is located in Eastern Bucharest.

The MSTS allows to study the only site in Romania where a complete set of strong motion records of previous Vrancea earthquakes is available: March 4, 1977, $M_w = 7.5$; PGA=1.9 m/s$^2$, August 30, 1986, $M_w = 7.2$, PGA=0.9 m/s$^2$, and May 30, 1990, $M_w = 6.9$, PGA=0.8 m/s$^2$ [9]. The normalized response spectra of these events are depicted in Figure 1.

Three-component accelerometers are installed in the free field at the surface and will be installed at various depths (30 m, 100 m, 180 m). Geotechnical parameters of the subsurface soil layer are controlled to a depth of 180 m.
The key objectives of the research activities at the MSTS are:

- Non-linear soil response analysis
- Calibration of the geotechnical and seismological parameters for seismic microzonation studies
- Verification of seismic hazard and seismic risk analysis
- Performance of rubber bearings during expected moderate to strong earthquakes

The test building was designed as a rigid concrete block of 100 t supported by high damping rubber bearings on a rigid concrete footing of 72 t. The stiffness of the rubber bearings and the mass of the structure has an eigenperiod of 1.1s to 1.6s. This period range represents maximum spectral accelerations during the 1977 and 1986 events (Figure 1). To record all 6 degrees of freedom (DOF) and the relative movement between footing and top of the concrete block, six 3-component accelerometers are installed as shown in Figure 1.

*Figure 1: Concept (top) and dynamic range (bottom) of test building*
Figure 2: Preparations for the 180 m borehole at INCERC (left). Concept of the MSTS (right)

Figure 2 shows the preparations for the drilling of the 180 m borehole at the MSTS (left) and illustrates the arrangement of test building and boreholes (right) [10].

2 A constitutive law for rubber-like materials

The conventional methods for earthquake-resistant structural design use high strength or high ductility concepts to mitigate damage from seismic impacts. In the first case, corresponding to shear wall structures, generally the design is problematic in that their fundamental frequency of vibration is in the range of frequencies where earthquake energy is strongest, resulting in a very high floor acceleration, which may cause damage to equipment or machinery. The second, the capacity design method, incorporates that a part of the energy transmitted into the structure by an earthquake is dissipated by plastic deformations. The capacity method mostly used for flexible structures like frames, provided that plastic deformations occur in structural elements, which are designed to undergo such large deformations. Therefore, the design of such yielding zones has to be planned carefully.

However, this concept may lead to a very high interstory drift, causing P-Δ effects and damage to non-structural elements. Thus, the costs for retrofitting or strengthening after a strong earthquake can be very high. An alternative approach consists in isolating the structure base from the ground by using rubber bearings [6].
It has to be pointed out that the dynamic system consisting of rubber bearings supporting a structure is predominantly governed by the non-linear behaviour of the bearings. Therefore we focus on the development of a constitutive law describing the behaviour of the rubber bearings under cyclic loading.

In the framework of EC Project 7010 on Optimisation of design and performance of high damping rubber bearings for seismic and vibration isolation a task at the Institute for reinforced concrete structures, University of Karlsruhe, was set-up to develop detailed numerical models for the analysis of such bearings [4].

Following the most important formulations of a viscoelastic constitutive law for finite strains are illustrated, as they have been developed based on an approach by Simo (1987).

The constitutive law proposed makes the following assumptions [7], [13]:

- uncoupled volumetric and deviatoric response for finite strains
- viscoelastic properties represented by separable functions of time and strain
- no volumetric relaxation

The applicability of the first assumption to the high damping rubbers was confirmed in this work by tests examining the effect of hydrostatic pressure on the tensile behaviour.

The Simo law is here modified with a strain hardening function to cover the stiffening of rubber at higher strains, as observed experimentally, and a strain softening function to account for strain history effects.

Thus, the fully three-dimensional constitutive law takes the form:

$$\sigma(\tau) = K \ln(J) I + \int_{\tau=0}^{1} \Phi(t-\tau) \frac{d}{dt} \left[ g(\varphi) \cdot H \left( \| \text{dev} \mathbf{C} \| \right) \text{dev} \mathbf{C} \right] d\tau$$  (1)

Here, the deformation gradient $\mathbf{F}$ is decomposed into a pure volumetric $\frac{1}{J^3}$ and isochoric deformation $\mathbf{\Phi}$, respectively [13].

