Estimation method of requisite strength to control deformation of RC structure under earthquake motion and investigation of the accuracy

T. Mukai, H. Kinugasa, S. Nomura
Faculty of Science and Engineering Department of Architecture, Science University of Tokyo, Japan

Abstract

Recently, in earthquake resistant design, there is a tendency to evaluate a building by the performance. One method of the evaluation is to estimate maximum response deformation for a building modeled by single-degree-of-freedom (SDOF) system. In our previous studies [1], it was confirmed that EIV, which is defined as the intensity of energy input in stationary response, is almost equal to \( \frac{E_{\text{max}}}{T_e} \). \( E_{\text{max}}/T_e \) is the intensity of energy input in earthquake response. In this paper, then, energy absorption model (ES) of building is determined. The Requisite Strength (Py) to control deformation is calculated by solving the relationship, \( ES = ED \) (Total Input Energy). In order to calculate the Py to control maximum deformation, it is necessary to estimate the non-linear cyclic number (ND), ED and Maximum Deformation Ratio coefficient (MDR). ED and ND are estimated from elastic response analysis in 5% damping corresponding to changed period by structure's plasticity and MDR is estimated by maximum ductility factor and energy absorption capacity coefficient (\( \zeta \)). Inelastic response analysis for SDOF is carried out to investigate the accuracy on estimated Py by the proposed model. As a conclusion of these investigations, it is shown that the assumption on modeling of energy absorption is generally valid. And the accuracy of calculating Py (that is accuracy of estimated ED, ND and MDR) is almost high. For further improvement of the accuracy, it is important to estimate accurately the MDR, particularly in the case of JMAKOBE which caused strong one side plastic deformation.
1 Introduction

It is important to estimate responses in Performance Based Seismic Design. There is a maximum deformation as the index represents damages. In many estimating methods, 'Equivalent Linear Method' is widely used. But this method is not able to express the effect of cyclic response and ratio of maximum deformation in earthquake. The purpose of this paper is to propose the estimating method of maximum deformation under earthquake motion for SDOF, considering with cyclic response and ratio of maximum deformation. The accuracy of proposed method is also investigated by response analysis.

2 Concept of Energy Input Velocity (EIV)

Building response will be different by a type of input earthquake motions. For instance, it could be understood when we compare the responses to strong earthquakes in a long-distance seismic center with those to medium earthquakes in a near-distance seismic center. Considering with it, we define EIV as the index of intensity of energy input in Fig.1. The left part of the Fig. shows time-history response displacement and input energy by the earthquake. The response is replaced to a stationary response with constant displacement amplitude as shown in right part. In actual response, $\Delta E_{\text{max}} / \Delta T$ represents the intensity of energy input and the value of $\Delta E$ changes in response every moment. $\Delta E_{\text{max}}$ is a maximum value of $\Delta E$ for the duration. In inelastic response, $\Delta T$ is applied equivalent period $T_e$ with equivalent stiffness $K_e$, not with yielding stiffness $K_y$ (see in Fig.2). In replaced response with constant displacement amplitude, EIV represents intensity of energy input and the value of $\Delta E$.

Figure 1: Definition of EIV
Figure 2: Type of Stiffness (Period)

Figure 3: Modeling of energy absorption

is unique for input motions. EIV is shown in eqn (1).

\[
\text{EIV} = \frac{\text{ED}}{\text{ND} \times \Delta T}
\]
where: \(\text{ED} = \) Total Input Energy, \(\text{ND} = \) Cyclic Number in earthquake
When it is assumed that \(\Delta E_{\text{max}} / \Delta T\) would be equal to EIV, ND is given by:

\[
\text{ND} = \frac{\text{ED}}{\Delta E_{\text{max}}}
\]
Thus, ND is an index that represents the characteristics of input motion.