Furthermore, the right Cauchy-Green-Tensor $\mathbf{C}$ for the isochoric deformation is given by:

$$\mathbf{\bar{C}} = J^{-\frac{2}{3}} \mathbf{C} = \mathbf{F}^T \mathbf{F}$$
The first term in the constitutive law covers the volumetric stress, while the deviatoric part is described by a convolution integral containing:

- a relaxation function

$$\Phi(t) = G_\infty + (G_0 - G_\infty) \cdot e^{-t/v}$$

with a long and a short shear time modulus $G_\infty$ and $G_0$ and a characteristic relaxation time $v$.

- a strain softening damage function

$$g(\varphi) = \beta + (1 - \beta) \cdot \frac{1 - e^{\varphi/\alpha}}{\varphi / \alpha}$$

with the material parameters $\alpha$ and $\beta$ and the norm of the Cauchy-Green-tensor $\varphi(\tau) = \max_{t \leq T} \| \text{dev}C \|$.

- a strain hardening function

$$H(\| \text{dev}C \|) = \delta \cdot \| \text{dev}C \|^2 + 1$$

with a scalar parameter $\delta$.

The material parameters: $K$, $G_0$, $G_\infty$, $\nu$, $\alpha$, $\beta$ and $\delta$ were determined for loading histories KOD ($f = 0.1$ Hz) and KOS (strain rate $7\%$/$s$). Both loading histories are up to a rubber shear deformation of $200\%$ ($\tan\gamma = 2$); where $\tan\gamma = S/T$. Here, $S$ means the total rubber thickness and $T$ the horizontal displacement.

The values of the material parameters appropriate to each test condition are listed in Table 1 [4].

Table 1. Material parameters for viscoelastic model

<table>
<thead>
<tr>
<th>Loading History</th>
<th>$K$ [MPa]</th>
<th>$G_0$ [MPa]</th>
<th>$G_\infty$ [MPa]</th>
<th>$\nu$ [s]</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>KOS</td>
<td>1000</td>
<td>3.5</td>
<td>1.3</td>
<td>5.0</td>
<td>0.1</td>
<td>0.2</td>
<td>0.02</td>
</tr>
<tr>
<td>KOD</td>
<td>1000</td>
<td>3.5</td>
<td>1.3</td>
<td>0.5</td>
<td>0.1</td>
<td>0.2</td>
<td>0.02</td>
</tr>
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</table>

3 FE-Simulation of rubber bearings compared with test

An algorithm proposed by Kim et al (1991) was used to evaluate the convolution integral [7]. Minor changes to stiffness parameters in Table 1 were made to take into account the difference in strain rate experienced by the rubber in the bearing test.

An example of a test result and the FE-analysis for a bolted bearing, $D = 250$mm and a total rubber thickness $T = 60$ mm, are shown in Figure 3 [4].
There is a good correspondence between the test results and the FE-analysis. The predicted loops are seen to reproduce the observed behaviour reasonably well. The FE-model of the rubber bearing is generated with 20-node quadratic continuum elements for the rubber layers and the two anchor plates [1].

Furthermore the response of time histories of two earthquakes with a different dominant frequency content, Friaul 1976 (3-5 Hz) and Vrancea 1986 (0.7-1 Hz) shown in Figure 4, has been calculated to test the calibrated KOD parameters. The loading mass amounts to 33 tons, which represents a third of the total mass of the test building for each bearing. Figure 4 illustrates that the low-frequency Vrancea time history is amplified, which corresponds to the design of the test building, and the "high-frequency" Friaul time history is attenuated.

For the Friaul earthquake it is shown that the seismic loading imparted to a structure can be efficiently reduced. The need to take into account the soil conditions before using rubber bearings as base isolation is obvious. In particular regions of soft soil which mostly reveal low frequency response to an earthquake are critical [6].

Figure 3: Bearing test, D = 250mm, T = 60mm, Loading history KOD; Rubber shear rate 1.8%/s (left) and FE-Simulation (right)

Figure 4: Simulated response of Friaul 1976 and Vrancea 1986 earthquakes
Figure 5 shows the spectral amplitudes and spectral ratio for the simulated response of the used bearing for the Vrancea 1986 earthquake.

Figure 5: Spectral amplitudes (left) and spectral ratio (right) calculated for the Vrancea 1986 earthquake

The peak in the calculated spectral ratio for the Vrancea 1986 event in Figure 5 shows, that for moderate earthquakes the predominant frequency is about 1 Hz, while the calculated maximum shear deformation goes up to tan $\gamma = 0.4$. For stronger earthquakes the predominant period is expected in the range of 0.7 Hz and tan $\gamma = 1.5$, due to loss of stiffness of the rubber bearings for larger shear deformations.

4 Non-linear site response

For an earthquake resistant design, the engineer has to be aware of the seismic risk at the site. This means, that dynamic soil response analysis is necessary for getting the design seismic input motion at the free surface or at the base of the structure.

In general, site effects are affected by the source mechanism, the amount of released energy in terms of magnitude, the duration of shaking, the frequency content of the ground motion, topography effects, lateral discontinuities and the material behaviour of the surficial soil layers [8].

An example of site effects in Bucharest, Romania is presented in Figure 1. It shows the normalized acceleration response spectra of the three strong earthquakes from 1977, 1986 and 1990 recorded at the INCERC location in Bucharest (Lungu et al., 1998). The long predominant period was experienced during the severe 1977 and the moderate 1986 earthquake, while for the smaller 1990 event this effect was not observed. Taking into account that the focal depth of Vrancea earthquakes lies in the range of 100 km to 150 km and the epicentral...
distance to Bucharest is about 150 km, it can be assumed that the influence of the fault rupture mechanism is very little and can therefore be neglected. The seismic hazard at the site is mainly affected by the focal depth and the magnitude [9]. Thus, the long period effect at this site can only be explained by the non-linear behaviour of the soil.

The modulus reduction behaviour is mainly affected by the strain level, the plasticity index \( I_p \) and the mean effective pressure \( p \) [8]. The effects of effective confining pressure and plasticity index were combined by Ishibashi and Zang (1993). The \( I_p = 0 \) modulus reduction curve from Figure 6 is very similar to the average modulus reduction curve that is commonly used for sands. This similarity suggests that the modulus reduction curve of Figure 6 may be applicable to both fine and coarse-grained soils [5]. But this suggestion has to be confirmed in the future by additional soil testing.

Figure 6: Secant modulus reduction curves with the appropriate damping as a function of shear strain for different effective confining pressures (right) and different plasticity indices for \( \sigma_0 = 1.0 \text{ KN/m}^2 \) after [5]

Another important parameter for a realistic evaluation of the soil response is the initial shear modulus \( G_0 \). In general the initial shear modulus can be derived from laboratory testing of the soil samples or from in situ VSP-Measurements. Based on their own test data and including experimental results obtained by other authors, solely resonant column tests were taken into account, a relation for the initial shear modulus is derived by Santos and Correia (2000). These empirical equations (5) proposed for an upper and lower bounded initial shear modulus are derived from the initial void ratio \( e \) and the effective confining pressure \( p \). The relations for the upper and lower bounds are given by [12]:

\[
G_{0}^{\text{upper}} = 8000 \cdot e^{-1.1} \cdot p^{0.5} \quad [\text{KN/m}^2]
\]

\[
G_{0}^{\text{lower}} = 4000 \cdot e^{-1.3} \cdot p^{0.5} \quad [\text{KN/m}^2]
\]

The proposed curves seem to be a consistent tool that can be applied for sands and clays as a guide for practical purposes.
5 A quasi-linear model for non-linear soil behaviour

One of the most widely used approaches to model sediment non-linearity is the 1-D equivalent-linear model, based on a secant stiffness formulation. Here, the sediment response is treated as linear-viscoelastic. However, the shear-wave velocities and damping levels are changed from their original weak-motion values to be compatible with test data, suggested for the particular strain level induced by the input motion [11].