3 Calculating method of requisite strength

3.1 Modeling of energy absorption and calculating process of requisite strength

Energy absorption for elasto-plastic structure is modeled as shown in Fig.3 by assuming that time-history response can be replaced by stationary response with constant displacement amplitude as mentioned above. ES (Total Energy
Absorption) can be given from sum of four absorption energies (Ey, Eds, Ec, Eh) as shown in eqn (3). Ey is elastic absorption energy, Eds is plastic one in first cycle and Ec is plastic one except Eds, Eh is viscous one. In eqn (3), $\xi$ is energy absorption capacity coefficient (see in Fig.4). In case of $\xi=0.5$, the energy absorption capacity of structure is high, where $h$ is damping factor.

$$ES = Ey + Eds + Ec + Eh$$

where:

- $Ey = 0.5Py\delta y$
- $Eds = 2Py(\delta D - \delta y)$
- $Ec = 4Py\xi(ND - 1)(\delta D - \delta y)$
- $Eh = 2\pi hPy\delta DND$

When $ED$ (Total Input Energy) is equal to $ES$ as eqn (4), $Py$ becomes the requisite strength.

$$ED = ES$$

Eqn (5) is given by solving eqn (4) about $Py$.

$$Py = \sqrt{\frac{ED \cdot \text{Key}}{0.5 + 2(\muave - 1) + 4\xi(ND - 1)(\muave - 1) + 2\pi h\muave ND}}$$

where: $\muave$ = average ductility factor (average of positive and negative maximum ductility factor)

Eqn (5) means that $Py$ to control the average ductility factor (that is amplitude) under an input motion is given. However in design, it is desirable to calculate $Py$ to control the maximum ductility factor ($\mu_{\text{max}}$). Considering with it, MDR is defined by:

$$MDR = \frac{\mu_{\text{max}}}{\muave}$$

It is necessary to estimate $ED$, $ND$ and $MDR$ for calculating $Py$ to control $\mu_{\text{max}}$.

3.2 The estimating method of $ED$ (Total Input Energy) and $ND$ (Cyclic Number in earthquake), $MDR$ (Maximum Deformation Ratio coefficient)

$ED$ and $ND$ are estimated by using spectra corresponding to period characteristics.

$ED$ can be estimated with applying Akiyama's method [2]. In Fig.5, VE which is converted $ED$ to velocity unit, in relation to period is shown. Period is used yielding period $T_{ey}$ in elastic response and equivalent period $T_e$ in inelastic response. A solid line is elastic value when damping factor is 5%. A
circle mark is inelastic value in each damping factor and each earthquake (EQi is simulated motions). The marks on solid line are not influenced to damping factor. Thus, ED can be estimated from elastic spectrum.

In Fig.6, the ordinates is ND which is given from eqn (2). The remarks are same to those in Fig.5. From Fig.6, value of ND is large in short period domain and small in long period one. ND means not only the characteristics of earthquake but also one of structure. In the other word, ND generally represents the cyclic number of structure and is also significant index. Thus, ND can be estimated from using elastic spectrum corresponding to changed period by
structure's plasticity.

Then, it is important to estimate $\mu_{ave}$ from $\mu_{max}$ with applying MDR. Ratio of the maximum deformation is caused by various factors and it is judged MDR has relation with $\mu_{max}$ and $\xi$. Residual ductility factor $\mu^*$ is represented as function of $\mu_{max}$ and $\xi$ (see Fig. 7 (a)) by:

$$\mu^* = \frac{(\mu_{max} - 1)}{\xi}$$

(7)

In Fig. 7 (b), it is shown a relation between MDR and $\mu^*$ in each $Tey$ (yielding period), each $\xi$, each earthquake. In case of JMAKOBE, MDR has more large value than the others. As a general tendency, in case of $\xi = 0.5$, MDR increases to the fixed value of $\mu^*$ and keeps at a constant one in more than the value. In case of $\xi = 0.125$, MDR is small value in wide range. It is difficult to estimate MDR separately. Then it is regarded tendency of $\xi = 0.5$ as important, MDR is given by the following eqn (8) and represented by the solid line in Fig. 7 (b).

$$\text{MDR} = 1 + 0.25\mu^*$$

(8)

4 Accuracy verification of calculating method

The procedure to calculate requisite strength ($P_{y}$) is shown in Fig. 8. Firstly, $Tey$ and $\xi$ and $h$ are determined as initial structural characteristics. After it is set up allowable $\mu_{max}$, $\mu_{ave}$ is given by using eqn (8) and eqn (6). Secondary, it is necessary to calculate $Tey$ so that ED and ND in inelastic response may be given. In the calculation, eqn (9) can be applied when skeleton curve is bi-linear and secondary stiffness is almost 0.

$$Tey = \sqrt{\mu_{ave}}$$

(9)

After the $Tey$ is replaced by $Tey$ from eqn (9), ED and ND are estimated by elastic spectra in 5% damping. At the same time, energy absorption of structure is determined. Finally, $P_{y}$ is given by solving as $ED$ is equal to $ES$.