Because the strain is not known a priori, the response is obtained in an iterative process. That is, shear-wave velocities and damping factors are successively adjusted to be compatible with the level of strain implied from the previous calculation until further iterations do not significantly change the result. This equivalent linear modelling produces a systematic shift in resonant peaks toward lower frequencies as the level of strain increases. It also predicts a too strong reduction in amplification factors at higher frequencies.

However, truly non-linear dynamic response analysis, in which the change of mechanical property is revised in each time increment, based on the tangent stiffness formulation, has more potential to simulate the realistic dynamic response of the surface ground during an earthquake than equivalent-linear methods.

This was the starting point to develop a 3-D non-linear constitutive relation, where the actual stress-strain loops are represented numerically. This relation implies an initial shear modulus (5), as a function of the confining pressure and the initial void ratio. For the stiffness degradation in shear at strain levels greater than $10^{-6}$, a simple damage function for the loading and unloading case is proposed.

The developed algorithm, including simple hysteretic rules for the loading/unloading case, was implemented in a general dynamic FE-Code [1].

Following, the basic equations of the quasi-linear model are illustrated.

The incremental stress response is derived from the well-known linear stress-strain relation for a Cauchy elastic material. The response is decoupled in a purely volumetric and deviatoric part:

$$\Delta \sigma_{ij} = \Delta p + \Delta s_{ij} = K_t \Delta \epsilon_{kk} \delta_{ij} + 2 G_t \left( \Delta \epsilon_{ij} - \frac{1}{3} \Delta \epsilon_{kk} \delta_{ij} \right)$$  \hspace{1cm} (6)

Here, $p$ is the volumetric and $s$ is the deviatoric stress tensor. $G_t$ and $K_t$ are the tangent shear and bulk modulus.
The separation of the constitutive formulation (6) into a volumetric and deviatoric part is very convenient but automatically precludes dilatancy effects, as it is the case for dense sands and overconsolidated clays under pure shear deformation. From the physically point of view, the proposed model can only give qualitative results for such soil types [2], [3].

It has to be pointed out, that the proposed model is applicable only for a limited class of materials, loadings, and stress paths, it may not be used as general model. Due to the circumstance, that it is rather difficult to get all parameters for more sophisticated constitutive relations in practice, this approach seems to be justified.

The general formulations of the tangent shear modulus $G_t$ with the appropriate damage functions for the loading and unloading case are given below.

For the loading case, $G_t$ is given by:

$$G_t = \frac{G_0}{1 + (g_1(J_{2E}, p)) \cdot \sqrt{J_{2D}}}$$

with the damage function $g_1(J_{2E}, p)$:

$$g_1(J_{2E}, p) = a_0 + a_1 \ln \sqrt{J_{2E}} + a_2 \cdot p$$

For the unloading case, $G_t$ is described as:

$$G_t = \frac{G_0}{1 + g_2(J_{2E}, \zeta)}$$

with the damage function $g_2(J_{2E}, p)$:

$$g_2(J_{2E}, p) = b_0 + b_1 \ln \sqrt{J_{2E}} + b_2 \cdot p$$

Here, $\zeta = \sqrt{J_{2D_{rev}}} - \sqrt{J_{2D_{max}}}$ is the deviatoric stress difference, $\sqrt{J_{2D_{max}}}$ is the maximum deviatoric stress at reversal for the unloading case, $J_{2D}$ and $J_{2E}$ are the stress and strain invariants, $p$ is the effective confining pressure and $a_0$, $a_1$, $a_2$, $b_0$, $b_1$, $b_2$ are material parameters for the appropriate damage functions.

As it has already been mentioned before, the bulk modulus $K$ is assumed to be constant for the results presented.
In Figure 7, results for a static numerical simulation for different maximum strain levels are shown. For the numerical simulation an increasingly sinusoidal loading history was chosen. All numerical simulations were performed with a three-dimensional 8-node linear element.

The parameters in the damage functions (8) and (10) were adjusted in order to fit the modulus degradation and the damping values for the depicted curves in Figure 7. As it can be seen, the model is able to fit the experimental results quite well.

Figure 7: Simulated hysteretic stress-strain-curves for max. strain levels at 1.0E-04 (top-left), 1.0E-03 (top-right) and 1.0E-02 (bottom-left). Comparison between experimental data and numerical simulations for the secant shear modulus with the appropriate damping (right-bottom)

Starting from the results obtained from the static simulation, a dynamic 2-D analysis with the same parameters for the initial shear modulus and the damage functions was performed. The finite element model is depicted in Figure 8. For the dynamic simulation, 8-node linear elements were chosen and plane strain boundary conditions perpendicular to the loading direction were assumed. For quite boundaries in the loading direction, infinite elements were used. The Vrancea 1990, INCERC-NS-Component was used as input motion. This motion, which is presented in Figure 10 (left), was scaled by a factor of 2. The normalized response spectra for the non-scaled time history is shown in Figure 1.

In Figure 8 (left), the appropriate modulus degradation and damping curves for the two soil layers are shown.
The first two eigenperiods of the soil profile presented in Figure 8 are $T_1 = 4 \frac{H}{\sqrt{\nu}} = 0.31\,s$ and $T_2 = 4 \frac{H}{3\sqrt{\nu}} = 0.1\,s$.

The initial material properties for the two soil layers are presented in Table 2. As aforementioned, the volumetric part of the constitutive relation (6) is assumed to be constant throughout the analysis.

Table 2. Initial material properties for the dynamic analysis

<table>
<thead>
<tr>
<th>Soil Layer</th>
<th>Void ratio $e$</th>
<th>$\rho$ [MPa]</th>
<th>$G_0$ [MPa]</th>
<th>$\rho$ [kg/cm$^3$]</th>
<th>$\nu_s$ [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.8</td>
<td>0.025</td>
<td>51</td>
<td>2.0</td>
<td>160</td>
</tr>
<tr>
<td>2</td>
<td>0.8</td>
<td>0.1</td>
<td>101</td>
<td>2.0</td>
<td>225</td>
</tr>
</tbody>
</table>

The results for the simulated stress-strain curves at various depths are depicted in Figure 9. In Figure 8, the appropriate elements are highlighted in white.
Figure 9: Simulated stress-strain curves for layer 2 at -14.75m (right bottom) and at -7.75m (left bottom) and for layer 1 at -7.25m (left top) and at -0.25m (right top).

As it can be seen from Figure 9 (left top), the highest modulus degradation and a permanent displacement occurs at the bottom of layer 1, while the material behaviour of the element near the surface is almost linear. The simulated acceleration at the surface is shown in Figure 10 (right).

Figure 10: Input motion (left) and simulated time history at the surface (right).

To compare the results with a 1-D equivalent-linear model, a computation was performed with the computer program SHAKE [11]. The effective shear strain for the equivalent-linear computation was taken as 65% from the maximum shear strain. Furthermore, the same input motion, modulus degradation and damping curves were used, as depicted in Figure 10 (left) and Figure 8 (left), respectively. The 5%-damped response spectra of the computed motion at the surface from both analysis are shown in Figure 11.
Figure 11: Comparison of the 5%-damped response spectra obtained from the FE-Simulation and with SHAKE [11].

Figure 11 compares the 5%-damped response spectra of both analysis. Here, the first two eigenperiods are presented as dotted lines. The results from both analysis show a good agreement in the high period range at maximum spectral accelerations. In contrast, SHAKE underestimate the spectral accelerations in the low period range dramatically. The numerical FE-Simulation clearly shows the shortage of the equivalent-linear method as before mentioned. It has to be pointed out, that the soil profile of the considered example is very simple. For more complex soil profiles, the difference might be more significantly.

6 Conclusion

The numerical results of the behaviour of rubber bearings under cyclic loading up to serviceability of \( \tan \gamma = 2 \) presented herein have shown that non-linear effects as well as the hysteretic behaviour can be simulated in good accordance to the test results with the proposed constitutive law.

Additionally, a new quasi-linear model for non-linear soil response analysis was presented. It has shown, that the proposed three-dimensional model is able to simulate the non-linearity in cyclic shear in the time domain quite well. The model is able to closely reproduce realistic shear modulus degradation and material damping experimentally obtained for clays and sands. Furthermore, the model is able to estimate permanent strains in non-linear soil response analysis. The future work will be focused on the dynamic response analysis of 2-D profiles with appropriate material properties of the soil layers, which can contribute a very useful input to further microzonation studies.
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