In this chapter, the two accuracy verifications on estimated $P_{y}$ are investigated. The one is validity of modeling on energy absorption (see in Fig. 3), the other is accuracy verification of estimating method (see in Fig. 8). The response analysis for SDOF systems is carried out each of verification and analysis parameters are listed in Table-1. Generally, it is used tri-linear model as Takeda hysteresis model for RC structure. But it is thought the difference of response displacement is not large between bi-linear and tri-linear model. Thus degrading bilinear model is used in Fig. 9. Damping is assumed to be proportional to the instantaneous stiffness.

4.1 The validity of modeling on energy absorption

To verify the validity of modeling energy absorption, MDR and ED and ND are given from response analysis. The Fig. 10 shows verification result for validity on modeling of energy absorption and $P_{y}/mg$ (m is mass and g is
Figure 7 (a): \( \mu^* \) (residual ductility factor) in relation to \( \mu_{\text{max}} \)

![Graph showing \( \mu^* \) in relation to \( \mu_{\text{max}} \)](image)

Figure 7 (b): Relation between MDR and \( \mu^* \)

![Graph showing relation between MDR and \( \mu^* \)](image)

Table 1: Analytical parameters for SDOF

<table>
<thead>
<tr>
<th>Input Motions</th>
<th>JMA KOBE 0.5%</th>
<th>EL Centro 0.5%</th>
<th>EQ1 0.5%</th>
<th>EQ2 0.5%</th>
<th>EQ3 0.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tey (yielding period) (s)</td>
<td>0.5, 1.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \xi ) (energy absorption capacity)</td>
<td>0.5, 0.125</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Py (yielding strength)</td>
<td></td>
<td></td>
<td>Py is set up so that ( \mu_{\text{max}} ) is from 1 to 6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( h ) (damping factor) (%)</td>
<td>0, 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 8: Estimation procedure of requisite strength Py

Figure 9: Hysteresis model
gravity acceleration) in relation to $\mu_{max}$ in case of JMAKOB in each damping (0 and 5%). The estimated value (white marks) well represents the set value (black marks) for response analysis and this is similar in case of any other input motions. Consequently, it is said the proposed assumption on modeling of energy absorption are valid.

4.2 The accuracy verification of estimating method

According to Fig. 8, $P_y$ is calculated by eqn (5). In the calculation, ED and ND are estimated by elastic spectra and MDR is calculated by eqn (8) and (6). The Fig. 11 shows verification result for accuracy on estimating method of ED and ND and MDR. The axes and remarks are same to those in Fig. 10, where the left part is the case of JMAKOB and the right part is that of EL Centro in 5% damping. The figures show high estimation accuracy except a part of JMAKOB ($T_{ey} = 0.5\, s$, $\xi = 0.5, 0.125$). Therefore, $P_y$ to control maximum deformation under an earthquake motion can be given in design.

5 Conclusions

In this paper, calculating method of $P_y$ (requisite strength) to control a maximum ductility factor for SDOF is proposed. This method is based on energy-balance and is considered the intensity of energy input. In this method, it is necessary for calculating $P_y$ to estimate ED (Total Input Energy), ND (Cyclic Number) and MDR (Maximum Deformation Ratio) in inelastic response. After the estimation is conducted, the accuracy of this method is verified. The following conclusions are obtained.

(1) ED and ND can be estimated by using elastic spectra corresponding to changed period by structure's plasticity. And MDR is estimated from maximum ductility factor and $\xi$ (energy absorption capacity coefficient).

(2) It is confirmed the assumption on modeling of energy absorption is valid (Fig. 10).

(3) It is shown that this method has generally enough accuracy except a part of JMAKOB ($T_{ey} = 0.5\, s$, $\xi = 0.5, 0.125$) (Fig. 11). In other words, it means that the accuracy of estimated ED, ND and MDR almost is high. For further improvement of the accuracy, estimation of MDR is important, particularly in case of pulse wave as JMAKOB.

References

Figure 10: Validity on modeling of energy absorption

Figure 11: Accuracy verification on estimated Py